iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 13 Issue: IV Month of publication: April 2025

DOI: https://doi.org/10.22214/ijraset.2025.69195

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 13 Issue IV Apr 2025- Available at www.ijraset.com

An Optimum Design for Composite Cylindrical Pressure Vessels

Badr Bedairi[1], Sultan Alqash[2], Badr Azzam[3]
Taibah University, Madinah, KSA

Abstract: Composite materials have found wide applications in different engineering fields; mechanical, aeronautical, civil and chemical. This is due to the attractive properties which these materials possess as the high strength, high modules, low density, high corrosion resistance and well-tailored as needed through fabrication. One of these mechanical applications is composite pressure vessels of different shapes; cylindrical, spherical, toroidal and elliptical.

In this paper, cylindrical pressure vessels fabricated from an inside liner (aluminum or polymeric material) reinforced with different composite overwrapped; glass, carbon and Kevlar mixed with epoxy resin have been considered.

Generally, the composite pressure vessels contain the inside liner to prevent the leakage of the inside fluid, where the composite fibers cannot prevent that leakage due to the presence of voids and gaps between the fibers. The fibrous materials are used to give a reinforcement to the liner for sustaining the resulting hoop and longitudinal stresses in the vessel wall.

A design optimization has been performed to get the optimal parameters (outer radius, fiber volume fraction and fiber orientation angle) governing the design objective which is minimizing the vessel weight. After formulation of the optimization problem, as objective and constraints functions, Kuhn-Tucker method has been used to declare the main conditions of optimum design. This technique is used since the problem is nonlinear with inequality constraints. Then, the optimum solution is obtained using MATLAB program and checked with Kuhn Tucker conditions.

Nomenclatures

s_q= hoop stress
s_r= radial stress
s_a= axial stress
t= shear stress
r= radius at a general position
P_i= internal pressure
P_e= external pressure
W= vessel weight
$\overline{w}$ =non-dimensional vessel weight
S_{cl}= composite laminate longitudinal strength
S_{ct}= composite laminate transverse strength
S_{cs}= composite laminate interface shear strength
q= fiber orientation angle (with hoop direction)
GRP= glass fibers reinforced epoxy
CRP= carbon fibers reinforced epoxy
KRP= Kevlar fibers reinforced epoxy

I. INTRODUCTION

Pressure vessels are leak-proof containers used in many mechanical and aerospace applications as: hydraulic cylinders, boilers, gas containers, oil pipelines and much more.

These vessels can take many configurations; cylindrical, spherical, elliptical, and toroidal manufactured from different engineering materials as: carbon steel, aluminum, stainless steel and recently from fibrous composite materials.

Although cylindrical pressure vessels are less durable than spherical ones (due to the high stress induced in their walls compared to spherical pressure vessels), but they are less expensive in production.

The design of these vessels depends on many different parameters. Some of these parameters are related to material such as: material strength, modulus and density. Some parameters are related to functions like pressure loading, temperature and type of fluid to be stored. Whereas, the other parameters are related to geometry as diameter, thickness and length. The most common material used for manufacturing pressure vessels is carbon steel. This is due to its excellent durability, versatility, and cost-effectiveness. Carbon steel can withstand high pressures and temperatures, making it ideal for a wide range of industrial applications

Recently, pressure vessels are made from composite materials due to the attractive properties of these materials as; low density, high strength/weight ratio, high modulus/weight ratio and high corrosion resistance [1].

The pressure vessels fabricated from composite materials generally comprise an inside polymeric liner reinforced by fibers wound around it to give the vessel the high strength and high modules [2]. Advanced fibers as: glass, Kevlar, carbon or hybrid fibers may be used for this purpose.

In this paper, the design of cylindrical pressure vessels subjected to internal pressures and fabricated from different composite fibrous materials has been optimized. This optimum design is based on the failure criteria of composite materials, and the simple formulae for determining the cylinder shapes by using mathematical optimization techniques.

II. DESIGN ANALYSIS

Figure (1) shows a cylindrical pressure vessel subjected to internal pressure P_i and of dimensions: Inside radius R_i, outside radius R_o and length L. The vessel is reinforced by fibrous material to produce an external pressure around the inside liner and so increase its capability to sustain the internal pressure [2].

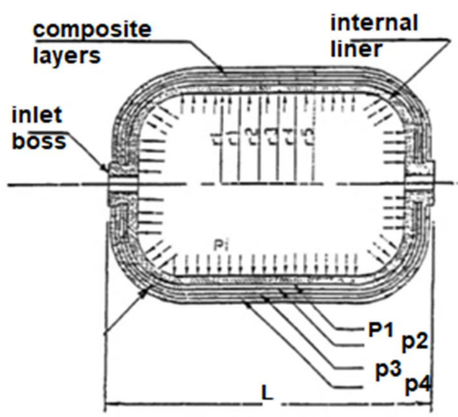


Figure (1) Composite cylinder with four reinforced layers

In this design, the optimum thickness of reinforced composite layers (which formed of a number of layers), optimum fiber volume fraction and optimum fiber orientation angle have been considered for different fibrous materials. This optimum design enables the pressure vessels designer to find the optimum design of such composite vessels reinforced with different fibrous materials.

Due to the internal pressure of the inside fluid and the external pressure resulting from the over-wrapped composite layers, the induced stresses in the vessel wall will be as follows [3, 4]:

$$
\begin{aligned}
\sigma_\theta &= \frac{P_i R_i^2[1 + (R_o/\mathrm{r})^2]}{(R_o^2 - R_i^2)} - \frac{P_e R_o^2[1 + (R_i/\mathrm{r})^2]}{(R_o^2 - R_i^2)} \\
\sigma_r &= \frac{P_i R_i^2[1 - (R_o/\mathrm{r})^2]}{(R_o^2 - R_i^2)} - \frac{P_e R_o^2[1 - (R_i/\mathrm{r})^2]}{(R_o^2 - R_i^2)} \qquad (1) \\
\sigma_a &= \frac{P_i R_i^2}{(R_o^2 - R_i^2)}
\end{aligned}
$$

Where;

s_q= hoop stress

s_r= radial stress

s_a= axial stress

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 13 Issue IV Apr 2025- Available at www.ijraset.com

Since the radial stress in the vessel wall is much lower than the other stresses (hoop and axial) and can be sustained by the inside liner, then the analysis will be considered only the hoop and axial stresses. Therefore, the overwrapped composite layers will be tasked to sustain the hoop and axial stresses only. Beside the inside liner, the resin matrix craze paths can join to create a leak path. The resin crazing generally becomes significant at a composite stress between 10 and 40 % of the ultimate fiber strength [3].

Also, the liner must be capable of straining or elongating with the composite layers during the effect of internal pressure. Since all the overwrapped layers (unidirectional laminate) wound around the inside liner with an orientation angle, q, and due to the applied hoop and axial stresses, the laminate direction stresses (s_l , s_t and t_{lt}), figure (2), can be obtained through the stress transformation matrix as follows [1]:

$$\begin{bmatrix} \sigma_l \\ \sigma_t \\ \tau_{lt} \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_\theta \\ \sigma_z \\ \tau_{\theta z} \end{bmatrix}$$

Where;

s_l= stress in the longitudinal direction of the composite laminate

s_t= stress in the transverse direction of the composite laminate

t_{lt}= shear stress on the composite laminate

t_{qz}= shear stress acting on the vessel wall (= 0)

c= cos (q)

s= sin (q)

From the above transmission matrix, the stresses in the composite laminate directions can be obtained as follows:

$\sigma_l = c^2\sigma_\theta + s^2\sigma_a$

$\sigma_t = c^2\sigma_a + s^2\sigma_\theta$

$\tau_{lt} = -cs\sigma_h + cs\sigma_a$

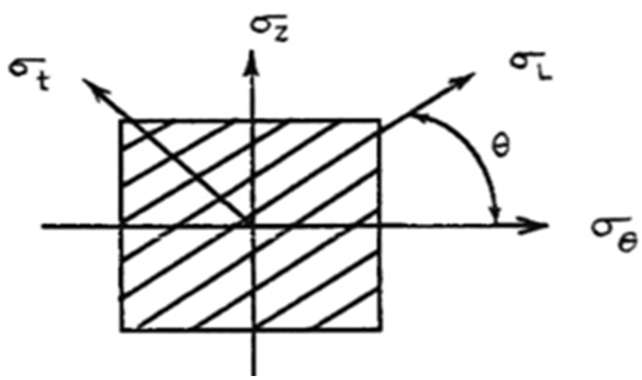


Figure (2) Orthotropic laminate with its material principal directions oriented by an angle q with reference coordinate axes

III. FAILURE ANALYSIS OF COMPOSITE MATERIALS

In composite materials, there are three criteria of failure; the first is the Tsai-Hill criterion, the second is the Tsai-Wu criterion and the third is the maximum stress theory.

The Tsai-Wu Criterion is a quadratic, interactive stress-based criterion that identifies failure, but does not distinguish between different modes of failure.

According to Tsai-Wu criterion, the failure occurs whenever the following condition is satisfied [5].

$$F_1\sigma_{11} + F_2\sigma_{22} + F_{11}\sigma_{11}^2 + F_{22}\sigma_{22}^2 + F_{66}\sigma_{12}^2 + 2F_{12}\sigma_{11}\sigma_{22} \geq 1.0 \qquad (2)$$

The various coefficients F_i and F_{ij} of the Tsai-Wu criterion are defined in terms of known/measured strengths of the composite material (see appendix A).

On other hand, for simplification of the analysis and according to the maximum stress theory [1], failure occurs when any one of the stresses in the principal material directions is equal to or greater than the corresponding ultimate strength. The failure due to σ_t resulted in failure of the matrix itself and the failure due to t_{tt} results in failure in the interface between the fibers and matrix. However, the failure due to σ_l is caused by the fiber fracture and results in complete ultimate failure.

Hence, in composite laminates the failure may be divided into three types: The first type is called first-ply (initial) failure due to the matrix itself failure, the second one is called bonding failure due to the interface failure and the third one is the ultimate failure (final) due to the fibers' breakage [6].

IV. ANALYSIS OF PRESSURE VESSELS MADE FROM DIFFERENT COMPOSITE FIBROUS MATERIALS

In this analysis, the inner radius of the vessel is taken as 100 mm for all the vessels and the applied internal pressure is assumed as 10 MPa. Regarding the external pressure applied on the composite vessels (due to the wound fibers), it is taken as 10% of the applied internal pressure [7]. This overwrapped pressure can be controlled through the winding tensioning force in the fibers during overwrapping.

In order to optimize the design of these pressure vessels, the vessel weight is chosen as the objective to be minimized. The weight is normalized w.r.t. an equivalent metallic one having the same inner radius and made of steel of 200 MPa allowable strength and 7800 kg/m^3 density. The outer radius of the metallic vessel is found as 105.2 mm (by Using lame's equation). By normalizing the composite vessel weight with respect to its equivalent metallic one, the non-dimensional weight of the composite vessel is obtained as follows:

$$\bar{w} = 120.5\,[R_o^2 - (100)^2]\,[\,v_f r_f + (1-v_f)\, r_m]$$

V. OPTIMIZATION PROBLEM FORMULATION

The fiber volume fraction is generally varying between 0.5 and 0.8 in most composite applications [8]. The angle of orientation is taken between 10^o and 80^o to sustain the hoop and longitudinal stresses at the same time. After substitution for stresses from equations (1), the optimization problem can be formulated as follows:

1) Objective Function

Minimize the vessel weight

$$f(Ro, v_f, \theta) = 120.5\,[R_o^2 - (100)^2]\,[\,v_f r_f + (1-v_f)\, r_m]$$

2) Such That

$$\begin{aligned} &\cos(\theta)^2\,\sigma_\theta + \sin(\theta)^2\,\sigma_a \le S_{cl} \\ &\sin(\theta)^2\,\sigma_\theta + \cos(\theta)^2\,\sigma_a \le S_{ct} \\ &-\cos(\theta)\sin(\theta)\sigma_h + \cos(\theta)\sin(\theta)\sigma_a \le S_{cs} \\ &0.5 \le v_f \le 0.8 \\ &10^o \le \theta \le 80^o \end{aligned} \qquad (3)$$

A. *Types of fibrous Materials used in this Study*

Three types of fibrous materials: E-glass fibers, HM-Carbon fibers and 49-kevlar fibers mixed with epoxy resin are used. The mechanical properties of these fibers and resin are given in table (1) [9].

Table (1) Mechanical properties and density of fibers and matrix [9]

Fibrous Material *Mech. Property*	E-glass fibers	HM-carbon fibers	49-kevlar fibers	Epoxy resin
S_{ut} *(MPa)*	1950	2100	1400	70
S_{uc} *(MPa)*	1900	2000	1400	70
E (GPa)	76	400	76	3.5
r (kg/m³)	2540	1900	1400	1210

B. Optimum Design for Composite Pressure Vessels reinforced by GRP

By Substitution of the glass fibers strength=1950 MPa, the epoxy matrix strength=70 MPa and the interface shear strength= 49.1 MPa, the preceding problem formulation can be rewritten as below. Three parameters (R_o, v_f & q) govern that optimum design. That optimization problem is nonlinear optimization problem with three inequality constraints and two bounded constraints.

Objective function:

Minimize

$$f(\mathrm{Ro}, v_f, \theta) = 120.5 * (10)^{-9} \, (R_o^2 - (100)^2) * (1330\, v_f + 1210)$$

Such that:

Inequality constraints

$$g_1 = \frac{8\, R_0^2\, (\cos\theta)^2 + 10(100)^2}{(R_o^2 - (100)^2)} \leq 1880\, v_f + 70$$

$$g_2 = \frac{8\, R_0^2\, (\sin\theta)^2 + 10(100)^2}{(R_o^2 - (100)^2)} \leq 70$$

$$g_3 = \frac{-8\, R_0^2\, (\sin\theta)\,(\cos\theta)}{(R_o^2 - (100)^2)} \leq 49.1$$

Bounded Constraints

$$0.5 \leq v_f \leq 0.8$$

$$10^0 \leq \theta \leq 80^0 \tag{4}$$

C. Optimization Solution Method

Since this problem is nonlinear with inequality and bounded constraints, the **Kuhn-Tucker** technique is reasonable for solving such problems. The necessary conditions for the inequality and bounded constraints problem is written in the standard form as follows [10]:

1) Lagrange Equation

$$L(R_o, v_f, \theta, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7) = 120.5 * (10)^{-9} \, (R_o^2 - (100)^2) * (1330\, v_f + 1210)$$
$$+ \lambda_1 \left[\frac{8\, R_0^2\, (\cos\theta)^2 + 10(100)^2}{(R_o^2 - (100)^2)} - 1880\, v_f - 70\right] + \lambda_2 \left[\frac{8\, R_0^2\, (\sin\theta)^2 + 10(100)^2}{(R_o^2 - (100)^2)} - 70\right] +$$
$$+ \lambda_3 \left[\frac{-8\, R_0^2\, (\sin\theta)\,(\cos\theta)}{(R_o^2 - (100)^2)} - 49.1\right] + \lambda_4 [v_f - 0.8] + \lambda_5 [0.5 - v_f] + \lambda_6 [\theta - 80^o] + \lambda_7 [10^o - \theta] \tag{5}$$

2) Kuhn-Tucker Conditions

$$\frac{\partial L}{\partial R} = 241 * (10)^{-9} (1330 v_f + 1210) R_o + \lambda_1 \left[\frac{-16(100)^2 R_o (\cos\theta)^2 - 20(100)^2 R_o}{(R_o^2 - (100)^2)^2}\right]$$
$$+ \lambda_2 \left[\frac{-16(100)^2 R_o (\sin\theta)^2 - 20(100)^2 R_o}{(R_o^2 - (100)^2)^2}\right] + \lambda_3 \left[\frac{16\,(100)^2 R_o (\sin\theta)\,(\cos\theta)}{(R_o^2 - (100)^2)^2}\right] \tag{c1}$$

$$\frac{\partial L}{\partial v_f} = 160.265 * (10)^{-6} * [R_0^2 - (100)^2] - 1880\, \lambda_1 + \lambda_4 - \lambda_5 = 0 \tag{c2}$$

$$\frac{\partial L}{\partial \theta} = \lambda_1 \left[\frac{(16\, R_0^2\, (\sin\theta)\,(\cos\theta)}{R_o^2 - (100)^2}\right] + \lambda_2 \left[\frac{16\, R_0^2\, (\sin\theta)\,(\cos\theta)}{R_o^2 - (100)^2} + \lambda_3 \left[\frac{8 R_o^2 * [(\cos\theta)^2 - (\sin\theta)^2]\,)}{(R_o^2 - (100)^2)}\right] + \lambda_6 - \lambda_7\right.$$
$$= 0 \tag{c3}$$

$$\lambda_1\left[\frac{8\,R_0^2\ (\cos\theta)^2+10(100)^2}{(R_o^2-(100)^2)}-1880\,v_f-\ 70\right]=0 \quad (c4)$$

$$\lambda_2\left[\frac{8\,R_0^2\ (\sin\theta)^2+10(100)^2}{(R_o^2-(100)^2)}-70\right]=0 \quad (c5)$$

$$\lambda_3\left[\frac{-\,8\,R_0^2\ (\sin\theta)\,(\cos\theta)}{(R_o^2-(100)^2)}-\ 49.1\right]=0 \quad (c6)$$

$$\lambda_4\left[v_f-0.8\right]=0 \quad (c7)$$

$$\lambda_5\left[0.5-v_f\right]=0 \quad (c8)$$

$$\lambda_6\left[\theta-80^o\right]=0 \quad (c9)$$

$$\lambda_7\left[10^o-\theta\right]=0 \quad (c10)$$

$$\frac{8\,R_0^2\ (\cos\theta)^2+10(100)^2}{(R_o^2-(100)^2)}-1880\,v_f-\ 70\ \leq 0 \quad (c11)$$

$$\frac{8\,R_0^2\ (\sin\theta)^2+10(100)^2}{(R_o^2-(100)^2)}-70\leq 0 \quad (c12)$$

$$-\frac{(8\,R_0^2\ +2(10)^5)\,(\sin\theta)\,(\cos\theta)}{(R_o^2-(100)^2)}-49.1\leq 0 \quad (c13)$$

$$v_f-0.8\leq 0 \quad (c14)$$

$$0.5-v_f\leq 0 \quad (c15)$$

$$\theta-80^o\ \leq 0 \quad (c16)$$

$$10^o\ -\theta\leq 0 \quad (c17)$$

$$\lambda_1\,,\lambda_2\,,\ \lambda_3\,,\lambda_4,\lambda_5\,,\lambda_6\,,\lambda_7\ \geq 0 \quad (c18)$$

VI. OPTIMUM DESIGN SOLUTION WITH MATLAB

To solve that optimization problem and get the optimal values for parameters governing the vessel design, the above 18 Kuhn-Tucker conditions must be satisfied.

Moreover, the solution obtained must satisfy the condition of equation (2) to indicate that no-failure occurs according to Tsai-Wu theory.

Due to the difficulty of solving that problem and getting the optimum solution satisfying those inequality constraints, therefore, the toolbox MATLAB will be used to find the optimum solution.

MATLAB Optimization Toolbox uses the Kuhn-Tucker method and other concepts that have been developed over a period of time [11]. The program is used to solve such nonlinear equations obtained as Karush–Kuhn–Tucker (KKT) with optimality conditions for constrained optimization problems, see Appendix (B). It will also help in analyzing the problem formulation if the solution process reports a failure in solving the problem.

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 13 Issue IV Apr 2025- Available at www.ijraset.com

A. Optimum Fiber Orientation

In order to apply the MATLAB program to solve that optimization problem, an initial starting solution point must be given to the program to ensure that the optimization algorithm has been proven to converge to a global minimum. This depends on the lower and upper bounds of the variables and may be done by solving a similar problem with known solutions. The lower and upper bounds of the vessel radius are assumed as 105 to 110 mm, respectively. Whereas, the fiber orientation angle is assumed between 10^o and 80^o. Also, to facilitate the solution, the fiber orientation angle in the composite laminate has been optimized separately to get its optimum value using the following equation (6). This angle is obtained by making the fibers oriented in the direction of the maximum resultant load to take a major part of the stress resulting from this load [2].

Therefore, the fiber orientation angle can be obtained as follows, (refer to figure (2)):

$$\tan^2(\theta) = \frac{\sigma_a}{\sigma_\theta} \qquad (6)$$

By substitution for s_q and s_a from equations (1) one can obtain q as 35^o (approximately) (with hoop direction).

VII. RESULTS AND DISCUSSIONS

Using MATLAB code, for the pressure vessels reinforced by fiber glass, the optimum parameters could be obtained as: ***108.9 mm*** the outer radius and ***0.6*** the fiber volume fraction with ***35º*** for the fiber orientation angle. The non-dimension weight is ***0.42***. The optimization problem: objective function and constraints is represented graphically in figure (3) showing the optimum parameters of R_o and v_f.

Similarly, same results are obtained for both the carbon fibers and Kevlar fibers, i.e. ***108.9 mm*** for the outer radius and ***0.5*** for the fiber volume fraction with ***35º*** for the fiber orientation. Whereas, the non-dimensional weight for carbon fibers and Kevlar fibers was ***0.35*** and ***0.29,*** respectively at ***60%*** fiber volume fraction as shown in figure (4).

The obtained results shows that, the type of fiber doesn't affect the optimum design parameters of the composite vessels but affects only the vessel weight. Where, the vessels made of 49-Kevlar give the lightest weight of all the fibrous composite vessels.

On the other hand, vessels overwrapped by carbon fibers composites can sustain higher stresses than the other two types of fibers, due to their high longitudinal strength, which enable them to carry fluids of high pressures with reasonable weights.

The failure index **F.I.** for those composite vessels has been calculated using equation (2) at different fiber orientation angles and represented graphically in figure (5) for GRP. The results show that no failure can occur for orientation angles less than **38º**. Otherwise, the failure tends to occur at higher angles of orientation. This indicates that orientation angles of **35º** with hoop direction (**55º** with longitudinal direction) maintain the vessel safety against laminate principal stresses. The results are compared with experimental results given in paper [6] and a good agreement is obtained.

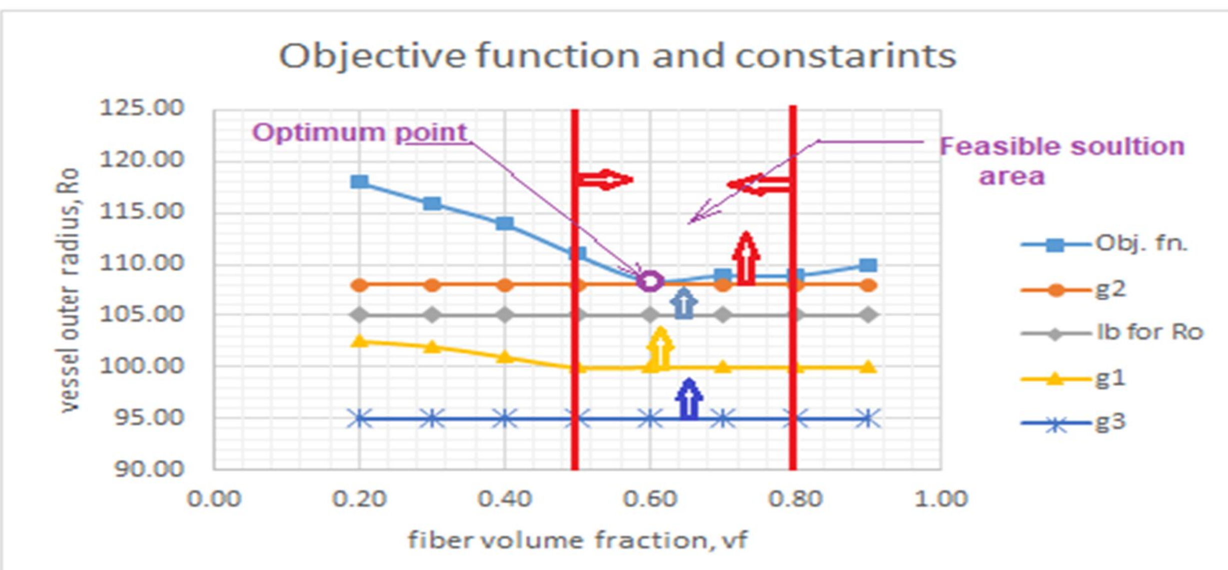


Figure (3) Optimization problem; objective function and constraints

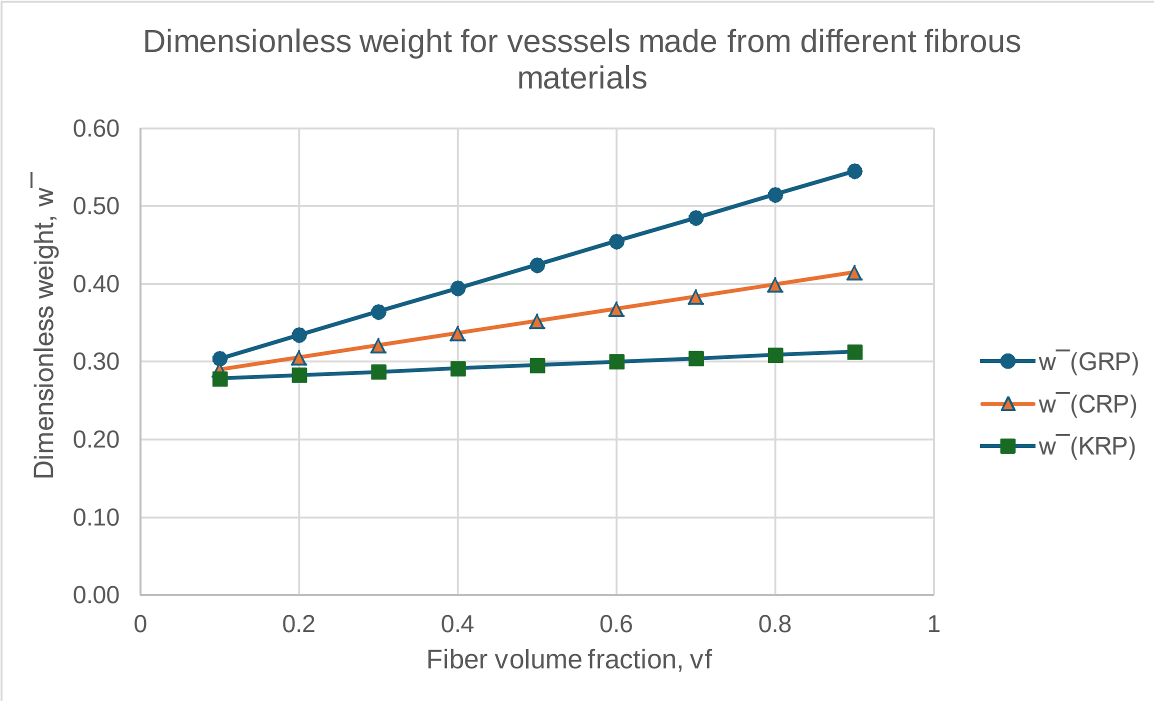


Figure (4) Dimensionless weight for vessels reinforced by different advanced fibers

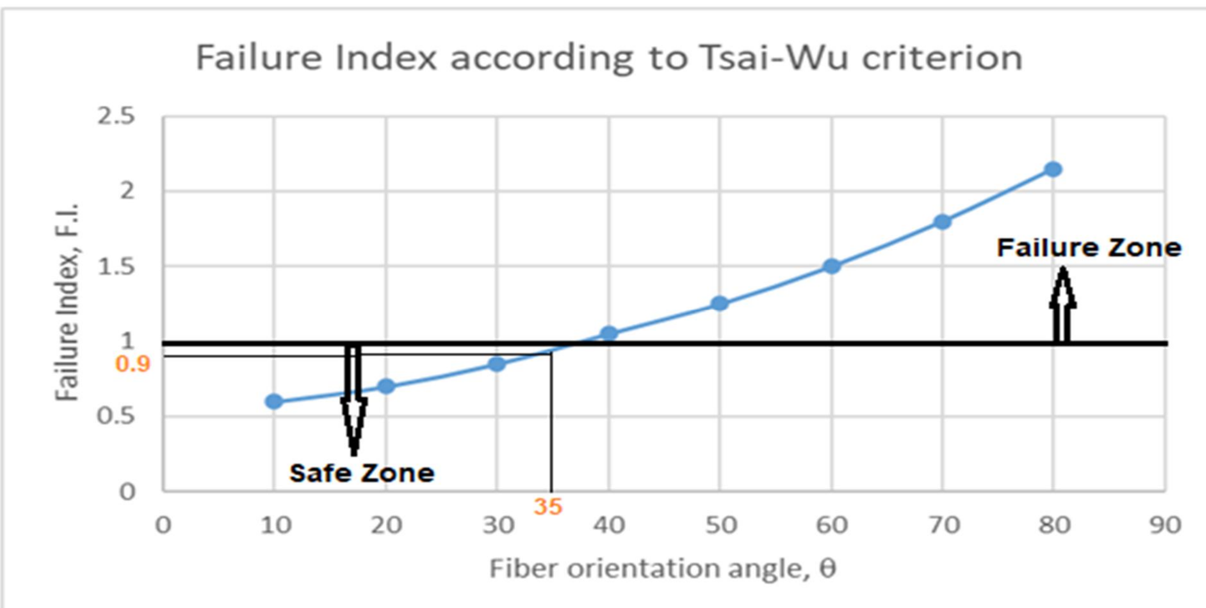


Figure (5) Failure index for GRP composite vessels at different fiber orientation angles

VIII. CONCLUSIONS

Throughout the optimization work performed in this paper, the following conclusions can be withdrawn:

1) Composite pressure vessels can carry fluids with higher internal pressures than steel ones of the same dimensions with light weights.
2) These types of pressure vessels must use an inside liner (polymeric or aluminum) to sustain the radial stresses and thus prevent the fluid leakage through the gaps between fibers.
3) The failure analysis performed in this paper is based on the *Maximum stress theory* and verified by the *Tsai-Wu theory*.

4) Three different advanced fibrous materials; GRP, CRP and KRP mixed with epoxy as matrix are used in the study to reinforce the pressure vessels.
5) Composite pressure vessels overwrapped by Kevlar fibrous material have shown the lowest weight among the three considered fibrous materials (carbon or glass).
6) The failure of the composite pressure vessels passes by three stages:
- The first stage is the interface failure due to the bond failure between the fibers and the matrix, the second failure stage is the matrix failure. These two stages of failure cause the initial failure of vessels.
- The third stage is the failure due to the fiber breakage and leads to final failure of the vessel.
7) The more predominant factor affects the design of these composite vessels is the fiber strength.
8) The optimum winding angle of the fibers around the vessel is 35^o with the circumference direction (i.e., 55^o with the longitudinal direction) for all the types of fibrous materials.
9) The optimum fiber volume fraction is found as 60% for all the types of fibrous materials.
10) For the studied design case, the optimum outer radius of the vessel is found as 10% more than the inside radius, i.e. the wall thickness is around 10% of the vessel radius.
11) Depending on the thickness of the composite layer (which compromise the matrix and the immersed fibers thickness), one can get the optimum number of layers.
12) As a final conclusion, in the studied case, if the thickness of the composite layers is 1 mm, the optimum number of composite layers will be 10 layers with stacking lamination code as: 5 [$\pm 35^o$].

REFERENCES

[1] Agrawal, B. D., "Analysis and Performance of Fiber Composites", John Wiley & Sons Inc., USA, 1990.
[2] Azzam, B. S., M. A. A. Mohammed, M. O. A. Mokhtar, and F. A. Kolkailah, "A Theoretical and Design Analysis of the Filament Wound Composite Pressure Vessels", Journal of Science and Engineering of Composite Materials, Vol. 4, No.2, 1995.
[3] Johns, R. H. and A. Kaufman, "Filament Overwrapped Metallic Cylindrical Pressure Vessels", in AIAA/ASME Seventh Structures and Materials Conference, AIAA, pp. 52-63, 1966.
[4] Shigley, J. E. ,G. Budynas and J. K. Nisbett, "Mechanical Engineering Design", McGraw-Hill Inc., USA, 2020.
[5] Tsai, S. W. and E. M. Wu, "A General Theory of Strength for Anisotropic Materials", Journal of Composite Materials. vol. 5, pp. 58–80, 1971.
[6] Azzam, B. S., M. A. A. Mohammed, M. O. A. Mokhtar, and F. A. Kolkailah, "Analytical, and Experimental Studies of the Filament-Wound Composite Pressure Vessels," ICAC'95, Nottingham, UK, 1995.
[7] Azzam, B. S., M. A. A. Mohammed, M. O. A. Mokhtar, and F. A. Kolkailah, "Optimum Design of the Filament-wound Composite Pressure Vessels", ATMAM '94 in Concordia, Canada, 1994.
[8] Elmoselhy, S. A., B. S. Azzam, and S. M. Metwalli "Theoretical and Numerical Analysis of Laminated Fibrous Composite E-Springs for Vehicles Suspension Systems", Journal of SME-2004, #TP04PUB77, USA, 2004.
[9] Abou-Taleb, A. S., B. S. Azzam, B. El-Hadidi and S. M. Metwalli., "An Optimum Design of A Horizontal Axis Wind Turbine Composite Blades Structure", 2nd Inter. Conference on Renewable Energy: Generation and Applications" ICREGA'12 March 4-7, 2012.
[10] Arora, J. S., "Introduction to Optimum Design", 2nd ed., Elsevier Academic Press, 2010.
[11] Venkataraman, P., "Applied Optimization with MATLAB Programming", John Wiley, New York. 2002.
[12] Vanderplaats, G. N. and N. Yoshida, "1985. Efficient Calculation of Optimum Design Sensitivity", AIAA Jornal, 23 (11), 1798–1803.
[13] Koussios, S., "Design of Cylindrical Composite Pressure Vessels: Integral Optimization", 17th International Conference on Composite Materials - Edinburgh Duration: 27 Jul 2009 → 31 Jul 2009.

APPENDICES

Appendix (A)

Tsai-Wu Failure Criterion

$$F_1\sigma_{11} + F_2\sigma_{22} + F_{11}\sigma_{11}^2 + F_{22}\sigma_{22}^2 + F_{66}\sigma_{12}^2 + 2F_{12}\sigma_{11}\sigma_{22} \geq 1.0 \quad \text{for No-failure}$$

$$F_1 \equiv \frac{1}{S_{11}^+} - \frac{1}{S_{11}^-}$$

$$F_2 \equiv \frac{1}{S_{22}^+} - \frac{1}{S_{22}^-}$$

$$F_{11} \equiv \frac{1}{S_{11}^+ S_{11}^-}$$

$$F_{22} \equiv \frac{1}{S_{22}^{+}S_{22}^{-}}$$

$$F_{66} \equiv \frac{1}{S_{12}S_{12}}$$

S_{11}^{+} = Value of σ_{11} at longitudinal tensile failure

S_{11}^{-} = Value of σ_{11} at longitudinal compressive failure

S_{22}^{+} = Value of σ_{22} at transverse tensile failure

S_{22}^{-} = Value of σ_{22} at transverse compressive failure

S_{12} = Absolute value of σ_{12} at longitudinal shear failure

The interaction coefficient F_{12} can be defined in one of two different ways. If a biaxial failure stress ($\sigma_{11} = \sigma_{22} = \sigma_{biax}$) is used, F_{12} is computed as:

$$F_{12} \equiv \frac{1}{2\sigma_{biax}^{2}}\left[1 - \left(\frac{1}{S_{11}^{+}} + \frac{1}{S_{11}^{+}} + \frac{1}{S_{22}^{+}} + \frac{1}{S_{22}^{+}}\right)\sigma_{biax} + \left(\frac{1}{S_{11}^{+}S_{11}^{+}} + \frac{1}{S_{22}^{+}S_{22}^{+}}\right)\sigma_{biax}^{2}\right]$$

Otherwise, the interaction coefficient F_{12} is computed as

$$F_{12} = f^{*}\sqrt{F_{11}F_{22}}$$

where f* is a user-specified constant, $-0.5 \le f^{*} \le 0$. If the Tsai-Wu criterion is selected for analysis, you must specify the coefficient f*.

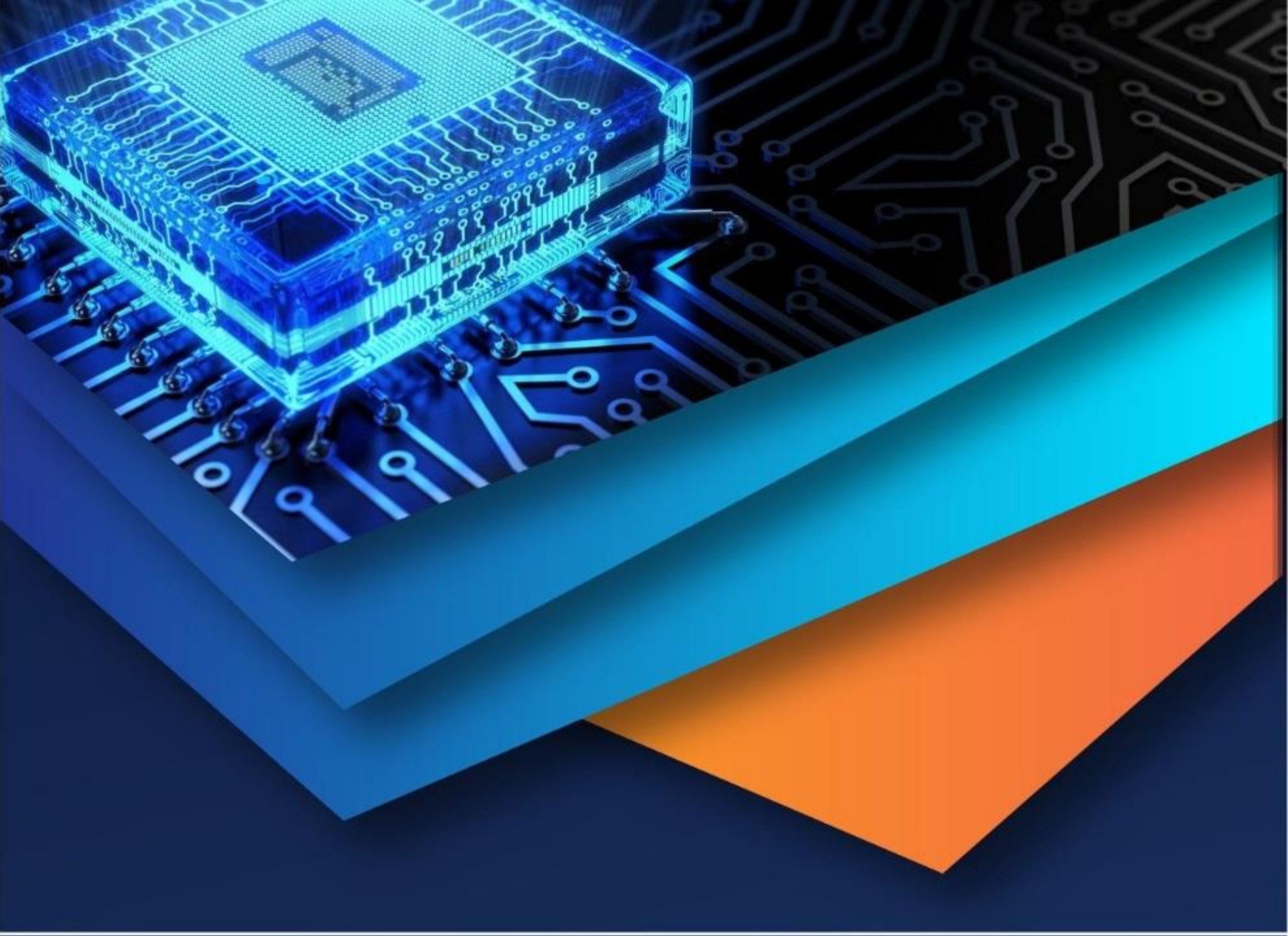
APPENDIX (B)

MATLAB code

```
function f=objective(x)
f=120.5*(10)^(-9)*(x(1)^2-10000)*(1330*x(2)+1210);
end
function [c,ceq]=constraints(x)
c=[(8*(x(1)^2)*(0.67)+100000)*((x(1)^2-10000)^(-1))-1880*x(2)-70;(8*(x(1)^2)*(0.32)+100000)*((x(1)^2-10000)^(-1))-70;-
(x(2)^2)*0.47*((x(1)^2-10000)^(-1))-49.1];
ceq=[];
end
lb= [102,0.5];
ub= [110,0.8];
x0=[102 0];
[x,fval,exitflag]=fmincon(@objective,x0,[],[],[],[],lb,ub,(@constraints));
```

MATLAB Results

A	[ ]	0×0	double
Aeq	[ ]	0×0	double
b	[ ]	0×0	double
beq	[ ]	0×0	double
exitflag	1	1×1	double
fval	*0.2971*	1×1	double
lb	[102,0.50]	1×2	double
options	1×1 Fmincon	1×1	optim.options.Fmincon
ub	[110,0.8000]	1×2	double
x	[*108.9146,0.6000*]	1×2	double
x0	[102,0]	1×2	double



10.22214/IJRASET

INDEX COPERNICUS

45.98

IMPACT FACTOR:
7.129

TOGETHER WE REACH THE GOAL

IMPACT FACTOR:
7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY