Effect of Reinforcement Profile On Creep Rates in a FG Disc

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Abstract: In the present study, we have investigated effect of varying reinforcement profile on creep rates in a Functionally Graded (FG) Rotating Disc. The disc under observation is made of Al-SiC\(_p\) composite. The SiC\(_p\) particle content is decreasing radially for negative value of reinforcement gradation index (n) while it is increasing for the positive value of n. It is concluded that the creep rates in the FGM disc reduces significantly with decreasing value of reinforcement gradation index.

Key words: Creep, variable thickness, FGM

I. INTRODUCTION

FGMs are heterogeneous composite materials with variation of volume content continuously from one side to the other [1-2]. Rotating disc is very important due to its extensive use in engineering applications [3]. Singh and Ray [4] investigated steady state creep in a rotating isotropic FGM disc of constant thickness by using Norton’s power law. Gupta et al. [3] extended the work to investigate the creep rates for a constant thickness rotating FGM disc operating under thermal gradient. Several authors investigated creep response in rotating FGM disc but using linear thickness profile [3, 5]. In the present study we have investigated effect of reinforcement profile on creep rates in rotating FGM disc.

II. DISC PROFILE AND DISTRIBUTION OF REINFORCEMENT

Let us consider a rotating disc (\(a =31.75 \text{ mm}\) and \(b =152.4 \text{ mm}\)) rotating at 15000 rpm. The thickness of the disc is varying along radius (\(r\)) as given by,

\[
h(r) = h_b \left(\frac{r}{b}\right)^k
\]

where \(k = -0.5\) and \(h_b = 20.13 \text{ mm}\) are the gradation index and outer thickness respectively.

The SiC\(_p\) content in the FGM disc is assumed to vary with radial distance as,

\[
V(r) = V_b \left(\frac{r}{b}\right)^n
\]

where \(n\) is gradation index.

On equating equal SiC\(_p\) content in constant thickness and variable thickness FGM disc, we get the SiC\(_p\) content at the outer radius (\(V_o\)),

\[
V_o = \frac{(2 + k + n)b^n V_{avg} (b^{2k} - a^{2k})}{(2 + k)(b^{2k+n} - a^{2k+n})}
\]

The density of disc material is assumed to vary as given by,

\[
\rho(r) = \rho_b \left(\frac{r}{b}\right)^{n_b}
\]

The effective strain rate (\(\dot{\varepsilon}\)) of the disc material is described by the threshold stress (\(\sigma_o\)) based law [5] as given by,

\[
\dot{\varepsilon} = \left[M(r) (\bar{\sigma} - \sigma_o(r))\right]^{5}
\]

where
The constitutive equations between stresses \((\sigma_r, \sigma_\theta)\) and strain rates \((\dot{\varepsilon}_r, \dot{\varepsilon}_\theta)\) for an isotropic disc under plane stress condition are given by [3],

\[
\dot{\varepsilon}_r = \frac{\dot{\varepsilon}}{2\sigma} \left[2\sigma_r - \sigma_\theta\right]
\]

\[
\dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}}{2\sigma} \left[2\sigma_\theta - \sigma_r\right]
\]

(7)

According to von Mises yield criterion, the effective stress \((\bar{\sigma})\) is given by,

\[
\bar{\sigma} = \frac{1}{\sqrt{2}} \left[\sigma_\theta^2 + \sigma_r^2 + (\sigma_r - \sigma_\theta)^2\right]^{1/2}
\]

(8)

Considering the equilibrium of forces acting on an element of a variable thickness disc, one may get the following equilibrium equation [6],

\[
\frac{d}{dr} \left[ r h(r) \sigma_r \right] - h(r) \sigma_\theta + \rho(r) r^2 h(r) \omega^2 = 0
\]

(9)

where \(\rho (r)\) is the density of FGM disc at any radius \(r\).

The disc is assumed to operate under free-free boundary conditions [3],

\[
\sigma_r = 0 \quad \text{at} \quad r = a \quad \text{and} \quad \sigma_r = 0 \quad \text{at} \quad r = b
\]

The equilibrium eq. (9) is solved along with set of constitutive eqs. (7) by following the procedure given in [5] to obtain the distribution of stresses and strain rates in the FGM disc.

### III. RESULTS AND DISCUSSION

A code has been developed for the calculations. The effect of varying reinforcement gradation index \((n)\) has been investigated on the creep response (Refer Table 1).

<table>
<thead>
<tr>
<th>(n)</th>
<th>SiC(_p) Content (vol %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(V_a)</td>
</tr>
<tr>
<td>0.5</td>
<td>11.51</td>
</tr>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>-0.5</td>
<td>33.39</td>
</tr>
</tbody>
</table>
It is clear above from Fig. 1 that radial strains in the FGM disc are lowest in FGM disc with lowest gradation index \((n = -0.5)\) as compare FGM disc \((n = 0.5)\) and composite disc \((n = 0)\). The effect of increasing PG on the tangential strain in the FGM disc is similar to radial strain.

### IV. CONCLUSIONS

The study has led to the following conclusions:

A. The creep response of the FGM disc with decreasing SiC\(_p\) content along the radius is superior to a similar FGM disc with decreasing SiC\(_p\) content along the radius.

B. The creep life can be significantly improved with decreasing SiC\(_p\) content along the radius.

### REFERENCES


