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A Fuzzy AHP-Based Analysis of Context-Dependent t-Norms in Adaptive Fuzzy Automata for Uncertainty-Driven Decision-Making Systems

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Abstract: Adaptive fuzzy automata allow us to model sequential processes through a formally formalised mechanism wherein the state transitions are realised as are the recognition outcomes and terminal decisions not by crisp values but various graded values. Selecting an aggregation law for uncertain evidence remains a consistent challenge for modellers. Traditional fuzzy automata typically use a constant t-norm, which reflects either minimum or product operation. Thus, it propagates uncertainty according to one invariant semantics of conjunction. In this paper, we present a framework based on fuzzy analytic hierarchy process (fuzzy AHP) for context-dependent t-norm selection relevant to adaptive fuzzy automata. The presented model, on behalf of t-norms, represents a multi-criteria decision problem whose attributes are robustness, continuity, interpretability, sensitivity to weak evidence, computational speed (complexity), being fit for high-stakes use, and adaptability. This paper defined a formal context-dependent fuzzy automaton and then proposed a Fuzzy AHP procedure through triangular fuzzy numbers, geometric-mean fuzzy weights, defuzzification, consistency checking, and final t-norm ordering. A computational automaton is illustrated showing how the accepted grade of an input word is changed by different t-norms. A context-sensitive selector can justify switching among Godel/minimum, product, Lukasiewicz, Hamacher, and Schweizer-Sklar t-norms. Papers provide a clear-cut mathematical bridge between fuzzy automata theory and multi-criteria decision-making for systems that are driven by uncertainty.
Keywords: Fuzzy AHP; Context-Dependent t-Norms; Adaptive Fuzzy Automata; Uncertainty-Driven Decision-Making; Fuzzy Aggregation; Multi-Criteria Decision-Making; Computational Intelligence.

I. INTRODUCTION

A. Fuzzy Automata and Graded Recognition

In fuzzy automata, a transition, initial, final and recognized language may have grades of membership in the unit interval, instead of taking binary truth values, which are true or false. The automaton is appropriate for reasoning tasks over sequences of incompletely, vaguely, noisily, or linguistically expressed information. The basic idea is taken from fuzzy set theory, in which class membership is represented by degrees rather than sharp inclusion or exclusion (Zadeh, 1965). Once this graded view is incorporated into automata theory, a word being recognized is no longer "accept" or "reject" but rather an acceptance grade which is obtained by aggregating the degrees of possible transitions to states and the terminal membership values (Wee & Fu, 1969; Mordeson & Malik, 2002).

B. Role of t-Norm Aggregation

The aggregation rule is primary, not a technical choice. The grade of a path in a fuzzy automaton is a function of the transitions which are included in the path. The Godel or minimum t-norm interprets a chain of evidence through its weakest component, the product t-norm resembles multiplicative combination of independent confidence values, and the Lukasiewicz t-norm imposes a threshold-sensitive form of joint satisfaction. More general t-norm families, such as Hamacher and Schweizer-Sklar families, show parameterized aggregation behaviour. According to the literature on triangular norms, these operators are variously continuous, nilpotent, strict, Archimedean, compensative (Klement et al, 2000; Schweizer & Sklar, 1960). A fuzzy automaton that uses a different t-norm is not simply using a different computational formula; it is using a different logic of reachability.

C. Limitations of Fixed t-Norm Computation
In classical fuzzy automata, computations are assumed to be done with a constant t-norm. While being mathematically convenient, this assumption is justified due to the associativity of composition that renders path aggregation independent of bracketing and makes the studying of recognized fuzzy languages simpler. Variable interpretations of conjunction are common among adaptive decision systems. When the risk level is high, a clinical triage system may need a conservative conjunction, while multiplicative aggregation is necessary when symptoms are statistically independent. Under conditions like malicious activity or low reliability, sensor fusion could switch between resembling the product of measure like combinations to minimum of those combinations. A smart controller might appreciate computational efficiency under normal operating circumstances, while valuing robustness and continuity during the instability. A constant t-norm is unable to express these varying semantics without an exogenous redefinition of the model.

D. Fuzzy AHP-Based Selection Rationale
Given the multi-dimensions and complexity of the situation, a well-structured decision support system cannot be random-choiced or ad hoc. The Analytic Hierarchy Process is a widely used method for deriving priority scales from pairwise judgments (Saaty 1977, 1980, 2008). Fuzzy AHP enhances the rationale of AHP by enabling the representation of assessments through fuzzy symbols in the form of fuzzy numbers. Fuzzy numbers can effectively capture the vagueness that originates from expert evaluation (Van Laarhoven & Pedrycz, 1983; Buckley, 1985; Chang, 1996). Given the circumstances, no single criterion may be found which uniquely defines the best aggregation operator. While robustness, continuity, interpretability, sensitivity to weak evidence, efficiency, suitability for high-risk environments, and adaptability are all relevant, their importance varies less markedly with decision context.

E. Research Gap and Objectives
The primary assertion of this research paper postulates that a context-dependent fuzzy automaton must not switch its aggregation operation. A mathematically defensible adaptive automaton should have a definite specification stating how to observe the context, how to evaluate candidate t-norms, and how the selected t-norm will be involved in the composition of the transition. Using expert judgments for t-norm decision making can be done with fuzzy AHP, as it makes the expert judgment transparent by converting qualitative judgments into normalized priority weights and allowing you to rank candidate t-norms under an explicit criteria hierarchy. The outcome is an automaton whose context-sensitivity is decision-justified rather than merely heuristic.
The study has four objectives to achieve. It first introduces a formal framework for adaptive fuzzy automata that has a context-indexed t-norm selection mechanism. In the second section, there is a Fuzzy AHP model to rank five candidate t-norms against. The illustration provides a computational example showing how an input word on a specified plant obtains different acceptance grades. The fourth part speaks of the theoretical and practical implications of establishing a link between fuzzy automata and decision-making in the presence of uncertainty in multi-criteria decision systems. Central motive is conceptual and mathematical. All numerical matrices and transition grades which are used in the Fuzzy AHP and in the example of automata, are marked as computational illustrative data, and not empirical data.

II. REVIEW OF LITERATURE

A. Fuzzy Sets, Fuzzy Relations, and Fuzzy Topological Reasoning
The current study's starting point is the theory of fuzzy sets that substituted sharp membership for a graded membership function from a universe into the unit interval (Zadeh, 1965). The idea was extended to L-fuzzy sets with degrees taking values in more general lattices, by Goguen (1967). These foundations make possible to regard vagueness as a mathematical object having internal order, algebra and topology rather than a mere statistical error. Fuzzy openness, fuzzy neighbourhoods, compactness, and convergence in fuzzy topology can be formally studied through the families of fuzzy subsets that are collectively closed (or open) (Chang 1968, Lowen 1976, Pu & Liu 1980).
The evolution of states cannot be reduced to a mere numerical recursion, which makes fuzzy topological reasoning relevant for adaptive automata. Upon updating a state distribution, a fuzzy automaton modifies the graded observability of properties allocated to states. Instability occurs in decision systems when minor perturbation to either context or transition reliability produces a discontinuous change in acceptance grade. Continuity and convergence are not mere ornamental concepts; they are essential to understanding why an adaptive fuzzy computation continues to be reliable for small changes in evidence or operating regime (George & Veeramani, 1994).

B. Fuzzy Automata and Sequential Uncertainty

Fuzzy automata were proposed to denote finite-state systems whose transition degrees and recognition outcomes are not restricted to crisp values. The early work of Santos (1968) and of Wee and Fu (1969) relates automata-theoretic structure to max-min composition and learning models. Subsequent interventions provided a more methodical explanation of fuzzy automata, fuzzy languages, reduction, determinism as well as algebraic representation (Mordeson & Malik, 2002). Bělohlávek (2002) and Doostfatemeh and Kremer (2005) proposed using fuzzy automata for determining deterministic behavior. Doostfatemeh and Kremer (2005) argued that the current definition of fuzzy automata is not general enough to allow proper assignment and updating of the membership value of the state.

A major mathematical direction in fuzzy automata theory studies membership values in complete residuated lattices or lattice-ordered monoids. The fuzzy finite automata and fuzzy regular expressions were linked by Li and Pedrycz (2005) via lattice-ordered monoid values and determinization methods of fuzzy automata was developed over complete residuated lattices in Ignjatović et al. (2008). The algebraic properties of L-fuzzy finite automata have been considered by Jin et al. The automaton is directly influenced by the algebra of truth values in these works. The current article is based on the above insight but focuses on a different modelling issue. Thus, the aggregation algebra may need to vary across contexts, and we should justify the choice of local algebra with a decision procedure.

C. Triangular Norms as Fuzzy-Conjunction Semantics

Triangular norms are operations that are performed on the closed interval between 0 and 1. Here, the closed interval refers to the range of 0 and 1 together with all the values lying between. They satisfy laws such as associativity and identity law. The reason for not selecting a t-norm is because of computational convenience. The function is key to determining how evidence accumulates along some path, whether a weak evidence is able to dominate the result, whether aggregation is strict or nilpotent, whether small perturbations in the input grades lead to stable changes in the output grades and so on.

The Godel, or minimum t-norm, T_G(x,y)=min(x,y) allows for the strongest possible cautious interpretation because the aggregate cannot be stronger than the weakest. When grades are seen as independent confidence-like quantities, the product t-norm is natural. The Lukasiewicz t-norm, T_L(x,y)=max(0,x+y-1), returns zero whenever the combined support is too weak to yield a positive conclusion. Hamacher and Schweizer-Sklar families allow for parameterized adjustment within conjunctive regimes. The distinct semantics of these operators legitimize allowing the adaptive automaton to choose any of them based on context. Yet, for this, there must be a mechanism for the choice to be transparent and reproducible.

D. Fuzzy AHP and Multi-Criteria Selection under Imprecision

AHP breaks down a decision problem into a hierarchy with the goal, criteria, and alternatives, followed by pairwise comparison requiring the decision maker to derive priority weights (Saaty, 1977, 1980). The consistency ratio is a diagnostic that assesses the coherence of pairwise judgments. In situations where experts are unable to provide exact ratios, fuzzy extensions of AHP model judgments have been proposed in the form of intervals or triangular fuzzy numbers (Van Laarhoven & Pedrycz, 1983; Buckley, 1985). An extent analysis approach suggested by Chang (1996) is popularly adopted in Fuzzy AHP applications whereas, Mikhailov (2002) was using fuzzy programming ideas to deal with uncertain priorities in the selection of partners.

Fuzzy AHP is the appropriate methodology for the present problem because the choice of t-norm is neither entirely empirical nor entirely axiomatic. For instance, the product aggregation will be very appropriate under independent evidence but less appealing in a high-risk situation where a weak component should under-one dominate the decision. Interpretable and conservative, minimum aggregation can over-preserve high acceptance grades when used on long sequences. A fuzzy AHP model makes these trade-offs explicit by asking how important is each criterion, and by how strongly the t-norm satisfies each criterion. The decision layer of a fuzzy automaton with adaptivity in the present study is Fuzzy AHP.

III. MATHEMATICAL PRELIMINARIES

A. Fuzzy Sets and Fuzzy Relations

Let X be a non-empty universe. A fuzzy set A on X is a mapping mu_A: X -> [0,1], where mu_A(x) expresses the degree to which x belongs to A. If mu_A(x)=1, x is fully included in A; if mu_A(x)=0, x is fully excluded; intermediate values indicate graded membership. For two fuzzy sets A and B on X, A is contained in B when mu_A(x) <= mu_B(x) for every x in X. Standard fuzzy union and intersection are usually represented by a t-conorm and a t-norm, respectively, although the present paper focuses on conjunction through t-norms.

A fuzzy relation R from X to Y is a fuzzy subset of X x Y, represented by a function R: X x Y -> [0,1]. If R is a relation from X to Y and S is a relation from Y to Z, their t-norm-based composition is defined by the supremum over intermediate elements. This is the relational operation behind fuzzy transition composition in an automaton.

$$(R \circ_T S)(x,z) = \sup_{y \in Y} T(R(x,y), S(y,z)).$$

B. Triangular Norms

A triangular norm is a function T: [0,1] x [0,1] -> [0,1] such that, for all x,y,z in [0,1], T(x,y)=T(y,x), T(x,T(y,z))=T(T(x,y),z), T(x,y) <= T(x,z) whenever y <= z, and T(x,1)=x. These four properties express commutativity, associativity, monotonicity, and the identity condition. They ensure that the operator behaves as a mathematically coherent fuzzy conjunction.

The present paper uses five candidate t-norms: Godel/minimum, product, Lukasiewicz, Hamacher, and Schweizer-Sklar. They are selected because they represent distinct aggregation semantics. The minimum t-norm is conservative; product is multiplicative; Lukasiewicz is threshold-sensitive; Hamacher is rational and parameter-sensitive; Schweizer-Sklar is a parametric family that can interpolate among aggregation behaviours. In the illustrative calculations, the Hamacher t-norm is used in its standard zero-parameter form and the Schweizer-Sklar family is represented by p=0.5.

$$T_G(x,y)=\min(x,y); \quad T_P(x,y)=xy; \quad T_L(x,y)=\max(0,x+y-1).$$

$$T_H(x,y)=xy/(x+y-xy), \text{ where the value is 0 if } x=y=0.$$

$$T_{SS,p}(x,y)=\max(0,x^p+y^p-1)^{(1/p)}, \text{ with } p=0.5 \text{ in the example.}$$

Table 1. Candidate t-norms and their decision semantics.

t-Norm	Formula	Interpretive role
Godel/minimum	T(x,y)=min(x,y)	Weakest evidence dominates; useful in conservative decisions
Product	T(x,y)=xy	Multiplicative aggregation; useful under independent evidence
Lukasiewicz	T(x,y)=max(0,x+y-1)	Threshold-sensitive; useful when insufficient combined support should vanish
Hamacher	T(x,y)=xy/(x+y-xy)	Flexible rational aggregation; useful in adaptive intermediate regimes
Schweizer-Sklar	T_p(x,y)=max(0,x^p+y^p-1)^(1/p)	Parameterized family; useful where the system must tune aggregation strength

C. Fuzzy Finite Automata

A fuzzy finite automaton over an input alphabet Sigma may be represented as A=(Q,Sigma,delta,sigma,tau,T), where Q is a finite non-empty set of states, delta: Q x Sigma x Q -> [0,1] is a fuzzy transition function, sigma: Q -> [0,1] is the initial fuzzy state membership, tau: Q -> [0,1] is the terminal membership, and T is the t-norm used for path aggregation. For a word w=a_1...a_m, the grade of a path q_0...q_m is obtained by composing the initial grade, transition grades, and terminal grade with the chosen t-norm. The recognized fuzzy language is the function L_A: Sigma* -> [0,1] assigning each word its acceptance grade.

When the same t-norm is used throughout the automaton, associativity allows a compact expression of path aggregation. If different t-norms are used at different stages, associativity across operators is not guaranteed. Consequently, a context-dependent automaton must explicitly fix the order of evaluation. This paper adopts a temporal left-fold convention because it respects the order in which the input word is processed.

$$L_A(w)=\sup_{q_0,\ldots,q_m} T(\sigma(q_0), T(\delta(q_0,a_1,q_1), \ldots, T(\delta(q_{m-1},a_m,q_m), \tau(q_m)) \ldots)).$$

D. Fuzzy AHP Concepts

In AHP, a pairwise comparison matrix A=(a_ij) is positive and reciprocal, meaning a_ij>0 and a_ji=1/a_ij. The priority vector is commonly estimated by the principal eigenvector or by the geometric mean method. The consistency index is CI=(lambda_max-n)/(n-1), and the consistency ratio is CR=CI/RI, where RI is Saaty's random index for a matrix of order n (Saaty, 1977, 1980). A CR below 0.10 is conventionally treated as acceptable for many decision problems, although the threshold is a practical guideline rather than a theorem.

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In Fuzzy AHP, an exact judgment a_ij is replaced by a triangular fuzzy number a_ij~=(l_ij,m_ij,u_ij), where l_ij <= m_ij <= u_ij. The modal value m_ij is the most representative judgment, while l_ij and u_ij express lower and upper bounds of plausible judgment. The reciprocal of a triangular fuzzy number is given by (1/u_ij,1/m_ij,1/l_ij). Buckley's geometric-mean approach computes fuzzy row means and then normalizes them to derive fuzzy weights (Buckley, 1985). Defuzzification converts each fuzzy weight into a crisp score, usually by a centroid formula.

r_i~ = (product_{j=1}^n a_ij~)^(1/n), w_i~ = r_i~ x (r_1~ + ... + r_n~)^(-1).
Defuzz(w_i~) = (l_i + m_i + u_i)/3, W_i = Defuzz(w_i~) / sum_k Defuzz(w_k~).

IV. PROPOSED FRAMEWORK: CONTEXT-DEPENDENT T-NORM ADAPTIVE FUZZY AUTOMATA

A. Formal Definition

A context-dependent t-norm adaptive fuzzy automaton is defined as A_c=(Q,Sigma,C,delta,sigma,tau,gamma,Phi), where Q is a finite state set, Sigma is a finite input alphabet, C is a non-empty context set, delta: C x Q x Sigma x Q -> [0,1] is a context-indexed fuzzy transition function, sigma and tau are initial and terminal fuzzy sets, gamma: C x Sigma x [0,1]^Q -> C is a context-update function, and Phi is a t-norm selector. The selector Phi maps the current context and the Fuzzy AHP decision profile into one t-norm from the candidate set T={T_G,T_P,T_L,T_H,T_SS}.

The automaton therefore has two coupled dynamics. The first is the state-membership recursion, which updates a fuzzy state distribution after each input symbol. The second is context evolution, which updates the operating regime from evidence quality, risk level, reliability, or another observable contextual descriptor. The t-norm used at step k is not chosen manually after the fact; it is selected by Phi based on the Fuzzy AHP ranking under the active context.

mu_{k+1}(q') = sup_{q in Q} T_{c_k}(mu_k(q), delta(c_k,q,a_{k+1},q')).
c_{k+1} = gamma(c_k, a_{k+1}, mu_{k+1}), T_{c_k}=Phi(c_k, W, S).

B. Left-Fold Semantics and Loss of Global Associativity

The model deliberately uses left-fold aggregation when different t-norms appear in the same computation. Even though each individual t-norm is associative, a sequence containing different t-norms is not globally associative. For example, applying the minimum t-norm at one step and the product t-norm at another generally yields a different value if the order of grouping is changed. This matters in automata because the accepted grade of a word must be reproducible and unambiguous.

The temporal left-fold convention can be read as follows. At each input symbol, the current state membership and transition degree are combined using the t-norm selected by the current context. The resulting distribution becomes the input to the next stage. This convention is consistent with sequential decision-making because the system does not retroactively change the way previous evidence was aggregated. It also makes the computation implementable by dynamic programming.

(((x_1 T_{c_1} x_2) T_{c_2} x_3) ... T_{c_{m-1}} x_m), not an unordered product of grades.

C. Monotonicity and Finite-Word Stability

The suggested path aggregation shows monotonic behaviour for every transition grade of a fixed context sequence. The monotonicity of every t-norm in the family guarantees this property immediately. If a change in degree goes up while all other quantities and contexts are held constant, no path grade can go down. It is important for decision support to ensure monotonicity as it will prevent the model from decreasing an acceptance grade when evidence strengthens.

Stability of finite-word is attainable on ordinary continuity assumptions. When transition degrees which depend on the context vary over the context parameter continuously and every selected t-norm is continuous, acceptance of a fixed finite word have the form of finite compositions of continuous maps and finite suprema. Finite Supremum of Continuous Real Valued Functions is Continuous on Finite State Space. Small perturbations in transition reliability or context member-ship, will not cause abrupt jumps in the computed acceptance grade, as long as the context selector does not implement discontinuous switching, with-out a tie-breaking rule.

D. Decision-Theoretic t-Norm Selection

The central novelty of the framework is the t-norm selector Phi. Instead of repairing one unique conjunction operator, the selector tests candidate t-norms against a criterion vector. The baseline criteria are robustness under uncertainty, continuity and convergence stability, interpretability, sensitivity to weak evidence, computational efficiency, suitability for high-stakes decisions, and adaptability to changing contexts. A context c is capable of modifying either the criteria weights or the t-norm suitability scores or both. This allows a single automaton architecture to function under different aggregation semantics in different decision regimes.

A two-level system is a result. The fuzzy automaton at the lower level computes state evolution and word acceptance. The layer of Fuzzy AHP decision is higher that justifies which t-norm to be used as. The lower-level offers a compute sequentially; the upper-level allows accountable operator selection. This disconnect prevents an adaptive fuzzy model from developing a common weakness, namely, changing rules without justification for the rational change.

V. APPLICATION OF FUZZY AHP FOR T-NORM SELECTION

A. Goal, Criteria and Alternatives of the Problem

The aim of the decision is to identify the best t-norm which can be used in an adaptive fuzzy automaton under uncertainty-induced decision. There is one goal, seven criteria and five alternatives for decision making. There are five alternatives, namely Godel/minimum, product, Lukasiewicz, Hamacher, and Schweizer-Sklar t-norms. The seven criteria reflect mathematical and decision-theoretic requirements rather than application-specific convenience. Strength is a measure of resistance to noise. Smoothness and convergence stability encompass numerical and topological regularities. The interpretability relates to the capacity to explain the resulting acceptance grade to a decision-maker. Weak evidence intolerance therefore refers to how strongly poor evidence should reduce output. The ability to execute repeated finite-state updates efficiently. High-risk suitability aggregates conservatively with a view to safety. Whether the t-norm can be shifted or tuned across regimes is adaptivity

All numerical values in this section are fictitious computational data. This case studies were selected to illustrate the Fuzzy AHP process. The t-norms should not be viewed as measurements or rankings in practice. In actual implementation, the domain experts will elicit the alternative scores and criteria weights, tested on validation data or learned subject to constraints.

Table 2. Criteria used for Fuzzy AHP-based t-norm selection.

Code	Criterion	Operational interpretation
C1	Robustness under uncertainty	Ability to preserve reliable decision behaviour under noisy, vague, or conflicting evidence
C2	Continuity and convergence stability	Continuity of aggregation and stability of iterated fuzzy state updates
C3	Interpretability of decision output	Ease with which users can understand why the accepted grade changes
C4	Sensitivity to weak evidence	Degree to which weak transition evidence appropriately constrains the result
C5	Computational efficiency	Computational cost of repeated transition composition in finite-state updates
C6	Suitability for high-risk decision contexts	Suitability for conservative decisions in safety-critical or risk-sensitive contexts
C7	Adaptability to changing context	Ability to accommodate changing contexts or parameterized aggregation policies

B. Fuzzy Pairwise Comparison Matrix

A baseline risk-aware expert profile was first specified as a coherent modal priority vector. The corresponding reciprocal comparison matrix was obtained by a_ij=w_i/w_j. This produces a perfectly consistent modal matrix and makes the calculation reproducible. Each modal comparison was then fuzzified into a triangular fuzzy number.

For a modal ratio r>1, the associated triangular number is (0.8r,r,1.2r); for r<1, the reciprocal construction is used. This gives an interval of judgment around the modal value while preserving reciprocity. The purpose is not to claim that these are final expert judgments but to provide a transparent demonstration of Fuzzy AHP mechanics.

The modal pairwise comparison matrix is shown below. The values indicate the relative importance of the row criterion over the column criterion. For example, the value 2.62 in the row C1 and column C5 means that robustness under uncertainty is treated as about 2.62 times as important as computational efficiency under the baseline risk-aware profile.

Table 3. Modal pairwise comparison matrix for the seven criteria (illustrative computational data).

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**
C1	1.00	1.17	1.62	1.75	2.62	1.24	1.91
C2	0.86	1.00	1.38	1.50	2.25	1.06	1.64
C3	0.62	0.72	1.00	1.08	1.62	0.76	1.18
C4	0.57	0.67	0.92	1.00	1.50	0.71	1.09
C5	0.38	0.44	0.62	0.67	1.00	0.47	0.73
C6	0.81	0.94	1.31	1.42	2.12	1.00	1.55
C7	0.52	0.61	0.85	0.92	1.38	0.65	1.00

The consistency check for the modal comparison matrix gives lambda_max=7.000, CI=0.000, RI=1.32, and CR=0.000. Since CR is below 0.10, the matrix is acceptable under the conventional AHP consistency guideline. The zero value results from the fact that the modal comparison matrix is generated from a coherent priority vector. In practical applications, expert judgments would rarely be exactly consistent, but Fuzzy AHP still allows imprecision to be represented in the comparison intervals.

Table 4. Consistency diagnostic for the modal AHP matrix.

Measure	Value	Interpretation
Order of matrix	7	Seven criteria are compared
lambda_max	7.000	Principal eigenvalue of the modal reciprocal matrix
Consistency index (CI)	0.000	CI=(lambda_max-n)/(n-1)
Random index (RI)	1.32	Saaty random index for n=7
Consistency ratio (CR)	0.000	CR=CI/RI; acceptable because CR<0.10

C. Fuzzy Weights and Defuzzification

The fuzzy geometric mean of each criterion row was computed and normalized using Buckley's Fuzzy AHP method. Each fuzzy weight is reported as a triangular number (l,m,u). The centroid defuzzification value is then normalized so that the final criteria weights sum to one. The results show that robustness, continuity, and high-risk suitability dominate the risk-aware profile, while computational efficiency receives the smallest weight. This does not mean efficiency is unimportant; rather, it is less important than stability and risk-sensitivity for the baseline decision environment.

Table 5. Fuzzy AHP criteria weights and normalized defuzzified priorities.

Code	Criterion	Fuzzy weight (l,m,u)	Defuzzified	Normalized weight
C1	Robustness under uncertainty	(0.146, 0.210, 0.293)	0.216	0.208
C2	Continuity and convergence stability	(0.126, 0.180, 0.253)	0.186	0.179
C3	Interpretability of decision output	(0.092, 0.130, 0.185)	0.136	0.130
C4	Sensitivity to weak evidence	(0.086, 0.120, 0.171)	0.126	0.121

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Code	Criterion	Fuzzy weight (l,m,u)	Defuzzified	Normalized weight
C5	Computational efficiency	(0.058, 0.080, 0.116)	0.084	0.081
C6	Suitability for high-risk decision contexts	(0.120, 0.170, 0.240)	0.177	0.170
C7	Adaptability to changing context	(0.079, 0.110, 0.158)	0.116	0.111

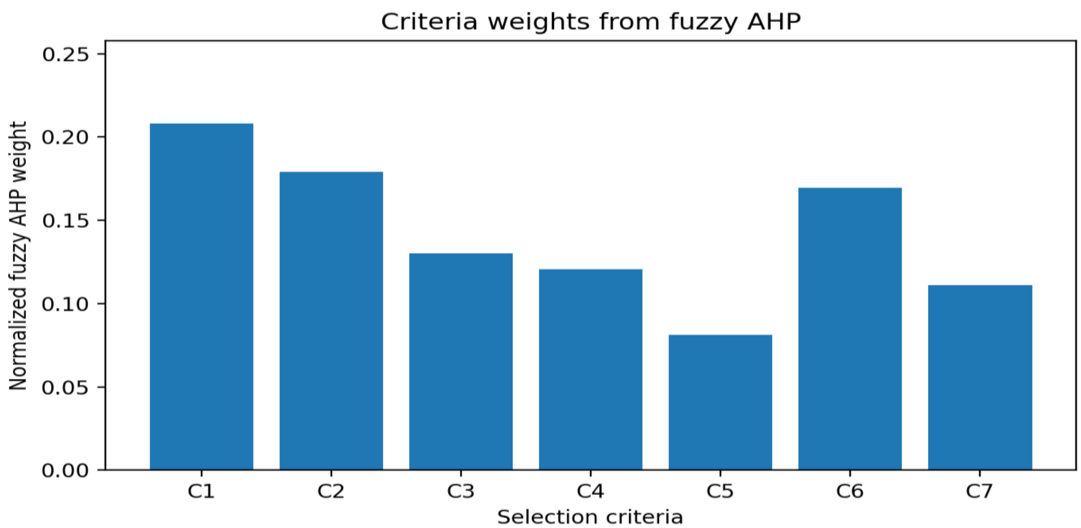

Figure 1. Criteria weights from the Fuzzy AHP calculation.

D. Candidate t-Norm Scoring and Final Ranking

After deriving criteria weights, each candidate t-norm was scored against each criterion using a suitable scale normalized on a 0 to 1 scale. These scores reflect fictive computational data grounded in the known qualitative behaviour of the t-norms. The minimum t-norm is an efficient and robust aggregation operator that is easy to interpret. It is suitable for dealing with high-risk cases as it is sensitive to weak evidence. The total score is controlled by the least degree of transition. The product t-norm is characterized by a high degree of continuity and efficiency and under independence assumptions it is interpreted as particularly meaningful.

The Lukasiewicz aggregation is strong for threshold-sensitive systems as a low combined support can turn into zero. The Hamacher and Schweizer-Sklar t-norms exhibit a high degree of adaptability as their families allow flexibility in the adjustment of their aggregation behaviours.

The final priority score of each alternative can be defined as the weighted sum of suitability scores using the Fuzzy AHP criteria weights – normalized. The Godel/minimum secures the highest score under the baseline risk-aware profile. Smart follow schweizer-sklar en hamacher maar zij zijn veel adaptiever, maar ook capacitieiten sterkrobuster. Lukasiewicz is fourth in the global ranking but is more attractive when threshold behaviour is the focus. The risk profile related to the product is lower than the baseline, but is appropriate when acontextual independence is the dominant assumption.

Table 6. Candidate t-norm suitability scores and weighted priority values (illustrative computational data).

Alternative	C1	C2	C3	C4	C5	C6	C7	Priority
Godel/minimum	0.95	0.90	0.95	0.90	0.95	0.98	0.60	0.901
Product	0.78	1.00	0.82	0.72	0.96	0.60	0.82	0.806
Lukasiewicz	0.76	0.88	0.76	0.94	0.86	0.80	0.74	0.816
Hamacher	0.85	0.92	0.75	0.82	0.78	0.83	0.93	0.846
Schweizer-Sklar	0.88	0.88	0.72	0.86	0.70	0.85	0.96	0.846

Table 7. Final Fuzzy AHP ranking of candidate t-norms under the baseline risk-aware profile.

Rank	t-Norm	Priority score	Decision implication
1	Godel/minimum	0.901	Selected when this score is maximal under the active context profile
2	Schweizer-Sklar	0.846	Selected when this score is maximal under the active context profile
3	Hamacher	0.846	Selected when this score is maximal under the active context profile
4	Lukasiewicz	0.816	Selected when this score is maximal under the active context profile
5	Product	0.806	Selected when this score is maximal under the active context profile

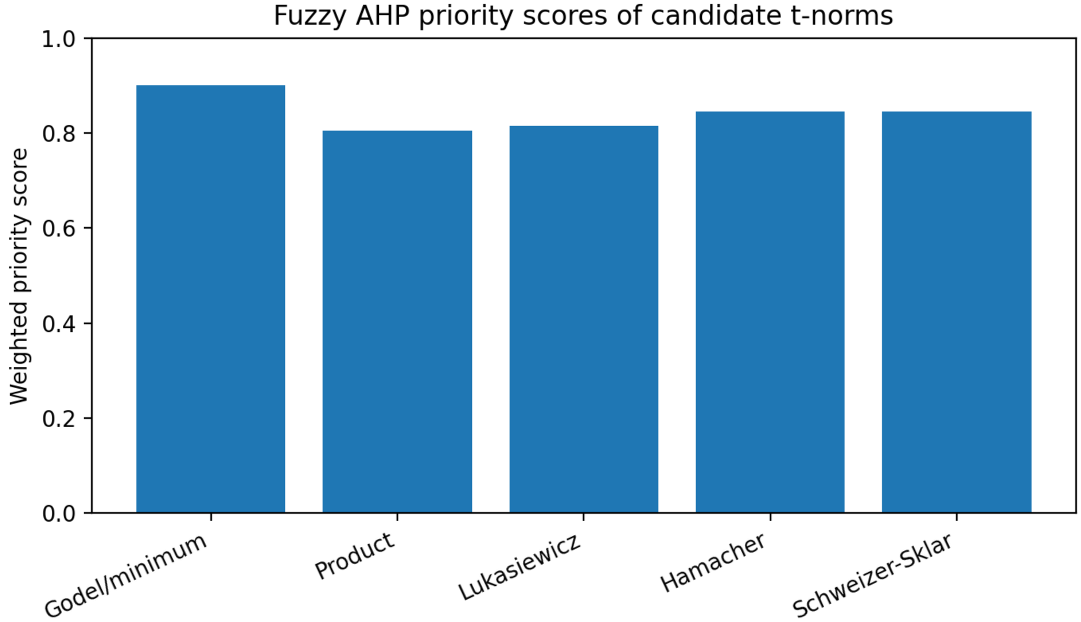

Figure 2. Weighted Fuzzy AHP priority scores of candidate t-norms.

E. Contextual Selection Rule

The baseline ranking should not be read as a universal instruction to use the minimum t-norm in every situation. The purpose of context dependence is precisely to allow the criteria weights or suitability scores to change when the decision context changes. In a high-risk context, robustness and weak-evidence sensitivity dominate; therefore the Godel/minimum t-norm is reasonable. In an independent-evidence context, the product t-norm is preferred because it expresses multiplicative confidence accumulation. In a threshold-sensitive decision system, the Lukasiewicz t-norm is suitable because it can suppress acceptance when combined support remains below a threshold. In an open-world adaptive context, Hamacher or Schweizer-Sklar t-norms are attractive because they offer flexible families for interpolation across aggregation regimes.

Thus, Fuzzy AHP does not simply rank t-norms once. It provides a reproducible decision grammar: define criteria, elicit context-specific weights, score alternatives, check consistency, defuzzify priorities, and select the t-norm whose priority is highest under the active context. This grammar is essential for accountability because the automaton can report not only the accepted grade of a word but also the reason for the aggregation law used at each step.

Table 8. Context-dependent t-norm selection logic supported by Fuzzy AHP.

Decision context	Dominant decision concern	Preferred t-norm	Reason
High-risk or safety-critical	Avoid over-acceptance when any component is weak	Godel/minimum	Weakest evidence controls the result; conservative and interpretable
Independent evidence aggregation	Combine independent confidence-like signals	Product	Multiplicative semantics match independent support aggregation
Threshold-sensitive reasoning	Reject when combined support is insufficient	Lukasiewicz	Nilpotent behaviour creates an explicit threshold effect
Adaptive/open-world uncertainty	Tune aggregation across changing regimes	Hamacher or Schweizer-Sklar	Parameterized or flexible behaviour supports context-dependent adjustment

VI. ILLUSTRATIVE COMPUTATIONAL EXAMPLE

A. Automaton Specification

This section presents a small fuzzy automaton to illustrate how the proposed framework operates. The numerical values are illustrative computational data. Let Q={q0,q1,q2,q3}, Sigma={a,b}, initial membership sigma=(1.00,0.20,0.00,0.00), and terminal membership tau=(0.00,0.20,0.70,1.00). The word processed by the automaton is w=aba. Two transition matrices are used, one for input symbol a and one for input symbol b. The matrices are written with rows as current states and columns as next states. The transition values are not empirical measurements; they are chosen to make the computation reproducible and to show the consequences of different t-norms.

Table 9. Initial and terminal fuzzy state memberships.

State	Initial sigma	Terminal tau
q0	1.00	0.00
q1	0.20	0.20
q2	0.00	0.70
q3	0.00	1.00

Table 10. Context-indexed transition degrees used in the illustrative automaton.

Input/From	q0	q1	q2	q3
a: q0	0.15	0.80	0.30	0.05
a: q1	0.00	0.60	0.75	0.20
a: q2	0.00	0.00	0.65	0.85
a: q3	0.00	0.00	0.00	0.90
b: q0	0.40	0.00	0.35	0.00
b: q1	0.00	0.20	0.80	0.50
b: q2	0.00	0.30	0.50	0.90
b: q3	0.00	0.00	0.20	0.95

B. Fixed t-Norm Computation

For a fixed t-norm, the state distribution is updated by the sup-t composition rule. Starting from sigma, the automaton reads a, then b, then a. The final acceptance grade is the supremum over terminal states of the t-norm between the final state membership and terminal membership. Because the same transition structure is evaluated under different t-norms, any change in accepted grade is caused by the aggregation semantics rather than by a change in the graph.

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Table 11. Final state distribution and acceptance grade for word aba under fixed t-norms.

Fixed t-norm	q0	q1	q2	q3	Acceptance grade
Godel/minimum	0.150	0.300	0.650	0.800	0.800
Product	0.009	0.096	0.416	0.544	0.544
Lukasiewicz	0.000	0.000	0.250	0.450	0.450
Hamacher	0.072	0.169	0.491	0.596	0.596
Schweizer-Sklar	0.000	0.014	0.354	0.505	0.505

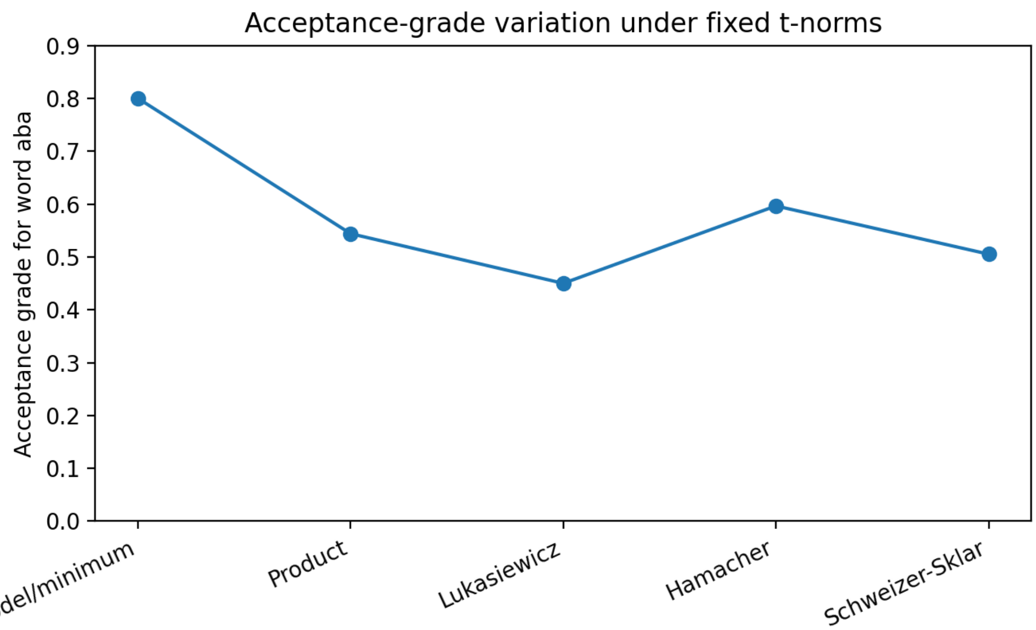

Figure 3. Acceptance-grade variation for the same word under fixed t-norms.

C. Context-Dependent Adaptive Computation

For the adaptive computation, the t-norm sequence is selected according to the context. The first symbol a is processed in a high-risk/caution context, so the minimum t-norm is used. The second symbol b is processed in an independent-evidence context, so the product t-norm is used. The final symbol a is processed in an adaptive/open-world context, so the Schweizer-Sklar t-norm is used. The final terminal combination is evaluated conservatively by the minimum t-norm, representing a high-risk terminal decision policy. The adaptive result is not designed to maximize the acceptance grade. It is designed to make the aggregation law match the decision context. In the illustrative example, the fixed minimum t-norm gives a high acceptance grade of 0.800 because it preserves the strongest feasible weakest-link path. The adaptive sequence gives an acceptance grade of 0.521 because the product step and Schweizer-Sklar step dampen the distribution in a context-sensitive manner. This illustrates the core point: adaptive t-norm selection changes the semantics of uncertainty propagation, not merely the numerical scale.

Table 12. State evolution under the adaptive context-dependent t-norm sequence.

Step/context	q0	q1	q2	q3
Initial	1.000	0.200	0.000	0.000
After a / Godel-minimum	0.150	0.800	0.300	0.200
After b / Product	0.060	0.160	0.640	0.400
After a / Schweizer-Sklar	0.000	0.030	0.368	0.521

Table 13. Fixed t-norms versus context-dependent adaptive sequence.

Policy	Acceptance grade	Interpretation
Godel/minimum	0.800	Fixed t-norm used at every step
Product	0.544	Fixed t-norm used at every step
Lukasiewicz	0.450	Fixed t-norm used at every step
Hamacher	0.596	Fixed t-norm used at every step
Schweizer-Sklar	0.505	Fixed t-norm used at every step
Adaptive sequence	0.521	Godel -> Product -> Schweizer-Sklar, with conservative terminal rule

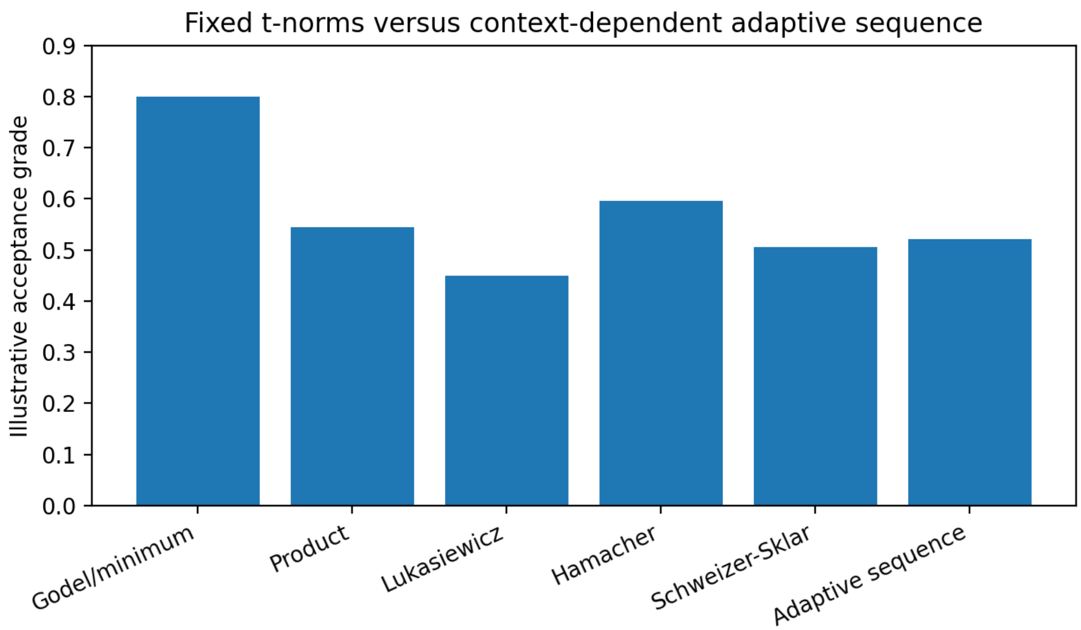

Figure 4. Comparison of fixed t-norm policies and the adaptive context-dependent sequence.

VII. RESULTS AND ANALYSIS

A. Criteria-Weight Pattern

The results of the Fuzzy AHP calculations yield a consistent decision profile for t-norm selection. The criteria that got the maximum normalized weight is robustness under uncertainty, followed by continuity and convergence stability and suitability for high-risk decision contexts. Moderate weights are given to interpretability, sensitivity to weak evidence, and adaptability; while the lowest weight is given to computational efficiency. This ordering is suitable for decision systems driven by uncertainty where avoiding conclusions that are unstable or unsafe is more important than slight savings on computation.

B. Final Ranking of Candidate t-Norms

Under the baseline profile, the final ranking is Godel/minimum first, Schweizer-Sklar second, Hamacher third, Lukasiewicz fourth and product fifth.

The difference between the two families, Schweizer-Sklar and Hamacher, is small, suggesting that the two flexible families are almost tied under the chosen suitability profile. The ranking also implies that product aggregation should not be dismissed altogether; rather it is less favoured under the baseline risk-aware profile because of the high weight of high-risk suitability and weak-evidence sensitivity. With a change in context to independent-evidence aggregation, the criteria weights can be recomputed so to select product as t-norm.

C. Acceptance-Grade Behaviour Across t-Norms

This example of an automata shows that by considering a different t-norm, the same transition graph has far different acceptance grades. In the given case, the maximum acceptance comes from the minimum aggregation as it retains the strongest weakest-link path. The lower grades of product and Lukasiewicz arise since they dampen or threshold the path more strongly during sequential composition. Hamacher and Schweizer-Sklar fall within these limits. The adaptive sequence does not act like a straightforward mean of fixed t-norms. It generates a result dependent on previous outcomes because at each step the current context t-norm is applied.

D. Decision Adequacy and Acceptance Magnitude

The outcome carries a significant interpretive consequence. Acceptance Grades in Decision Systems are not Necessarily Worse When Lower If the context requires a conservative treatment of unreliable evidence, or if the aggregation semantics change from independence to caution, that would be more appropriate. The proposed framework separates the numerical magnitude from decision adequacy. A t-norm can be said to be adequate if it matches the epistemic role of the context, not when it merely gives the largest accepted grade.

E. Summary of Computational Results

Table 14. Summary of main computational results.

Result component	Numerical outcome	Interpretation
Highest Fuzzy AHP criterion weights	C1=0.208, C2=0.179, C6=0.170	Robustness, stability, and high-risk suitability dominate the baseline profile
Top-ranked t-norm	Godel/minimum (0.901)	Conservative aggregation is preferred in the risk-aware baseline
Flexible t-norm alternatives	Schweizer-Sklar (0.846) and Hamacher (0.846)	Parameterized/flexible aggregation is nearly competitive
Highest fixed acceptance grade	Godel/minimum = 0.800	Weakest-link semantics preserve the strongest feasible path
Adaptive acceptance grade	0.521	Context-dependent aggregation dampens the path according to selected regimes

VIII. DISCUSSION

A. Decision-Theoretic Relevance of t-Norm Selection

The suggested model's primary advantage is that it considers the selection of t-norms as an internal problem of the automaton that is decision-theoretic in nature, rather than a once-off modelling decision taken externally to the system. In usual fuzzy automata, the chosen t-norm is usually assumed beforehand. This is not satisfactory for adaptive systems but is acceptable for theoretical studies of a fixed algebra. In contrast, the current framework allows the automaton to accommodate contextual change while remaining mathematically accountable. All switches in aggregation are attributable to a criterion-weight profile, alternative suitability scores, and a consistency-checked Fuzzy AHP calculation.

B. Mathematical Expressiveness and Stability

From a mathematical viewpoint, context dependence affords expressiveness but entails danger. The expressive power exists from the faculty of modeling different conjunction semantics in one finite-state architecture. The threat originates from the risk of losing global associativity and the potential for discontinuity if the selector undergoes a sudden shift. The continuity discussion and the left-fold convention tackle the issues by granting a specific order of computation and identifying where finite-word acceptance remains stable. A context-dependent model whose evaluation order is not fixed cannot be regarded as an automaton.

C. Transparency and Auditability
It improves transparency from a decision-support perspective Decision-makers are able to see why we prefer a t-norm in context. If there is a predominance of high risk suitability. If the independence of evidence prevails, then product aggregation is justified. If threshold sensitivity is dominant, Lukasiewicz aggregation can be justified. If flexibility prevails, the Hamacher or the Schweizer-Sklar families are justified. Because it is represented in tables, weights and rankings, rather than informal preference statements, the reasoning is auditable.

D. Interpretation of Acceptance-Grade Comparisons
The example also shows that acceptance-grade comparison is subject to misinterpretation. Having a high grade under one t-norm and a lower grade under another t-norm does not imply the superiority of one automaton over another. It means that different conjunction semantics have been applied to the same evidence of transition. When we have a highly risky system, under a context-sensitive t-norm it may be desirable for acceptance to be lower if it prevents that evidence from being recognized with overconfidence. Hence the proposed model supports epistemic discipline in adaptive fuzzy computation.

IX. CONTRIBUTIONS TO THEORY
A. Formalization of Context-Dependent Adaptive Automata
This study formalizes the context-dependent t-norm adaptive automaton with an explict t-norm selector, thus contributing to the theory of fuzzy automata. The model retains the finite-state structure of standard fuzzy automata, but it expands the aggregation layer so that conjunction semantics can change with context. Changing the algebra by which the degrees are composed is different from changing transition degrees.

B. Integration of Fuzzy Automata with Fuzzy AHP
The integration of fuzzy automata with Fuzzy AHP is the second contribution. Modern fuzzy automata theory provides a very potent tool for recognition, determinization, algebraic equivalence, and language representation. Fuzzy AHP provides a methodology for choosing between competing aggregation laws when several criteria matter. The combination produces a decision-justified automaton whose aggregation is thus (1) mathematically valid; (2) contextually rational.

C. Stability and Computational Semantics
There is a third contribution which clarifies stability issues. The author highlights the importance of the issues of context-dependence of t-norms namely left-fold semantics, monotonicity, finite-word continuity and context-switchin. The framework's articulation of these issues disallows the interpretation of AGGREGATE ADAPTIVELY as merely a verbal improvement. It illustrates how a modeller can build context dependence into the model with a well-defined computational procedure.

D. Reproducible Fuzzy AHP Demonstration
A further contribution is the set-up of reproducible illustrative Fuzzy AHP calculation. The matrices, weights, scores, rankings and examples of automata show how to apply the proposed framework. Though the numbers are illustrative, the approach transfers expert-elicited or data-calibrated settings.

X. PRACTICAL IMPLICATIONS
A. Medical Decision Support
In medical decision support, the same sequence of symptoms may require different aggregation semantics depending on the risk posed by the patient. A low-risk screening tool may tolerate product-like combinations of independent indicators, while an emergency triage context may require minimum-like caution. According to a Fuzzy AHP-enabled automaton, the explanation of the switch from one aggregation law to another could be documented.

B. Risk Assessment
In risk assessment, adaptive t-norm selection allows the decision system to be conservative when exposure, uncertainty, or severity of impact increases. When only one weak control has to constrain the overall safety grade, then minimum t-norm can be preferred. When there is changing risk and a parameterized response is required, then Hamacher or Schweizer-Sklar aggregation can be used.

C. Pattern Recognition and Sensor-Based Decision-Making

In pattern recognition and sensor-based decision-making, transition values are often derived from distorted signals. A product t-norm will be reasonable when the sensor outputs are independent and well-calibrated, whereas a more conservative t-norm is indicated when the signals are correlated, degraded or adversarial. The operator-selection rule, which translates engineering considerations is assisted by the Fuzzy AHP layer.

D. Smart Control and Fault Diagnosis

In smart control systems and fault diagnosis, the situation can change very quickly as the system goes from a normal runner to a warning and failure state. Context-dependent fuzzy automata can capture this dynamic change in a finite-state interpretable manner. The t-norm selector is incorporated in the controller logic, allowing the system to justify its choice between cautious, multiplicative, threshold-sensitive and adaptive aggregation.

E. Financial-Risk Simulation

The transition evidence for a subjective variable in a simulation could refer to financial risk indicators such as liquidity stress, repayment behaviour, leverage, and volatility.

F. Context-Sensitive Aggregation Policy

The policy t-norm depends on context to stop optimising from happening under bad regimes, while efficient aggregation still works in good regimes. The fuzzy AHP calculation provides a rationale document for the aggregation policy used in the decision model.

XI. LIMITATIONS AND FUTURE RESEARCH DIRECTIONS

A. Illustrative Nature of Computational Data

The first limitation relates to the illustrative computational data used for the numerical Fuzzy AHP and automata values. The expert panels, experiments, or operational datasets does not obtain them. Methodological demonstration is the purpose. In future investigations, experts from a specific domain should subjectively supply the criteria weights. Also, t-norm selection should be validated against actual decision outcomes.

B. Finite-State Modelling Constraint

Another limit stems from the finite-state assumption. Depending on the context – limits or no limits – the state space in which a finite fuzzy automaton operates can be either categorical or continuous. A generalization of the presented framework to infinite-state fuzzy transition systems will require stronger assumptions regarding the topologies, compactness conditions, and possibly measurable selection rules for context-sensitive t-norms.

C. Dependence on Expert Judgement

The third limitation relates to the reliance on expert judgment in fuzzy AHP. Pairwise comparisons may suffer from bias, inconsistency, and/or arbitrary selection of experts. While consistency checking does help, it does not eliminate subjectivity. Future analysis may involve collaboration between Fuzzy AHP and fuzzy TOPSIS, analytic network process, best-worst method and type-2 fuzzy extensions. Dynamic context learning might also modify criteria weights based on recorded performance while limiting selector for interpretability.

D. Uncertain Context Switching

The fourth limitation is that context switching is represented in a simplex deterministic fashion. The context itself is uncertain for many applications. This model can be richer in which one treats the context as a fuzzy distribution over regimes and aggregates transition updates across possible contexts. The proposed framework links to probabilistic fuzzy automata and fuzzy Markov decision processes, as well as learning-based uncertainty modelling.

E. Future Theoretical Extensions

Future studies may also look at equivalence, minimisation and bisimulation of context-dependent t-norm automata. It's possible for two automata to recognize the same fuzzy language under one context policy but not another. Theoretical problem concerning language equivalence of context-indexed aggregation is open. Another promising direction is the topological regularization of the selected t-norm so that the selected aggregation law depends smoothly on context.

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XII. CONCLUDING REMARKS

A. Synthesis of the Proposed Framework

A framework using fuzzy analytic hierarchy process is proposed to select t-norms of adaptive fuzzy automata. The proposed model overcomes a key limitation of a fixed-t-norm fuzzy automata: the premise that a single conjunction semantics is adequate in all decision contexts. By having the automaton implement a t-norm selector indexed to context, supported by Fuzzy AHP, aggregation law can change yet will be transparent.

B. Mathematical and Computational Implications

The mathematical framework presented a context-dependent adaptive fuzzy automaton. This clarifies how left-fold aggregation works under varying t-norms. It also addresses the issues of monotonicity and finite-word stability. The Fuzzy AHP application evaluated the t-norms based on seven criteria. Godel/minimum, product, Lukasiewicz, Hamacher, and Schweizer-Sklar In the illustrative calculation the dominant feature of the baseline profile was shown to be robustness, continuity and high-risk suitability which produces a conservative ranking. This indicates that the Godel/minimum t-norm is preferred. The computational automaton then showed how the same input word receives different acceptance degrees under fixed and adaptive t-norm policies.

C. Final Conclusion

Adaptive fuzzy automata, in a nutshell, should not informally change aggregation laws. There must be a mathematical specification and decision justification of context-dependent aggregation. Fuzzy AHP is essentially a tool developed to assist with decisionmaking. It allows t-norm choice to be transformed from an implicit modelling assumption to a component that is explicit, auditable and context-sensitive in uncertainty-driven decision-making systems.

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