Design of Low Pass Fir Filter Using Rectangular, Hanning And Kaiser Window Techniques

Kamlesh Sahu¹, Ayush Gavel², Pranay Kumar Rahi³

¹, ²BE Scholar, ³Assistant Professor

Department of Electrical & Electronics Engineering, Institute of Technology Korba Chhattisgarh, India

Abstract: Digital filter are widely used in the world of communication and computation. On the other hand, to design a Digital Finite Impulse Response (FIR) filter that satisfies all the required condition is challenging one. In this paper, design techniques of Low Pass filter using Rectangular, Hanning and Kaiser Window Techniques of order (15) are presented. It shown that filter design by using Rectangular Window Technique is better as it provides better result in terms of magnitude, phase, impulse, step responses and pole-zero plot. The analysis of magnitude, phase, impulse, step response and pole-zero plot of proposed FIR Low Pass filter is performed using MATLAB Simulation. [4]

Keywords- DSP, Digital filter, Low-Pass filter, FIR filter, Rectangular window, Hanning and Kaiser Window Techniques.

I. INTRODUCTION

Digital filter is essentially a system or network that improves the quality of a signal and/or extracts information from the signals or separates two or more signals which are previously combined. Digital filters are used in numerous applications e.g. control system, system for audio and video processing and communication systems. Now a day’s digital filters can be used to perform many filtering tasks are replacing the traditional role of analog filters in many applications. Digital filters can be applied to very low frequency signals, such as those occurring in biomedical and seismic applications very efficiently. In addition, the characteristics of digital filters can be changed or a adapted by simply changing the content of a finite number of registers, thus multiple filters are usually used to discriminate a frequency or a band of frequencies from a given signals which is normally a mixture of both desired and undesired signals. These are mainly two types of filter algorithms. They are finite impulse response filter (FIR) and infinite impulse response filter (IIR). In case of a FIR filter, the response due to an impulse input will decay with in a finite time. But for IIR filter, the impulse response never dies out. FIR filters are commonly known as non-recursive filters and IIR filters are known as recursive filters. These names came from the nature of algorithm used for these filters. A finite impulse response (FIR) digital filter is one whose impulse response is of finite duration. The impulse response is „finite” because there is no feedback in the filter if put in an impulse (that is, a single “1” sample followed by many “0” samples), zeroes will eventually come out after the “1” sample has made its way in the delay line past the entire coefficient. The structure of these algorithms uses a repetitive delay-and-add format that can be represented as “Direct Form-I Structure”. The advantage of FIR filter over IIR filters are, FIR filter has linear phase and easily to control where as IIR filter has no particular phase and difficult to control. FIR filter is stable and depends only on input. FIR filters consists of only zeroes and IIR filters consists of both poles and zeroes. FIR filters are filters having a transfer function of a polynomial in z-plane and is an all-zero filter in the sense that the zeros in the z-plane determine the frequency response magnitude characteristics. The Z transform of N point FIR filter is given by: [1]

\[ H(z) = \sum_{n=0}^{N} h(n)z^{-n}, \quad n = 0,1 \ldots \ldots N \]

Where N is the order of the filter which has (N+1) number of coefficients. h(n) is the filter’s impulse response. It is calculated by applying an impulse signal at the input. The values of h(n) will determine the type of the filter e.g. low pass, high pass, band pass etc. the values of h(n) are to be determine in the design process and N represents the order of the polynomial function. FIR filters are particularly useful for applications where exact linear phase response is required. The FIR filter is generally implemented in a non-recursive way which guarantees a stable filter. FIR filter design essentially consists of two parts:

Approximation problem.

Realization problem.

The approximation stage takes the specification and gives a transfer function through four steps. They are as follow:- A desired or ideal response is chosen, usually in the frequency domain.
An allowed class of filters is chosen.

A measure of the quality of approximation is chosen. iv. A method or algorithm is selected to find the best filter transfer function.

The realization part deals with choosing the structure to implement the transfer function which may be in the form of circuit diagram or in the form of a program.

In case of a FIR filter, it is an attractive choice because of the ease in design and stability. By designing the filter taps to be symmetrical about the center tap position, the FIR filter can be guaranteed to have linear phase. Finite impulse response (FIR) digital filters are known to have many desirable features such as guaranteed stability, the possibility of exact linear phase characteristic at all frequencies and digital implementation as non-recursive structure. Linear phase FIR filter are also required when time domain specifications are given. Traditionally, different technique exit for the design of digital filters [1].

A. Applications of DSP in area wise are as following

Telecommunication - Echo cancellation in telephone networks, equalization, telephone dialing application, modems, line repeaters, channel multiplexing, data encryption, video conferencing, cellular phone and FAX.

Military - Radar signal processing, sonar signal processing, navigation, secure communications and missile guidance.

Consumer electronics - Digital Audio/TV, electronic music synthesizer, educational toys, FM stereo application and sound recording applications.

Instrument and control – Spectrum analysis, position and rate control, noise reduction, data compression, digital filter, PPL, function generator.

Image processing- Image representation, image compression, image enhancement, image restoration and image analysis. Speech processing- Speech analysis methods are used in automatic speech recognition, speaker verification and speaker identification.

.Medicine- Medical diagnostic instrumentation such as computerized tomography (CT), X-ray scanning, Patient monitoring and X-ray storage/enhancement.

Signal filtering- Removing of unwanted background noise, removal of interference, separation of frequency bands and shaping of the signal spectrum. [5]

II. WINDOW METHOD

The desired frequency response of any digital filter is periodic in frequency and can be expanded in a Fourier series, i.e.

\[ H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)e^{-j\omega n} \]  \hspace{1cm} (1)

Where, \( h(n)=\frac{1}{2\pi} \int_{0}^{2\pi} H(e^{j\omega})e^{j\omega n} d\omega \) \hspace{1cm} (2)

The Fourier coefficient of the series \( h(n) \) are identical to the impulse response of a digital filter. There are two difficulties with the implementation of above equation for designing a digital filter. First, the impulse response is of infinite duration and second, the filter is non-causal and unrealizable. No finite amount of delay can make the impulse response realizable. Hence the filter resulting from a Fourier series representation of \( H(e^{j\omega}) \) is an unrealizable IIR filter. [5]

The windowing method requires minimum amount of computational effort; so window method is simple to implement. For the given window, the maximum amplitude of ripple in the filter response is fixed. Thus the stop band attenuation is fixed in the given window, but there is some drawback also of this method. The design of FIR filter is not flexible. The frequency response of FIR filter shows the convolution of spectrum of window function & desired frequency response because of this; the pass band & stop band edge frequency cannot be precisely specified. [4]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling Frequency(Fs)</td>
<td>48000Hz</td>
</tr>
<tr>
<td>Cut off Frequency(Fc)</td>
<td>10800Hz</td>
</tr>
<tr>
<td>Order(N)</td>
<td>15</td>
</tr>
</tbody>
</table>

**TABLE NO. 1**

A. Rectangular Window

The rectangular window (sometimes known as the **Boxer** or **Dirichlet window**) is the simplest window, equivalent to replacing all
but $N$ values of a data sequence by zeros, making it appear as though the waveform suddenly turns on and off: [5]

$$w(n) = 1.$$  

$$w_R(n) = \begin{cases} 
1, & \text{for } n \leq \frac{M-1}{2} \\
0, & \text{otherwise}
\end{cases}$$
Phase response of Rectangular Window

Magnitude (dB) and Phase Response of Rectangular Window

Impulse response of Rectangular Window
Step Response of Rectangular Window

Pole/Zero plot of Rectangular Window

Filter Coefficient of Rectangular Window
B. Hanning Window

The Hann window named after Julius von Hann and also known as the Hanning (for being similar in name and form to the Hamming window), von Hann and the raised cosine window is defined by (with hav for the haversine function): [5]

\[ W(n) = 0.5 \left(1 - \cos \left( \frac{2\pi n}{N-1} \right) \right) = \text{hav} \left( \frac{2\pi n}{N-1} \right) \]

The ends of the cosine just touch zero, so the side-lobes roll off at about 18 dB per octave.

- Time domain & frequency domain of Hanning Window
- Magnitude response of Hanning Window
- Phase response of Hanning Window
Magnitude (dB) and Phase Response of Hanning Window

Impulse response of Hanning Window

Step Response of Hanning Window
C. Kaiser Window

The Kaiser, or Kaiser-Bessel, window is a simple approximation of the DPSS window using Bessel functions, discovered by Jim Kaiser.

\[ w(n) = \frac{I_0(\pi \alpha \sqrt{1 - \frac{2n}{N-1} - \frac{1}{2}})}{I_0(\pi \alpha)} \]

Where \( I_0 \) is the zero-th order modified Bessel function of the first kind. Variable parameter \( \alpha \) determines the tradeoff between main lobe width and side lobe levels of the spectral leakage pattern. The main lobe width, in between the nulls, is given by \( 2\sqrt{1 + \alpha^2} \), in units of DFT bins, and a typical value of \( \alpha \) is 3.

Sometimes the formula for \( w(n) \) is written in terms of a parameter \( \beta \equiv \pi \alpha \)

Zero-phase version: \( w_0(n) = \frac{I_0(\pi \alpha \sqrt{1 - \frac{2n}{N-1} - \frac{1}{2}})}{I_0(\pi \alpha)} \) [5]
Time domain & frequency domain of Kaiser Window

Magnitude response of Kaiser Window

Phase response of Kaiser Window
Magnitude (dB) and Phase Response of Kaiser Window

Impulse response of Kaiser Window

Step Response of Kaiser Window
Pole/Zero plot of Kaiser Window

Filter Coefficient of Kaiser Window

D. Comparative analysis

Magnitude and Frequency Chart of Rectangular, Hanning and Kaiser Window technique
Phase and Frequency Chart of Rectangular, Hanning and Kaiser Window Technique

Magnitude and Frequency comparison of Rectangular, Hanning and Kaiser Window.

Phase and Frequency Comparison of Rectangular, Hanning and Kaiser Window.
Magnitude and Phase Responses Comparison of Rectangular, Hanning and Kaiser Window.

Impulse Response Comparison of Rectangular, Hanning and Kaiser Window

By using MATLAB 7.12.0 (R2011a) Simulation Technique we design all above simulation of LOW PASS FIR FILTER USING RECTANGULAR, HANNING AND KAISER WINDOW TECHNIQUES. All above figure of responses shows difference about Filters in the form of simulation which are Time domain & frequency domain, Magnitude response, Phase response, Impulse response, step response, Pole/Zero plot or Filter coefficient respectively.

Then we done comparative analysis of our filter which is combined figure shows at above simulation
In Hanning Window beta value is increase then main lobe width is increases but leakage factor is decreases. When leakage factor is 0.05% in Hanning Window then wider main lobe width (0.20313).

“In Interpretation of above Table-1 show parameter specification of windows designing of low pass filters using Rectangular, Hanning and Kaiser Window. Sampling frequency is 48000 Hz and cut off frequency is 10800 Hz. Filter order is 15.”

Window Techniques

<table>
<thead>
<tr>
<th>Frequency (KHz)</th>
<th>Rectangular Magnitude</th>
<th>Rectangular Phase</th>
<th>Hanning Magnitude</th>
<th>Hanning Phase</th>
<th>Kaiser Magnitude</th>
<th>Kaiser Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10242</td>
<td>-1.02566</td>
<td>0.01760</td>
<td>-1.02566</td>
<td>0.07405</td>
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<tr>
<td>2</td>
<td>0.26472</td>
<td>-2.03561</td>
<td>0.01176</td>
<td>-2.03561</td>
<td>0.18000</td>
<td>2.03561</td>
</tr>
<tr>
<td>3</td>
<td>0.25574</td>
<td>-3.03215</td>
<td>0.03798</td>
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<td>0.14990</td>
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<tr>
<td>4</td>
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<td>-4.01646</td>
<td>0.06425</td>
<td>-4.01646</td>
<td>-0.02091</td>
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</tr>
<tr>
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<td>0.04866</td>
<td>-5.02564</td>
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<tr>
<td>6</td>
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<td>-0.10255</td>
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<tr>
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<td>-16.24077</td>
<td>-12.01254</td>
</tr>
</tbody>
</table>

Magnitude, Phase and Frequency Result of Rectangular, Hanning and Kaiser Window techniques

IV. CONCLUSIONS

In this paper FIR low pass filter has been designed and simulated using Rectangular, Hanning and Kaiser Window technique has been compared. In Signal processing applications digital filters are more preferable than analog filters. The digital filters are easily designed and also easy to use in various types of signal filtering applications. The choice of technique to design the filter depends on the decision of designer whether to compromise accuracy of approximation or ease of design. In this view FIR filter has been designed using Modified coefficient of the Kaiser Window function and simulated with MATLAB programs. The results show that the filter design using Modified window function has a small main lobe width and sharp transition band compared to Rectangular, Hanning and Kaiser Window function. So that for same length this window function provides efficient results compare to Rectangular, Hanning and Kaiser Window function and this type of filters are very useful in spectral analysis and many other applications. The equivalent noise bandwidth of modified hamming window also reduces as compared to Rectangular Window, Hanning Window and Kaiser Window. On the basis of the desired filter characteristics, different parameters can be easily changed in the MATLAB program to meet the engineering requirements in the FIR filter design process.
REFERENCES


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[4] Sriti Pandey, 2Kamlesh Sahu, 3Pooja Singh Chandel, 4 Pranay Kumar Rahi; 1, 2, 3 B.E.Scholar, 4Assistant Professor, 1,2,3,4 Department of Electrical & Electronics Engineering, Institute of Technology Korba, Chhattisgarh, India (LOW PASS FILTER DESIGN AND ANALYSIS USING RECTANGULAR AND BARTLETT WINDOWS).


AUTORS

Kamlesh Sahu pursuing Bachelor of Engineering in Electrical and Electronics Engineering in 6th semester from Institute of Technology Korba, Chhattisgarh Swami Vivekananda Technical University, Chhattisgarh, India.

Ayush Gavel pursuing Bachelor of Engineering in Electrical and Electronics Engineering in 6th semester from Institute of Technology Korba, Chhattisgarh Swami Vivekananda Technical University, Chhattisgarh, India.

Pranay Kumar Rahi Received the Bachelors of Engineering degree in Electronics and Telecommunication Engineering from Government Engineering College, Guru Ghasidas University, Bilaspur, Chhattisgarh, India in 2004, working as a Assistant Professor in Electrical and Electronic Engineering of Institute of Technology Korba since 2008 and pursuing Masters of Engineering in Electronics and Communication Engineering from National Institute of Technical Teacher’s Training& Research, Punjab University, Chandigarh, India. Presently working as assistant professor in Department of Electrical and Electronic Engineering, Institute of Technology Korba, Chhattisgarh since 2008. He has authored More than 14 research publications and published journal papers and research papers in the leading International and National journal. His primary research interest includes Digital Signal Processing, VLSI Design, Control Systems and Digital Electronics and logic design.