Radiation Absorption and Chemical Reaction Effects on MHD Flow through Porous Medium Past an Exponentially Accelerated Inclined Plate

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Abstract: The present paper focused on the effect of radiation absorption and chemical reaction on MHD unsteady free convection flow past an exponentially accelerated inclined porous plate has been analyzed. By mean of perturbation technique the boundary layer equations are transformed in to ordinary differential equations. The velocity profiles, temperature profiles, concentration profiles are showed through graphically and evaluated numerically for skin friction for various parameter involving in the governing equations.

Keywords: MHD, Chemical reaction, Radiation absorption, Porous medium, Inclined plate

I. INTRODUCTION

The study of magnetohydrodynamic (MHD) viscous flows is important in industrial, technological and geothermal applications such as high-temperature plasmas, cooling of nuclear reactors, liquid metal fluids, MHD accelerators, Magnetohydrodynamic generators and accelerators. As a result, a significant amount of interest has been carried out to study the effects of electrically conducting fluids in the presence of a magnetic field on the flow and heat transfer aspects in various geometries. Chenna Kesavaiah and Sudhakaraiah [1] Effects of Heat and mass flux to MHD flow in vertical surface with radiation absorption, Ch Kesavaiah et. al. [2] Effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction, Chenna Kesavaiah and Satyanarayana [3] MHD and Diffusion Thermo effects on flow accelerated vertical plate with chemical reaction.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Possible heat generation effects may alter the temperature distribution and consequently, the particle deposition rate in nuclear reactors, electric chips and semiconductor wafers. Seddeek [4] studied the effects of chemical reaction, thermophoresis and variable viscosity on steady hydromagnetic flow with heat and mass transfer over a flat plate in the presence of heat generation/absorption. Patil and Kulkarni [5] studied the effects of chemical reaction on free convective flow of a polar fluid through porous medium in the presence of internal heat generation. Double-Diffusive Convection-Radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effects was studied by Mohamed [6]. Ch Kesavaiah et.al. [10] investigated an attempt has been made to study the effect of the steady two - dimensional free convection heat and mass transfer flow electrically conducting and chemically reacting fluid through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in the presence of a uniform magnetic field is presented. In the present analysis we have considered the heat generation (absorption) of the type $Q = Q'(T' - T'_w)$ Where $\frac{Q'}{\rho C_p}$ is the volumetric rate of heat generation (absorption).

The present study focuses on a few physical situations, particularly in seepage flow, in which the entire flow domain may not be exactly vertical/horizontal. Further, electrically conducting fluids are of usual occurrence due to contamination and industrial waste. The oscillatory motion is usual industries when the oscillating surfaces are embedded in a porous medium. Here, we have accounted for the effect of permeability of the medium through a linear Darcy model. The consideration of first order chemical reaction in the solutal equation contributes to enhance the number of parameters in flow model without compromising the possible occurrence of in physical situation.
II. FORMULATION OF THE PROBLEM

We consider an unsteady uniform MHD free convective flow of a viscous, incompressible and radiation absorption fluid past an exponentially accelerated inclined infinite plate with variable temperature embedded in a saturated porous medium. The $x$ – axis is taken along the plate and $y$ – axis is normal to the plate. Magnetic field intensity $B_0$ is applied in the direction perpendicular to the plate. The plate is inclined to vertical direction by an angle $\gamma$. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is very small. Initially, it is assumed that the plate and the surrounding fluid are at the same temperature $T_\infty$ and the concentration $C_\infty$.

\[ \frac{\partial u'}{\partial t'} = g \beta \left( T' - T_\infty \right) \cos \gamma + g \beta' \left( C' - C_\infty \right) \cos \gamma + \frac{\partial^2 u'}{\partial y'^2} = \frac{\sigma B_0^2}{\rho} u' - \frac{v}{K_p} u' \]  

(1)

\[ \rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + Q_l \left( C' - C_\infty \right) \]  

(2)

\[ \frac{\partial C'}{\partial t'} = \kappa \frac{\partial^2 C'}{\partial y'^2} = K_r \left( C' - C_\infty \right) \]  

(3)

The initial and boundary conditions are

\[ u' = 0, T' = T_\infty, C' = C_\infty \quad \text{for all} \quad y', t' \leq 0 \]

\[ u' = u_0 \exp \left( \alpha t' \right), T' = T_\infty + \frac{T'_w - T_\infty}{\nu} u_0^2 t', C' = C_\infty + \frac{C'_w - C_\infty}{\nu} u_0^2 t' \quad \text{at} \quad y' = 0, \alpha t' > 0 \]

\[ u' = 0, T' \rightarrow T'_w, C' \rightarrow C_\infty \quad \text{as} \quad y' \rightarrow \infty \]  

(4)
The boundary conditions for the temperature at the plate impose a linearity relation between temperature and time with a residual temperature \( T_\infty \) and having a constant slope \( \frac{u_0^2}{\nu} \) which depends upon square of the characteristic velocity and material property.

Similar explanation holds for concentration at the plate. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium.

On introducing the following non-dimensional quantities

\[
y = \frac{y}{y_0}, \quad U = \frac{u}{u_0}, \quad t = \frac{t}{t_0}, \quad a = \frac{a}{a_0}, \quad T = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{C - C_\infty}{C_w - C_\infty}
\]

\[
Gr = \frac{\nu g (T_w - T_\infty)}{u_0^3}, \quad Pr = \frac{\mu C_p}{\kappa}, \quad Gc = \frac{\nu \beta^* g (C_w - C_\infty)}{u_0^3}, \quad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}
\]

\[
K_p = \frac{u_0^2 K_p}{\nu^2}, \quad Kr = \frac{K r \nu}{u_0^2}, \quad Sc = \frac{\nu}{D}, \quad Q_i = \frac{Q_i \nu^2 (C_w - C_\infty)}{\rho C_p u_0^2 (T_w - T_\infty)}
\]

In equations (1), (2) and (3)

\[
\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial y^2} - M U - \frac{1}{K_p} U + Gr T \cos \gamma + Gc \cos \gamma
\]

\[
\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} + Q_i C
\]

\[
\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} - Kr C
\]

The initial and boundary conditions in dimensionless form are

\[
U = 0, \theta = 0, C = 0 \quad t \leq 0 \quad \text{for all } y
\]

\[
U = \exp(at), \theta = t, C = t, \quad \text{at } y = 0
\]

\[
U = 0, \theta \rightarrow 0, C \rightarrow 0 \quad \text{as } y \rightarrow \infty \quad t > 0
\]

Where \( u' \) is the velocity of the fluid along the plate in the \( x' \)-direction, \( t' \) is the time, \( g \) is the acceleration due to gravity, \( \beta \) is the coefficient of volume expansion, \( \beta^* \) is the coefficient of thermal expansion with concentration, \( T_n' \) is the temperature of the fluid near the plate, \( T_w' \) is the temperature of the fluid far away from the plate, \( T_\infty' \) is the temperature of the fluid, \( C' \) is the species concentration in the fluid near the plate, \( C_\infty' \) is the species concentration in the fluid far away from the plate, \( \nu \) is the kinematic viscosity, \( K_0 \) is the coefficient of kinematic visco-elastic parameter, \( \sigma \) is the electrical conductivity of the fluid, \( B_0 \) is the strength of applied magnetic field, \( \rho \) is the density of the fluid, \( C_p \) is the specific heat at constant pressure, \( K \) is the thermal conductivity of the fluid, \( \mu \) is the viscosity of the fluid, \( D \) is the molecular diffusivity, \( u_0 \) is the velocity of the plate, \( Gr \) is the thermal Grashof number, \( Gc \) is modified Grashof number, \( Pr \) is Prandtl number, \( M \) is the magnetic field, \( Sc \) is Schmidt number, \( Kr \) is Chemical reaction, \( K \) is Porous permeability respectively.
III. SOLUTION OF THE PROBLEM

Equation (6) – (8) are coupled, non-linear partial differential equations and these cannot be solved in closed form using the initial and boundary conditions (9). However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the fluid in the neighbourhood of the plate as

\[ U = U_0(y) + e^{\alpha_1 y} U_1(y) \]
\[ T = T_0(y) + e^{\alpha_1 y} T_1(y) \]
\[ C = C_0(y) + e^{\alpha_1 y} C_1(y) \]  

Substituting (10) in Equation (6) – (8) and equating the harmonic and non-harmonic terms, we obtain

\[ U_0'' - \beta_1 U_0 = -Gr T_0 \cos \gamma - Gc C_0 \cos \gamma \]  
\[ U_1'' - \beta_2 U_1 = -Gr T_1 \cos \gamma - Gc C_1 \cos \gamma \]  
\[ T_0'' = -Q_l Pr C_0 \]  
\[ T_1'' = -Q_l Pr C_1 \]  
\[ C_0'' - Sc Kr C_0 = 0 \]  
\[ C_1'' - \beta_1 C_1 = 0 \]  

The corresponding boundary conditions can be written as

\[ U_0 = 0, U_1 = 1, \theta_0 = 1, \theta_1 = 0, C_0 = 1, C_1 = 0 \quad \text{at} \ y = 0 \]  
\[ U_0 = 0, U_1 = 0, \theta_0 = 0, \theta_1 = 0, C_0 = 0, C_1 = 0 \quad \text{as} \ y \rightarrow \infty \]  

Where \( \beta_1 = Sc(Kr + at), \beta_2 = \left( M + \frac{1}{K_p} + at \right), \beta_3 = \left( M + \frac{1}{K_p} \right) \)

Solving Equations (11) - (16) under the boundary conditions (17) and we obtain the velocity, temperature and concentration distributions in the boundary layer as

\[ C_0 = t \ e^{\alpha_1 y}; C_1 = 0 \]  
\[ T_0 = D_1 e^{\alpha_1 y} + D_2; T_1 = 0 \]  
\[ U_0 = J_1 + J_2 e^{\alpha_1 y} + J_3 e^{2\alpha_1 y} + J_4 e^{3\alpha_1 y}; U_1 = 0 \]  

In view of the equation (10) becomes

\[ U = J_1 + J_2 e^{\alpha_1 y} + J_3 e^{2\alpha_1 y} + J_4 e^{3\alpha_1 y} \]  
\[ T = D_1 e^{\alpha_1 y} + D_2 \]  
\[ C = t \ e^{\alpha_1 y} \]

A. Coefficient of Skin-Friction

The coefficient of skin-friction at the vertical porous surface is given by
B. Coefficient of Heat Transfer

The rate of heat transfer in terms of Nusselt number at the vertical porous surface is given by

\[ \left( \frac{\partial T}{\partial y} \right)_{y=0} = D_1 m_4 \]

C. Sherwood number

\[ \left( \frac{\partial C}{\partial y} \right)_{y=0} = t m_4 \]

Table (1): Skin friction versus Gr

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<th>M</th>
<th>Kr</th>
<th>Sc</th>
<th>K</th>
<th>Pr</th>
<th>Q</th>
<th>R</th>
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Table (1) shows that numerical values of $M, Kr, Sc, K, Pr, Q, R$ versus $Gr$ for fixed values of $Pr = 0.71, t = 0.4, Q = 1.0, Ql = 1.0, Gc = 5.0, K = 0.5, Sc = 0.6, \gamma = \frac{\pi}{3}, a = 0.5, Kr = 0.5, R = 1.0$. It is observed that an increase in $M, Pr$ the skin friction decreases, but the reverse effect can be observed for the parameters $Kr, Sc, K, Q, R$.

### IV. RESULTS AND DISCUSSIONS

In order to get the physical insight into the problem, we have plotted velocity, temperature and concentration profiles and the following discussions are set out. The impact of several prevailing physical parameters such as convection parameters $Gr$ is the thermal Grashof number, $Gc$ is modified Grashof Number, $Pr$ is Prandtl Number, $M$ is the magnetic field, $Sc$ is Schmidt number, $Kr$ is Chemical Reaction, $K$ is Porous Permeability, $Q$ is radiation absorption and Pr is Prandtl number are elucidated through figures and tables. The effects of distinct governing parameters on velocity, temperature and concentration are revealed in figures (2) – (15). Figure (2) – (3) shows the influence of the thermal buoyancy force parameter Grashof number $(Gr)$ and modified Grashof number $(Gc)$, it is observed that increases $Gr, Gc$ in the velocity increases. This is due to the fact that buoyancy forces enhance fluid velocity and increase the boundary layer thickness with increase in the values of $Gr, Gc$. It can Figure (4) reveals that velocity of the fluid is increases, as accelerate the values of acceleration parameter. Figure (5) illustrate the variation of velocity distribution across the boundary layer for various values of permeability parameter $(K)$, the velocity increases with an increase in permeability parameter $K$. Figure (6) represent the influence of chemical reaction parameter $(Kr)$ on the velocity profiles respectively. Increasing the chemical reaction parameter $(Kr)$ the velocity decreases. Velocity profiles plotted in figure (7) for various values of magnetic parameter $(M)$. Figure (8) are plotted for different values of Prandtl number $(Pr)$ against velocity profiles. In this figure we can seen that an increment in Prandtl number $(Pr)$ produces a marked increase in the velocity. We notice that the velocity profiles decreases with increasing values of magnetic parameter. Effects of radiation absorption $(Q)$ parameter in figure (9). As increases radiation absorption parameter increases the thickness of the momentum boundary layer increases for velocity. Figure (10) display the effects of Schmidt number $(Sc)$ on velocity profiles respectively. As the Schmidt number increases the velocity decreases. This causes the concentration buoyancy effects to decreases yielding a reduction in the fluid velocity. The reductions and the velocity profiles are accompanied by simultaneous reductions in the velocity boundary layers. The temperature profiles chemical reaction parameter $(Kr)$ shown in figure (11). It is found that with the increase chemical reaction parameter $(Kr)$, the temperature increases. The temperature profiles for radiation absorption parameter $(Q)$ shown in figure (12). It is found that with the increase radiation absorption parameter $(Q)$, the temperature decreases. Figure (13) display the effects of Schmidt number $(Sc)$ on temperature profiles respectively. As the Schmidt number increases the temperature decreases. Figure (14) shows the chemical reaction parameter $(Kr)$ on concentration profiles. It is noticed here that the chemical reaction parameter $(Kr)$ decreases with increase in the $Kr$. The effect of the Schmidt number $(Sc)$ on the concentration shown in figure (15), it is observed that increase in Schmidt number $(Sc)$ lead to faster decrease in concentration of the flown field. The reductions concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

A. Appendix
\[ m_1 = -KrSc, m_2 = -\left( R - Q Pr \right), m_{12} = \beta_z, L_1 = \frac{Gr \cos \gamma t}{m_2^2 - \beta_z^2} \]

\[ L_2 = \frac{Gc \cos \gamma t}{m_2^2 - \beta_z^2}, L_3 = \left( e^{nt} - L_1 - L_2 \right), A_1 = \frac{Q_t Pr t}{m_2^2 - \left( R - Q Pr \right)} \]

\[ A_2 = (t - A_1) \]

REFERENCES


Figure (3): Velocity profiles for different values of $Gc$

$t=0.4; Q=0.5; Gr=15.0; M=5.0; K=1.0$
$Sc=0.6; y=\pi/6; a=0.5; Kr=0.5; Pr=0.7$
$Gc = 5, 10, 15, 20$

Figure (4): Velocity profiles for different values of $a$

$t=0.4; Q=0.5; Gr=15.0; Gc=15.0; M=1.0$
$K=1.0; Sc=0.6; y=\pi/6; Kr=0.5; Pr=0.7$
$a=0.1, 0.3, 0.5, 0.7$

Figure (5): Velocity profiles for different values of $K$

$t=0.4; Q=0.5; Gr=15.0; Gc=15.0; M=5.0$
$Sc=0.6; y=\pi/6; a=0.5; Kr=0.5; Pr=0.7$
$K=0.5, 1.0, 1.5, 2.0$
Figure (6): Velocity profiles for different values of $Kr$

$t=0.4, Q_x=0.5, Gr=15.0, Gc=15.0, M=5.0$
$K=1.0, Sc=0.6, \gamma=\pi/6, a=0.5, Pr=0.7$

$Kr=0.5, 1.0, 1.5, 2.0$

Figure (7): Velocity profiles for different values of $M$

$t=0.4, Q_x=0.5, Gr=10.0, Gc=5.0, K=1.0$
$Sc=0.6, \gamma=\pi/6, a=0.5, Kr=0.5, Pr=0.7$

$M=0.5, 1.0, 1.5, 2.0$

Figure (8): Velocity profiles for different values of $Pr$

$t=0.4; Q_x=0.5, Gr=15.0, Gc=15.0, M=5.0$
$K=1.0, Sc=0.6, \gamma=\pi/6, a=0.5, Kr=0.5$

$Pr=0.5, 0.7, 0.9, 1.1$
Figure (9): Velocity profiles for different values of $Q_l$

Figure (10): Velocity profiles for different values of $Sc$

Figure (11): Temperature profiles for different values of $Sc$
Figure (12): Temperature profiles for different values of $Q_l$

Figure (13): Temperature profiles for different values of $Sc$

Figure (14): Concentration Profiles for different $Kr$
Figure (15): Concentration Profiles for different $Sc$

$Sc=0.2, 0.4, 0.6, 0.8$

- Jyotsna Rani Pattnaik et al. ($t = 0.4$)
- Present Study ($t = 0.2$, $Kr = 1.0$)