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A Detailed Study of the Effect of Radii Ratio and Aspect Ratio of a Uniform Circular Annular Plate under Free and Forced Vibration in Thermal Environment

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Abstract: This paper deals with the free and forced vibration of annular circular plate under thermal environment using finite element analysis (FEA). The temperature is kept constant. The effect of Eigen frequency on radii ratio and aspect ratio is investigated for different annular circular plate under thermal environment. The free-free and clamped-free boundary condition is taken to analysis theses plates. The parametric studies are conducted and the results are analyzed and marked off.
Keywords: Circular Annular Plate, Concentric Plate, Aspect Ratio, Radii Ratio, Finite element modelling.

I. INTRODUCTION

Circular annular plates have many engineering applications. They are used in many structural components, i.e., building design, diaphragms and deck plates in launch vehicles, diaphragms of turbines, aircraft, and missiles, naval structures, nuclear reactors, construction of ships, automobiles, and the space shuttle, etc. Vibration characteristic circular annular plate has not been adequately examined though there is a substantial body of literature on the structural dynamics of circular annular plate under thermal environment. Limited vibration studies have considered either flexural vibration modes or rigid body piston motions of circular annular plate under thermal environment. In this article, the vibration response from a circular annular plate under thermal environment is proposed is proposed. In particular, the vibration behavior of circular annular plate from out-of-plane flexural modes is evaluated. For a complete investigation of the vibration characteristic of a circular annular plate under thermal environment, it is necessary to simultaneously consider both in-plane and out-of-plane vibrations. But the current analysis focuses only on the out-of-plane mode vibration of the plates. The vibration response of circular or annular plate is investigated by most of the researchers. The literature survey also suggested that the vibration response of circular annular plate under thermal environment has not been explored in depth. Hence, we find a research gap for vibroacoustic behavior of the uniform circular annular plate. The literature is purely based on the flexural response due to out-of-plane modes of different circular annular plates, which can highly influence the structural design. Several theories have been proposed by many researchers for vibration response of circular annular plate for its out of –plane modes. Abbasi et al. [2014] used the differential transforms method (DTM) exposed to non-uniform axisymmetric transverse loading to determine the static analysis of a circular plate resting on a Winkler elastic foundation. Abolghasemi et al. [2017] found the axisymmetric buckling of an annular plate using an asymptotic approximation technique based on the perturbation technique. Ansari [2016] used the Rayleigh-Ritz method based on classical plate theory to determine the forced reaction of polar orthotropic tapered circular plates resting on an elastic foundation. Askari et al. [2020] used closed-form geometric equations and the chain rule to analyze the eccentric annular plates for their free vibration as a novel mathematical method. Askari et al. [2020] used closed-form geometric equations, the chain rule, and the Rayleigh-Ritz method to estimate the characteristic vibration properties of the fluid-coupled plate structure. Bacciocchi et al. [2016] used the generalized differential quadrature method to investigate the vibration analysis of variable thickness plates and shells. Bahrami et al. [2015] analyzed the wave propagation in annular sector plates using the spectral strip method based on modified Bessel's equations. Barakat and Baumann [1968] determined the axisymmetric vibrations using the Ritz-Galerkin method for a thin circular plate with parabolic thickness variation. Bhardwaj and Lokendra [2020] applied the boundary characteristic orthonormal polynomials based on the Rayleigh-Ritz method to study the asymmetric vibration of polar orthotropic annular plates. Bhatnagar et al. [2019] investigated the comparative study for modal investigation of circular plates with different cutouts and end conditions by applying FEM. Bochkarev et al. [2020] did an experimental analysis and used FEM to investigate the natural and harmonic vibrations of plates interacting with air and fluid.

Chen et al. [2020] used the higher-order equations of vibration based on Mindlin plate theory to investigate the high-frequency vibrations of circular and annular plates. Chen et al. [2018] used FEM to determine the dynamic comparison of a rotating flexible annular disc under different edge boundary conditions. Chen [1997] used the finite element procedure to investigate the axisymmetric flexural response of circular annular plates having thickness variations arbitrarily. Dozio [2015] applied the unified Ritz formulation, numerical assessment and refined 2D theories to determine the free vibration response of annular plates. Duana et al. [2005] used the hypergeometric function to determine the transverse vibration of non-uniform annular plates. Duana et al. [2008] applied Mindlin plate theory to calculate the flexural modes of circular plates with a free edge. Filippov [2017] investigated the asymptotic and numerical analysis of free low-frequency ring-stiffened shell vibrations based on linear differential equations. Gorman [1993] applied the finite element method to investigate the natural frequencies of transverse vibration of polar orthotropic variable thickness annular plates.

The Chief objectives of this article are to investigate the influence of Eigen frequency on radii ratio and aspect ratio under thermal environment for different circular annular plates. The results are analyzed and marked off.

II. EQUATION OF PLATE MODELING

A. Equation of free vibration

The natural frequency along with the mode shape of the plate during modal analysis is obtained as

$$\left([K]-\omega^2[M]\right)\psi_{mn}=0 \quad (1)$$

In the above formulation, $[M]$ is said to be mass matrix and $[K]$ is said to be the stiffness matrix. The mode shape is represented by ψ_{mn} and the corresponding natural frequency of the plate is represented by ω denoted as rad/sec. Further, (λ^2) known as the non-dimensional frequency parameter which is obtained as

$$\lambda^2=\omega a^2\sqrt{\frac{\rho h}{D}} \quad (2)$$

Where D is said to be the flexure rigidity $=\frac{Eh^3}{12(1-\upsilon^2)}$, 'a' is said to be the outer radius, 'E' is said to be the Young's modulus of elasticity, 'υ' is said to be Poisson's ratio, 'h' is said to be the thickness of the plate and 'ρ is said to be the density of plate.

B. Geometric Modelling of Plate

The vibration characteristics of the plate are analyzed by applying the (FEA) Finite Element analysis. In ANSYS, the plate is modeled using Plane 185, which incorporates eight brick nodes, each with three degrees of freedom. The results that obtained numerically from the FEA are correlated through findings from published literature. The mass and total volume of the plate kept constant. The thickness is assumed to be uniform. In this research, the annular plate with outer radius = 151.5 mm, inner radius = 82.5 mm and thickness = 31.5 mm are taken for analysis. The different boundary conditions like free-free and clamped–free are used to analyze the plate. The out-of-plane structural modes are considered. The temperature is taken constant to create the thermal environment. The specification and other material property of the stiffener plate is reported in Table 1. The reference and coordinate systems of the plate are reported in Figure 1.

TABLE I
SPECIFICATIONS AND THE MATERIAL PROPERTIES OF AN ANNULAR CIRCULAR STIFFENER PLATE WITH UNIFORM THICKNESS

Dimension of the plate	Values
Outer radius (a) m	0.1515
Inner radius (b) m	0.0825
Radii ratio, (b/a)	0.54
Thickness ratio, (h/a),	0.21
Density, ρ (kg/m^3)	7905.9
Young's modulus, E (GPa)	218
Poisson's ratio, υ	0.305
Temperature	273K

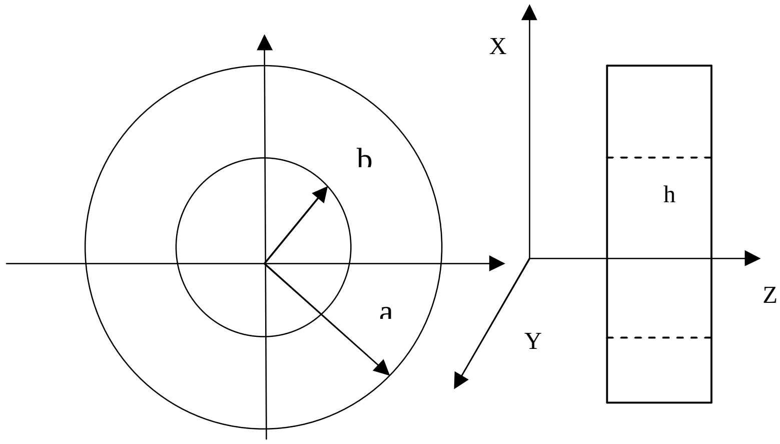

Fig.1 Coordinate and geometrical shape of annular circular plate with uniform thickness

III.RESULTS AND DISCUSSION

A. Effects of Plate Geometry on Natural Frequency

The effect of natural frequency (ω) on flexural response due to out of plane modes for plate is discussed in this section. The mass and volume of the plate is constant. Table II and Table III compared the first four natural frequency of annular plate with free-free and clamped-free boundary conditions. It is investigated from Table II and Table III that the natural frequency increases with higher model indices. This is because increases of stiffness with higher modes.

B. Vibrational Response under Free-Free and Clamped-Free Boundary Condition

The effect of natural frequency (ω) on flexural response due to out of plane modes for plate is discussed in this section. The mass and volume of the plate is constant. Table II and Table III shows the first four natural frequency of annular plate with free-free and clamped-free boundary conditions. It is also investigated from Table II and Table III that the natural frequency increases with higher model indices. This is because increases of stiffness with higher modes. It is also investigated that in compare to free-free boundary condition, clamped free vibration have higher frequency to vibrate the plate. This is because in clamped free boundary condition all the modes have higher stiffness in comparison to all the modes of free-free boundary condition.

TABLE II
VIBRATIONAL RESPONSE OF A UNIFORM CIRCULAR ANNULAR PLATE UNDER THERMAL ENVIRONMENT (T=273K) UNDER FREE-FREE BOUNDARY CONDITION.

BCS	Mode	Natural frequency, (ω)
F-F	0,2	1999.9
	0,1	5122.3
	0,3	5481.7
	1,1	7629.5
	0,4	9608.8

TABLE III
VIBRATIONAL RESPONSE OF A UNIFORM CIRCULAR ANNULAR PLATE UNDER THERMAL ENVIRONMENT (T=273K) UNDER CLAMPED-FREE BOUNDARY CONDITION.

BCS	Mode	Natural frequency, (ω)
C-F	0,0	2441.1
	0,1	2386.1
	0,2	3614.6
	0,3	6471.8
	0,4	10331.0

C. *Effect of Radii Ratio and Aspect Ratio*

The effect of natural frequency on vibration response of uniform plate due to different radii ratio with different boundary conditions as shown in this section. The mass and volume of the plate is kept constant. Table IV and Table V show the effect of different radii ratio on first four natural frequency of the uniform annular circular plate without stiffener with free-free and clamped-free boundary conditions. The mode shape for different radii ratio increases because the stiffness of the plate increases with increasing radii ratio. As the radii ratio increases form β = 0.1 to 0.6 the stiffness of the plate increases for different mode. So, we get higher frequency for the plate to vibrate. For Radii ratio 0.6 at (0, 1) mode there is a slight increase in natural frequency. This is due to the increases of the stiffness when we go from higher mode from (0, 2) to (0, 1). Further we see that for all the modes with different radii ratio there is a no any abrupt change in natural frequencies.

TABLE IV
EFFECT OF NATURAL FREQUENCIES OF DIFFERENT RADII RATIO (Β) ON NODES FOR DIFFERENT BOUNDARY CONDITION FOR CIRCULAR ANNULAR UNIFORM PLATE WITHOUT STIFFENER FOR FREE-FREE BOUNDARY CONDITION UNDER CONSTANT TEMPERATURE T=273K

BCS	Mode	Radii Ratio					
		β=0.1	β=0.2	β=0.3	β=0.4	β=0.5	β=0.6
F-F	(0,2)	2335.3	2263.3	2198.2	2162.7	1967.9	1817.8
	(0,1)	3601	3530	3656.7	4112.4	4394.6	5149.4
	(0,3)	5561.6	5472.8	5466.7	5608.1	5228.3	4890
	(1,1)	7510.4	7191.5	6826.1	6836.4	6856.8	7565.2
	(0,4)	9397.6	9260.9	9395.5	9663.5	9222	8786.4

TABLE V
EFFECT OF NATURAL FREQUENCIES OF DIFFERENT RADII RATIO (Β) ON NODES FOR DIFFERENT BOUNDARY CONDITION FOR CIRCULAR ANNULAR UNIFORM PLATE WITHOUT STIFFENER FOR CLAMPED-FREE BOUNDARY CONDITION UNDER CONSTANT TEMPERATURE T=273K

BCS	Mode	Radii Ratio					
		β=0.1	β=0.2	β=0.3	β=0.4	β=0.5	β=0.6
C-F	(0,0)	982.1	1621.1	2374.4	3433.1	4845.5	7364.5
	(0,1)	1519.3	1930	2581.9	3624.6	4968.6	7409.6
	(0,2)	2404.2	2598.7	2912.2	4158.1	5733.5	8403.7
	(0,3)	5564.6	5516.6	5760.3	6508.1	7385.8	9814.9
	(0,4)	9407.5	9264.9	9476.9	10079.3	10459	12364

Further from the Figures 2 and 3, we see that when we increasing the radii ratio the natural frequency increases for different radii ratio because as we go from lower frequency to higher frequency the stiffness of the plate increases. So, we get higher frequency to vibrate the plate. In compare to free-free boundary condition it is seen that for clamped free vibration there is a higher frequency in compare to free-free boundary condition. Because in clamped free boundary condition all the modes have higher stiffness in comparison to all the modes of free-free boundary condition. Form the Figure 2, it is seen that on increasing the radii ratio there is a stiff rise of natural frequencies from 0.1 to 0.4 modes. Because the clamped free vibration has higher stiffness in comparison to free-free boundary condition.

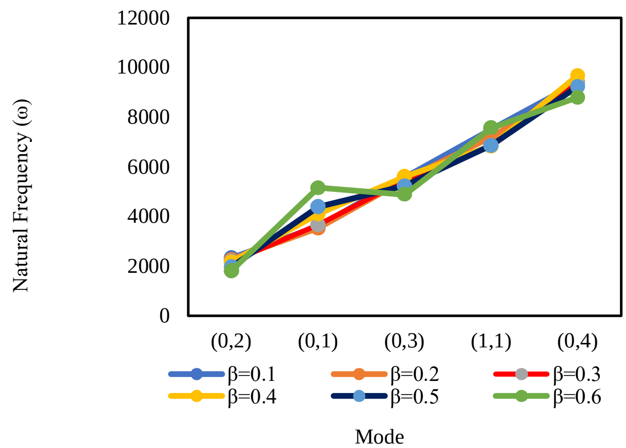

Fig. 2 Comparison of natural frequencies with different modes of a circular annular uniform plate having different radii ratio in free-free boundary condition under constant temperature T=273K

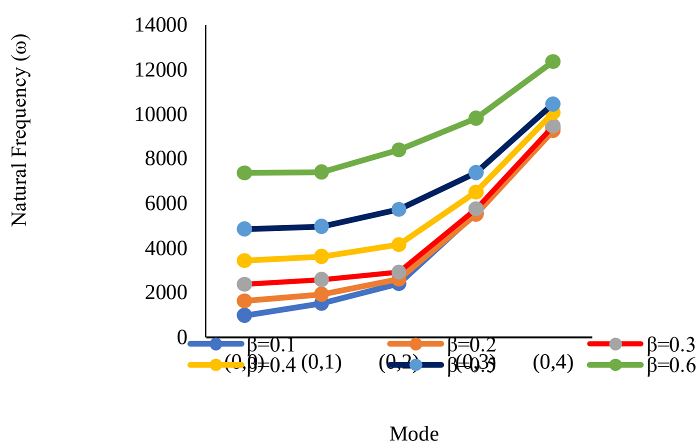

Fig 3 Comparison of natural frequencies with different modes of a circular annular uniform plate having different radii ratio without stiffener in clamped-free boundary condition under constant temperature T=273K

The effect of natural frequency on vibration response of uniform plate due to different aspect ratio with different boundary conditions is shown in this section. The mass and volume of the plate is kept constant. Table 6 and Table 7 show the effect of different aspect ratio on first four natural frequency of the uniform annular circular plate with free-free and clamped-free boundary condition.

TABLE VI
EFFECT OF NATURAL FREQUENCIES OF DIFFERENT ASPECT RATIO (H/A) ON NODES FOR DIFFERENT BOUNDARY CONDITION FOR CIRCULAR ANNULAR UNIFORM PLATE WITHOUT STIFFENER FOR FREE-FREE BOUNDARY CONDITION UNDER CONSTANT TEMPERATURE T=273K

BCS	Mode	Aspect Ratio					
		(h/a)=0.1	(h/a)=0.2	(h/a)=0.3	(h/a)=0.4	(h/a)=0.5	(h/a)=0.6
F-F	(0,2)	1177.3	1861.9	2301.3	2748.4	3149.3	3335.5
	(0,1)	3278.9	4725.9	5151.5	5894.2	6487.7	6670
	(0,3)	3452	5017	5854.8	6760.1	7557.4	7969.4
	(1,1)	5157.2	7109.1	7648.3	8355.7	8659	8312.3
	(0,4)	6452.9	8988.4	10235	11425	12481	13042

TABLE VII
EFFECT OF NATURAL FREQUENCIES OF DIFFERENT ASPECT RATIO (H/A) ON NODES FOR DIFFERENT BOUNDARY CONDITION FOR CIRCULAR ANNULAR UNIFORM PLATE WITHOUT STIFFENER FOR CLAMPED-FREE BOUNDARY CONDITION UNDER CONSTANT TEMPERATURE T=273K

BCS	Mode	Aspect Ratio					
		(h/a)=0.1	(h/a)=0.2	(h/a)=0.3	(h/a)=0.4	(h/a)=0.5	(h/a)=0.6
C-F	(0,0)	3983	5649.2	7042.6	8042.1	8263.3	8190
	(0,1)	4008.7	5750.3	7105.5	8119.6	8438.8	8286.6
	(0,2)	4846.2	6442.9	7754.8	8801.5	8967.7	8905.6
	(0,3)	6067.2	8100	9363.2	10659	11273	11272.4
	(0,4)	7932.2	10926	12490	13918	14873	15045

From the Table VI and Table VII, it is seen that on increasing the aspect ratio for different modes, the frequency increases. When we increase the aspect ratio from 0.1 to 0.6 there is an increase in the natural frequency. Also, from the table we see that we increase the mode form (0, 2) to (0, 4) also we see the increase in natural frequency there. This is due to the fact that when the aspect ratio increases the stiffness of the plate increases because the thickness is increasing. So, the higher level of frequency is required to vibrate the plate at higher aspect ratio.

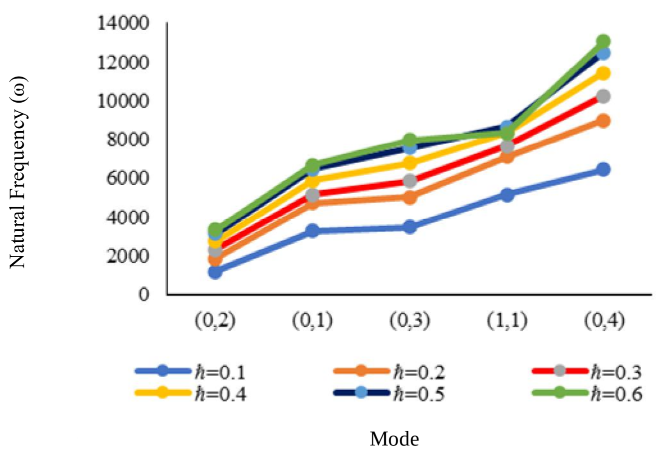

Fig. 4 Comparison of natural frequencies with different modes of a circular annular uniform plate having different aspect ratio without stiffener in free-free boundary

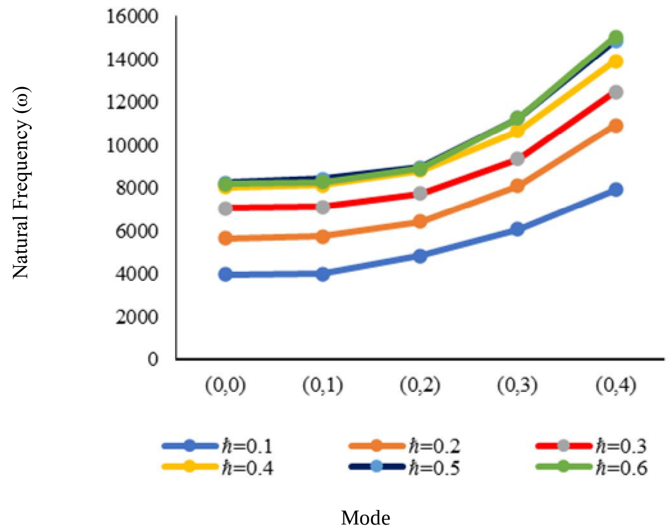

Fig. 5 Comparison of natural frequencies with different modes of a circular annular uniform plate having different aspect ratio without stiffener in clamped-free boundary condition under constant temperature T=273K

From the Figure 4 and Figure 5, we see that for the aspect ratio 0.5 we get the lower frequencies for different mode. However, for different aspect ratio we see the higher natural frequencies in respect to 0.5. From mode shape (0, 2) there is a stiff rise in the natural frequency due to the increase of the stiffness of the at (0, 1) mode and then the mode shape is clearly thin to be constant form (0, 2) to (1, 1) because here we find that the increase in frequency does not have any effect on the aspect ratio. However, form (1, 1) to (0, 1) there is a stiff rise in the natural frequencies due to the stiffness of the plate increase. In compare to free-free boundary condition it is seen that for clamped free vibration there is a higher frequency in compare to free-free boundary condition. Because in clamped free boundary condition all the modes have higher stiffness in comparison to all the modes of free-free boundary condition. Form the Figure 5, it is seen that on increasing the aspect ratio there is a stiff rise of natural frequencies from 0.2 to 0.4 modes. Because the clamped free vibration has higher stiffness in comparison to free-free boundary condition.

IV. VALIDATION OF STUDY

In this paper the natural frequency parameter λ2 of a uniform annular circular plate without stiffener is validated with the result of Lee &Singh [2005]. It is clear form Table VIII and Table IX that the result obtained is similar of the published result.

TABLE IX

COMPARISON AND VALIDATION OF NATURAL FREQUENCY (Ω) OF A UNIFORM ANNULAR CIRCULAR PLATE WITH FREE-FREE BOUNDARY CONDITION OBTAINED IN THE PRESENT WORK WITH THAT OF LEE AND SINGH [2005]

Plate	Mode	Natural frequency, (ω)	
		Lee and Singh [2005]	Present work
Uniform plate b/a = 0.54 h/a = 0.21	(0,2)	3.82	3.83
	(1,0)	8.85	8.82
	(0,3)	10.59	10.02
	(1,1)	15.42	13.70

TABLE X
COMPARISON AND VALIDATION OF NATURAL FREQUENCY (Ω) OF A UNIFORM ANNULAR CIRCULAR PLATE WITH CLAMPED–FREE BOUNDARY CONDITION OBTAINED IN THE PRESENT WORK WITH THAT OF LEE AND SINGH [2005]

Plate	Mode	Natural frequency, (ω)	
		Lee and Singh [2005]	Present work
Uniform plate b/a = 0.54 h/a = 0.21	(0,0)	13.61	13.49
	(0,1)	13.43	13.50
	(0,2)	15.28	14.12
	(0,3)	16.81	16.67

V. CONCLUSIONS

In this chapter the free and forced vibration from out-of-plane modes of annular circular plate with different boundary conditions are analyzed. The result of this chapter matches well with the published results. The natural frequency of the uniform circular plate with free-free and clamped-free boundary condition increases with higher modal indices. With an increasing radii ratio for free-free and clamped-free boundary condition the natural frequency increases with different mode. It is also investigated that with an increasing aspect ratio for free-free and clamped-free boundary condition the natural frequency increases with different mode.

VI. ACKNOWLEDGMENT

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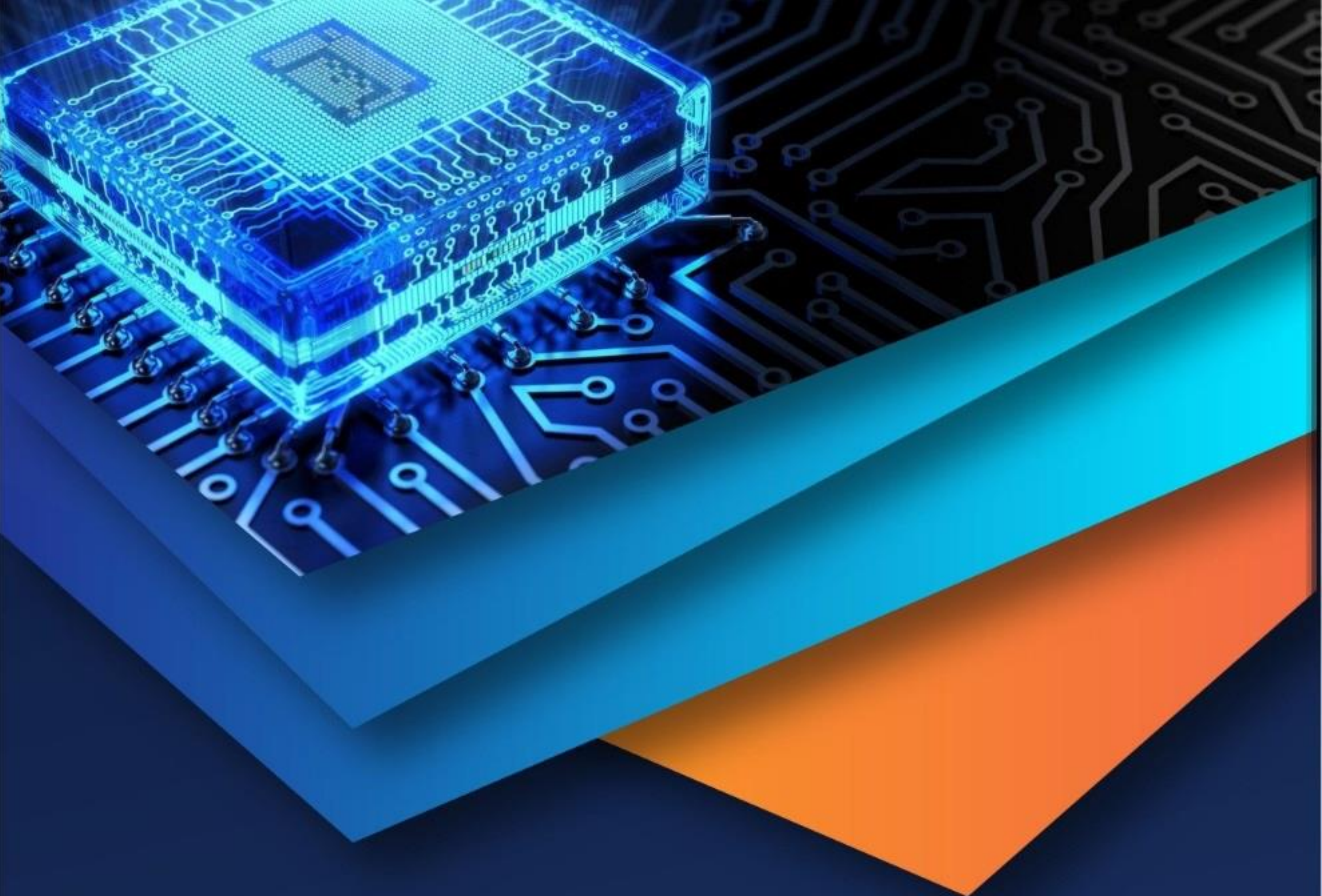

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