Region Based Segmentation in Presence of Significant Intensity Inhomogeneity using Legendre Polynomials

M. Rangaswamy¹, M. Sunitha², M. MD. Zaid khan³, D. Pavani⁴, J. Naga Ratnam⁵

¹²³⁴⁵ Department of ECE, Brindavan Institute of Technology and Science

Abstract: we propose a region based segmentation method which is capable of segmenting objects in presence of significant intensity inhomogeneity. Present method use some form of local processing to tackle intra region inhomogeneity, which further makes such methods susceptible to local minima. Here we present a framework which generalizes the traditional Chan-Vese algorithm. In contrast with existing local techniques, we represent the illumination of the regions of interest in a lower dimensional subspace using set of pre-specified basis functions. This representation can accommodate heterogeneous objects, even in presence of noise. We compare our results with several techniques like biological/biomedical images with tubular or filamentous structures. As the, we achieve a 44% increase in performance, which demonstrates efficacy of the method.

I. INTRODUCTION

Active contours [1]–[4] are popular for image segmentation. Active contours have ability to elastically deform and delineate object boundaries with sub-pixel accuracy. Furthermore, the energy optimization framework, which serves as the basic platform for most active contour based techniques, can be handle easily to introduce ad-additional constraints based on shape, appearance etc. to assist segmentation. Geometric active contours are used when the application requires the propagating curves to be able to adapt to the varying topology of the underlying object by automated splitting or merging.

The geometric active contours may be divided in two types. Edge based techniques and region based techniques. Edge based techniques [2], [5], [6] perform curve evolution geometrically, with the stopping criteria controlled by edge dependent features extracted from the image. However, in many applications where the edge information is unreliable, region based techniques are used. Chan and Vese [3] proposed a level set formulation to minimize the Mumford Shah functional [7] for segmentation. The Chan-Vese frame-work models the image as a set of constant illumination regions and performs a two-class segmentation by computing the optimal partition which hereby satisfies the constant illumination constraint. The authors also propose a multi-phase variant [8] of their approach to perform multi-class grouping.

Fig. 1. Segmentation results of Chan-Vese (blue)[3] and LBF model [9] (cyan) vs L2S (yellow) on a simulated image. The second row shows segmentation when a Gaussian noise of std. dev. 0.1 is added. (a) Original image, (b) Chan- Vese [3], (c) Li et al. [9], (d) L2S.

The constant illumination assumption is been challenged in applications where the signal intensity is inhomogeneous. This is observed frequently in many medical and biological imaging applications like magnetic resonance (MR) imaging, X-ray, ultrasound, confocal and electron microscopy, etc. While edge based techniques are better matched for heterogeneous images
because they are unable to tackle fragmented object boundaries or blurred edges, which we can see in medical applications. In this letter we propose an edge independent segmentation approach Legendre Level Set which is robust to variations in intensity levels. State of the art techniques that tackle inhomogeneity typically require some form of local processing. However, while a global method like Chan-Vese’s is insufficient in handling large scale intensity variations, a strictly local approach may lead to undesired segmentation artifacts, especially in presence of noise. Such artifacts are shown in Fig. 1. We aim to eradicate these issues by proposing a generalized solution for region based segmentation in presence of significant intensity variation and additive noise.

II. EXISTING METHOD SCHAN-VESE MODAL

Chan and Vese proposed a level set formulation to minimize the Mumford Shah functional for segmentation. The chan-vese framework models the image as a set of constant illumination regions and performs a two-class segmentation by computing the optimal partition which satisfies the constant illumination constraint. In this method we use an edge independent segmentation approach Legendre Level Set (L2S). This method is robust to variations in intensity levels. In this method we find the foreground and background of the image by using scalar constants c1 and c2. Chan-vese energy functional can be written as:

\[ \varepsilon(\Theta, c_1, c_2) = \int_\Omega |f(x) - c_1|^2 H_\epsilon(\Theta) \, dx + \int_\Omega |f(x) - c_2|^2 (1 - H_\epsilon(\Theta)) \, dx \]  

Where \( F(x) \) is the image defined over the domain \( \Omega \).

![Fig. 2. The set of nine 2D Legendre basis functions. The 2D polynomials are computed from 1D functions of degree 2.](image)

The level set function is defined to be positive inside the zero level set and negative outside it. Zero level set represents the object boundaries. The intensity level creates segments in a manner such that foreground and background are best approximated by the intensity levels c1 and c2. This method is incapable of handling spatially varying illumination. A solution was proposed where the scalars c1 and c2 are replaced with smooth functions. Whereas smoothing is an expensive process, since it requires two PDEs at each stage of the level set. In order to overcome all these problems we use Legendre level set polynomials.

A. Chunming Li Modal

Recently, Li et al. [9] introduced a region scalable model to limit the energy functional. The region limitation is controlled by the scale of a Gaussian kernel, which is manually tuned for less performance. Efforts are being made to incorporate the region statistics for segmentation [10], [11]. These methods are difficult to initialization and relatively less sensitive to noise. So the authors Lankton et al. [12] demonstrated that global statistics may not be the best resort for segmenting inhomogeneous objects. Instead, the authors generalize the local region based methods, by proposing generic energy functional capable of performing segmentation using different region based criteria. However, one drawback of their approach is that it requires additional local computation, that increases the risk of being stuck within local minima. Feng et al. [13] proposed a method for tomographic reconstruction by using a low order parametric model to represent object texture. However, the algorithm is used for tomography reconstruction which are difficult to generalize. Recently a method was proposed to model the foreground and background by a linear function [14]. This approach is an advancement over the model of Chan-Vese, but does not work out for nonlinear illumination change. From the above discussion we observe that a majority of these approaches work on local information only. While localizing the segmentation energy is essential in dealing with inhomogeneity and a generic global framework is also necessary to avoid the local minima problem. We proposed to model the foreground and background illumination by a set of Legendre basis functions. This model allows the region intensities to be represented in a lower dimensional subspace, which provides smooth approximation. Low dimensional signal representation has been used in a different context in the literature, primarily to accommodate previous shape for segmentation [15], [16]. However, the shape based information assists segmentation, such techniques require an atlas of pre-registered objects, which are unavailable for general purpose segmentation. We further show that our model (called L2S) is simple, since we achieve a stable, closed form solution at each iteration, that allows faster processing.
A. Suvadip Modal

The traditional Chan-Vese functional (1) can be reformulated and generalized by replacing the scalars $c_1$ and $c_2$ by two smooth functions $c^m_1(x)$ and $c^m_2(x)$. These functions are used to model the intensity in the two regions separated by the zero level set curves of $\emptyset$. The essence of our approach is embedded in computing these functions. To preserve the smoothness and flexibility of the functions, we represent them as a linear combination of a few Legendre basis functions. To preserve the smoothness and flexibility of the functions, we represent them as a linear combination of a few Legendre basis functions. Mathematically, we can write $c^m_1(x) = \sum_k \alpha_k p_k(x)$ and $c^m_2(x) = \sum_k \beta_k p_k(x)$. Here $p_k$ is a multidimensional Legendre polynomial, which can be written as the outer product of the one dimensional counterparts. The 2-D polynomial is computed as $c(x,y) = p_k(x)p_k(y), x, y \in \Omega \subset [-1,1]^2$. $p_k$ is a one dimensional Legendre polynomial of degree $k$ defined as

$$p_k(x) = \frac{1}{2^k k!} \sum_{i=0}^{k} \binom{k}{i} (x - 1)^{k-i} (x + 1)^i$$

(2)

The highest degree of the 1D bases is denoted by $m$. Hence, for the 2D case, we would represent the regions by a linear combination of a set of $(m+1)^2$ 2D Legendre basis functions.

B. Choice of Basis Function

Our primary objective is to perform segmentation in the presence of significant intensity variation. In many applications (e.g. phlebotomy via ultrasound), it is valid to assume that the intra-region inhomogeneity field (in homogeneous tissue, for example) is smoothly varying. Therefore, we seek a smooth representation of the region approximating functions. The set of Legendre polynomials provides a good platform for such representation, since the basis functions are smoothly changing themselves. Legendre polynomials have got a great application in certain aspects of image processing in designing illumination models for object tracking[17] and generating shape signatures for supervised segmentation [18]. Using Legendre polynomials model intensity based appearance allows the estimating functions to vary spatially, but at the same time variation is constrained by the inherent smoothness of the polynomials. A demonstrative example of nine 2D Legendre functions is shown in Fig. 2. Note that the zero degree $(m=0)$ version of the Legendre polynomial reduces to the traditional Chan-Vese model, thus presenting a generalized framework.

C. Optimization of the Energy Functional

Let us denote $p(x) = (p_0(x), \ldots, p_0(x))^T$ as the vector of Legendre polynomials, $A = (\alpha_0, \ldots, \alpha_N)^T$ and $B = (\beta_0, \ldots, \beta_N)^T$ are the coefficient vectors for the two regions. $N = (m+1)^2$ is the total number of basis functions. We can now rewrite the modified version of (1) in matrix form as follows:

$$\varepsilon^m(\emptyset, A, B) = \int_\Omega |f(x) - A^T P(x)|^2 m_1(x) \, dx + \lambda_1 \|A\|^2 + \int_\Omega |f(x) - B^T P(x)|^2 m_2(x) \, dx + \lambda_1 \|B\|^2 + \int_\Omega \delta_\emptyset(\emptyset) \frac{\partial v}{\partial \emptyset} \, dx$$

(3)

In (3), the last term introduces smoothness in the zero level curve, which is regulated by the parameter. Let us also denote $m_1(x) = H_1(\emptyset)$ and $m_2(x) = 1 - m_1(x)$. The non negative regularizing parameters $\lambda_1, \lambda_2$ can be selected using cross validation techniques to avoid over fitting. The energy functional (3) is optimized using alternating minimization. In the first step, to find optimal $A$ and $B$, we perform $\frac{\partial \varepsilon^m}{\partial A} = 0$ and $\frac{\partial \varepsilon^m}{\partial B} = 0$. A closed form solution $\hat{A}$ and $\hat{B}$ is obtained as

$$\hat{A} = [K + \lambda_1]^{-1} P$$

(4)

$$\hat{B} = [L + \lambda_2]^{-1} Q$$

(5)

denotes a matrix. The $N \times 1$ vectors and are obtained as $P = \int_\Omega p(x) f(x) m_1(x) \, dx$ and $Q = \int_\Omega p(x) f(x) m_2(x) \, dx$ respectively. $[K]$ and $[L]$ are $N \times N$ Gramian matrices [19], whose $(i,j)$ entry are obtained as

$$[K]_{ij} = \langle \sqrt{m_1(x)}(P)(x), \sqrt{m_1(x)}(P)(x) \rangle$$

(6)

$$[L]_{ij} = \langle \sqrt{m_2(x)}(P)(x), \sqrt{m_2(x)}(P)(x) \rangle$$

(7)
0 \leq i, j \leq N. denotes the inner product operator. With the updated coefficient vectors, we can now minimize (3) with respect to $\Theta$ borrowing techniques from variational calculus. The curve evolution is performed by numerically solving the following partial differential equation.

$$\frac{\partial \Theta}{\partial t} = \left[ -|f(x) - \bar{A}^T P(x)|^2 + |f(x) - \bar{B}^T P(x)|^2 \right] \partial_{\Theta} (\Theta) + \nu \partial_{\Theta} (\Theta) \text{div} \left( \frac{\Theta}{|\Theta|} \right) \tag{8}$$

$\delta_{\epsilon}(\Theta)$ is a regularized version of the Dirac delta function. We solve (8) using gradient descent and initializing $\Theta| \epsilon = 0 = \Theta_0$ and $\frac{\delta_{\epsilon}(\Theta)}{|\Theta|} \frac{\partial_{\Theta}}{\partial_{\Omega}}$ at the domain boundary.

IV. RESULTS & DISCUSSIONS

The surface approximate for foreground and background are obtained by computing $\bar{A}^T P(x)$ and $\bar{B}^T P(x)$ Since the coefficient vectors are available in closed form, it makes our algorithm fast and effective. The amount of intensity variation is governed by the coefficient vectors which are computed automatically. However, computing the coefficient vectors require a matrix inversion step. Here we show that the matrices $[K]$ and $[L]$ are invertible when the heaviside function is suitably regularized. Since $[K]$ is a gramian matrix, it is a Gramian matrix, it is full rank iff the polynomials $\alpha_i(\Omega)$, $(i = 1, \ldots, N)$ are linearly independent themselves, it is easy to show that the linear independence holds if $0 < \alpha_i(\Omega) < 1$ A similar argument holds for analyzing the invertibility of $[L]$.

In [3], the authors propose a regularized version of the Heaviside function as $\mathcal{H}(y) = \frac{1}{2} \left(1 + \frac{2}{\pi} \tan^{-1} \left(\frac{y}{\epsilon}\right)\right)$. By this definition, the functions $m_1(\Omega)$ and $m_2(\Omega)$ and bounded in $(0,1)$, which makes the matrices invertible. However, inverting the above mentioned matrices may still be prone to numerical error when $\sqrt{m_1(\Omega)}$ is small. The regularizing constants $\lambda_1$ and $\lambda_2$ contribute to make these matrices well-conditioned. Furthermore, the regularization terms are necessary to avoid over-fitting. In most situations, we find that only a few (typically 16) 2-D Legendre functions are sufficient to model the region intensity. However, image noise may lead to over-fitting of the polynomials to the image segments, which may disrupt segmentation as propagating level set may settle at local minima. The scalars $\lambda_1$, $\lambda_2$ produce a damping effect by constraining the $L_2$ norm of the bases coefficients, thereby favoring interior regions approximated by smooth functions.

Fig. 3. (a) The top row shows eight images from our dataset. The yellow polygon indicates level set initialization. The second, third
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and fourth rows show segmentation output of Chan-Vese [3] (green), Lankton et. al. [12] (white) and C. Li et.al. [9] (cyan). The fifth row shows segmentation results using L2S (yellow). Segmentation accuracy as calculated via the Dice index and computational time for each method are plotted in (b) and (c) respectively.

To demonstrate the ability of the proposed method, we have performed experiments on a dataset of 32 images. The data set consists of a set of synthetic images with added noise and simulated intensity inhomogeneity and a set of biomedical images which consists of blood vessels using magnetic resonance angiogram (MRA), neurons and dendrite spines imaged by confocal microscope and finally, a set of ultrasound images of human blood vessels.

Our algorithm requires specification of few parameters, namely the Legendre polynomial degree and the regularizing constants $\lambda_1$ and $\lambda_2$ in (3). We have experimentally verified that the intensity variation in the images can be modeled by using 1-D Legendre polynomials of (highest) degree three. We found that this algorithm is relatively difficult for the selection of this value, but a higher degree polynomial requires an inversion of a larger matrix, which makes the computation more expensive. To estimate the value of $\lambda_1$ and $\lambda_2$, we have a method called leave one out cross validation on each of the four categories in our dataset. The cross validation is performed over the values of $\{0.1,...,10\}$ in multiples of 2. For simplicity, we have chosen for $\lambda_1 = \lambda_2$ every experiment. The value which yields the highest average Dice coefficient for each dataset is chosen for the experimentation.

Automated selection of the contour smoothness parameter in (3) is significant. Typically $\lambda_1 \geq \lambda_2$, where a higher value produces smoother contour. As a rule of thumb, one may wish to set to a relatively higher value if the noise level in the image is high. For our experiments, we observe that the set of ultrasound images and the simulated noisy images require larger values of $\lambda$. For all these images, we select $\lambda$. For the less noisy images, it is typically set in the range 0.05 to 0.2.

To observe the performance of L2S, we compare our approach with those of Chan-Vese [3], Lankton et al. [12] and Li et al. [9]. We use an freely available tool called CREASEG to evaluate the performance. Thus Visual segmentation results for two images from each category in the dataset are presented in Fig. 3(a) for demonstration. Results of our algorithm are shown in yellow, in the fifth row. The Dice coefficient [21] is used to measure the results of segmentation. The Dice index $D \in [0,1]$ between the experimentally determined segmentation and the ground truth is given by $D(R_1,R_2) = \frac{2 \text{Area}(R_1 \cap R_2)}{\text{Area}(R_1) + \text{Area}(R_2)}$. A Dice value which is closer to 1 indicates superior performance. The quantitative performance is shown in Fig. 3(b). We also present the computation time in form of CPU cycles (Fig. 3(c)).

In this entire dataset we can observe L2S yields an average Dice score of 0.9, compared to 0.62, 0.57 and 0.7 for the methods described in [3], [12] and [9] respectively. Computationally, our method performs [12] and [9] on average. It may be noted that the apparent low convergence time of [3] often is a result of convergence at local minima.

V. CONCLUSION

A novel framework for segmentation in presence of significant intra region illumination variation is presented. Qualitative and quantitative results and comparison with the state of the art techniques suggest robustness of our approach. Here we have focused on bi-level segmentation, although extension to a multi-level framework appears straightforward. Also, our formulation allows easy incorporation of a priori shape information, which may enhance performance in select cases. However, like most level set methods, L2S is somewhat biased towards contour initialization. In past, there have been efforts to make such algorithms robust against initial curve placement [21]. Incorporating L2S to such a framework would be an interesting future research topic. Using Legendre polynomials for region intensity approximation provides an elegant solution. However, without further analysis it is difficult to comment on the optimality of this choice of basis. Effectiveness of other polynomialssuch as splines or wavelets [22] needs further investigation. In select cases, it may also be possible to learn a compact set of bases for representation. We wish to investigate the effectiveness of these approaches in the future.

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Mr. M Rangaswamy, has pursued B Tech in Madana palli Institute of Technology & Sciences and M Tech from in Madana palli Institute of Technology & Sciences. He is working as Associate Professor in BITS Kurnool.

Ms. M. Sunitha is pursuing B Tech in Brindavan Institute of Technology & Sciences in the department of ECE.

Ms. D. Pavani is pursuing B Tech in Brindavan Institute of Technology & Sciences in the department of ECE.
Mr. M. MD Zaid khan is pursuing B Tech in Brindavan Institute of Technology & Sciences in the department of ECE

Mr. J. Naga Ratnam is pursuing B Tech in Brindavan Institute of Technology & Sciences in the department of ECE