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A Magnitude Ranking Approach for a Green Inventory Model with Carbon Tax and Carbon Cap and Trade Policies Under Uncertainty

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Abstract: Abating carbon emission which is emitted from manufacturing sectors and organizations is the important act to be performed at present to safeguard our environment. So to dwindle such emissions, in the last couple of decades both government and sectors are focusing on it by implementing new techniques and policies. Carbon tax and Carbon cap and trade are the two policies initiated at present by many firms and discussed by researchers in production inventory system. Most importantly facing unpredictability among parameters in a green EOQ model is the most efficient need in today's scenario of changing life condition and technical advancement among customers. So this model concentrates in a way to remove ambiguity in few parameters through fuzzy idea by considering them as trapezoidal fuzzy numbers. The optimal solution is found by using magnitude ranking method. Numerical example is given to illustrate the model.
Keywords: Carbon tax policy, Carbon cap and trade policy, Trapezoidal fuzzy number, Magnitude Ranking method, Uncertainty.

I. INTRODUCTION

Global warming, resulting from industrial greenhouse gas emissions, particularly emission of carbon, poses a significant threat to the environment and the survival of human civilization. Supervisory bodies throughout the globe has enacted numerous carbon policies to limit such emissions. The current policies such as carbon tax and carbon cap-and-trade policy are the fundamental policies which comes under the process of reducing huge amount of emissions from manufacturing sectors. Hence Green Inventory Management with the analysation of green features prompts sectors to shine economically and most importantly to function in a way to safeguard the environment. Any unit of CO_2 emissions is subject to a fee under a carbon tax policy. Businesses are required to pay taxes for each emission unit even though they are free to emit CO_2 as needed. Carbon emissions are regarded as extra sources for financial expenses under this approach. Companies are given a carbon emission threshold, or "carbon cap," under carbon cap-and-trade policies. If businesses produce fewer than what has been designated they are able to sell any unused carbon to different firms, but if a corporation intends to emit excess carbon beyond the limit, they are able to purchase the additional carbon from other firms. So policies promote a way for the success of certain actions.

A several production-inventory models that take various carbon regulations into account was reviewed by Ghosh et al in 2020.A production-inventory model is optimized under two distinct carbon policies, and a hybrid carbon policy is proposed under random demand by Ghosh et al in 2021.Taking into account environmental emissions under carbon tax and cap-and-trade laws, a green inventory model is developed by Hasan et al in 2023.They demonstrated the concavity of the expected model through the provision of necessary mathematical derivations and explained the benefits of investing in green technology and enacting a carbon price. In 2024, Muthusamy et al analysed a production-inventory optimization system with deteriorating item shortages and the impact of carbon emission policies combined with green technologies. Hence many researchers have get into the idea of carbon policies in the supply chain system.

Apart from the judgement of optimal solutions in a green inventory system, a proper outcome of a supply chain system during unpredictable situations is a notable one as the entire world specifically industrial areas at present are facing vague state because of natural calamities, wars and socio, economic conditions. So to dealt with this, incorporation of fuzzy in green inventory system is needed. As fuzzy is a best tool in rectifying unclearness among parameters, it helps to determine the best result for the success of sectors through mathematical models associated with green inventory and fuzzy methods. An Eco-Friendly Demand-Based Sustainable Green Inventory System with Partial Backlog Under Fuzziness is examined by Durga Bhavani et al in 2022. In their model they changed the cost parameters to Pythagorean fuzzy numbers in order to alleviate their imprecision.

An analysis of the effects of green investments using two green warehouse inventory systems with pentagonal fuzzy storage capacity and two dispatching strategies was done by Bhavani et al in 2023. The present study discusses a green inventory management with carbon tax and carbon cap and trade policies using magnitude ranking approach for trapezoidal fuzzy numbers.In which it converts several cost parameters to trapezoidal fuzzy numbers to dealt with unpredictability.

A. Definitions:

Fuzzy Set:

A fuzzy set $\tilde{A}$ defined on a Universe of discourse X may be written as a collection of ordered pairs, $\tilde{A} = \left\{ (x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}} \in [0,1] \right\}$, where each pair $(x, \mu_{\tilde{A}}(x))$ is called a singleton and the element $\mu_{\tilde{A}}(x)$ belongs to the interval [0,1].The function $\mu_{\tilde{A}}(x)$ is called as membership function.

Trapezoidal Fuzzy Number:

The fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$,where $a_1 < a_2 < a_3 < a_4$ and defined on R is called the trapezoidal fuzzy number, if the membership function is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & : x < a_1 \text{ or } x > a_4 \\ \dfrac{(x - a_1)}{(a_2 - a_1)} & : a_1 \le x \le a_2 \\ 1 & : a_2 \le x \le a_3 \\ \dfrac{(x - a_4)}{(a_3 - a_4)} & : a_3 \le x \le a_4 \end{cases}$$

B. Methodology:

Magnitude Ranking Method:

If $A = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number, then the defuzzification formula of magnitude ranking method is given by,

$$P(\tilde{A}) = \frac{(a_1 + 5a_2 + 5a_3 + a_4)}{12}.$$

Assumptions:

- An integrated vendor-buyer supply chain with two tiers is taken into account.
- Demand is not a constant.
- Rate of production, cost of ordering, cost of setting items, demand and emission of carbon for holding inventory at the buyer and at the vendor are found to be uncertain.
- Number of shipments is constant.

Notations:

Crisp Parameters:

$\hat{C}_e -$ carbon emission cap.

$u_t -$ carbon emission tax for each unit.

$u_{pt} -$ price for selling unit carbon.

$D -$ Demand.

$C_o -$ cost of ordering items.

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P_s – cost of setting items per production.

R_p – rate of production.

I_p – production rate per unit item.

T – Lead time.

B – percentage of demand that is backordered during the stock out period.

$D(s)$ – anticipated shortage in demand at the ending of each buying cycle.

b_s – the cost of the buyer's shortage per unit.

P – the profit made by the buyer per unit.

H – cost of holding items per unit of the buyer.

H_V – cost of holding items per unit of the vendor.

e – emission of carbon for production per unit item.

C_α – emission of carbon for holding inventory at the buyer per unit.

C_v – emission of carbon for holding inventory at the vendor per unit.

r_p – reorder point.

Q_{ct} – Order Quantity with carbon tax policy.

Q_{cct} – Order Quantity with carbon cap-and-trade policy.

N – number of shipments.

TC_{ct} – Total cost with carbon tax policy.

TC_{cct} – Total cost with carbon cap-and-trade policy.

Fuzzy Parameters:

$\tilde{D}$ – Fuzzy Demand.

$\tilde{R}_p$ – Fuzzy rate of production.

$\tilde{P}_s$ – Fuzzy cost of setting items per production.

$\tilde{C}_o$ – Fuzzy cost of ordering items.

$\tilde{C}_\alpha$ – emission of carbon for holding inventory at the buyer per unit under fuzzy.

$\tilde{C}_v$ – emission of carbon for holding inventory at the vendor per unit.

$\tilde{Q}^*_{ct}$ – Fuzzy Optimum Order Quantity with carbon tax policy.

$\tilde{Q}^*_{cct}$ – Fuzzy Optimum Order Quantity with carbon cap-and-trade policy.

$P\left(T\tilde{C}_{ct}(Q_{ct})\right)$ – Fuzzy total cost with carbon tax policy.

$P\left(T\tilde{C}_{cct}(Q_{cct})\right)$ – Fuzzy total cost with carbon cap-and-trade policy.

Green Inventory model under two cases:

- Case 1: With Carbon tax policy:

The total cost of the green supply chain model with the addition of several costs under carbon tax policy is given by,

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$$TC_{ct}(Q_{ct}) = \frac{P_s D}{NQ_{ct}} + I_p D + H_V.\frac{Q_{ct}}{2}\left[N\left(1-\frac{D}{R_p}\right)-1+\frac{2D}{R_p}\right]+\frac{C_o D}{Q_{ct}}+H\left[\frac{Q_{ct}}{2}+r_p-DT+(1-B)D(s)\right]+$$
$$\frac{b_s D}{Q_{ct}}D(s)+\frac{P(1-B)D}{Q_{ct}}D(s)+u_t\left[\begin{array}{l} eD+C_\alpha\left[\frac{Q_{ct}}{2}+r_p-DT+(1-B)D(s)\right]+ \\ C_v.\frac{Q_{ct}}{2}\left[N\left(1-\frac{D}{R_p}\right)-1+\frac{2D}{R_p}\right]\end{array}\right]$$

---------------(1)

Differentiating eqn (1) with respect to Q_{ct} and equating to zero gives the Optimum Order Quantity under the carbon tax policy and it is given by,

$$Q_{ct} = \sqrt{\frac{2D\left(C_o+\frac{P_s}{N}+b_s D(s)+P(1-B)D(s)\right)}{H+u_t C_\alpha+(H_V+u_t C_v)\left[N\left(1-\frac{D}{R_p}\right)-1+\frac{2D}{R_p}\right]}}$$

- Case 2: With Carbon cap-and-trade policy:

The total cost of the green supply chain model with the addition of several costs under carbon cap-and-trade policy is given by,

$$TC_{cct}(Q_{cct}) = \frac{P_s D}{NQ_{cct}} + I_p D + H_V.\frac{Q_{cct}}{2}\left[N\left(1-\frac{D}{R_p}\right)-1+\frac{2D}{R_p}\right]+\frac{C_o D}{Q_{cct}}+H\left[\frac{Q_{cct}}{2}+r_p-DT+(1-B)D(s)\right]+$$
$$\frac{b_s D}{Q_{cct}}D(s)+\frac{P(1-B)D}{Q_{cct}}D(s)+u_{pt}\left[\begin{array}{l} eD+C_\alpha\left[\frac{Q_{cct}}{2}+r_p-DT+(1-B)D(s)\right]+ \\ C_v.\frac{Q_{cct}}{2}\left[N\left(1-\frac{D}{R_p}\right)-1+\frac{2D}{R_p}\right]-\hat{C}_e\end{array}\right]$$

---------------(2)

Differentiating eqn (2) with respect to Q_{cct} and equating to zero gives the Optimum Order Quantity under the carbon cap-and-trade policy and it is given by,

$$Q_{cct} = \sqrt{\frac{2D\left(C_o+\frac{P_s}{N}+b_s D(s)+P(1-B)D(s)\right)}{H+u_{pt} C_\alpha+(H_V+u_{pt} C_v)\left[N\left(1-\frac{D}{R_p}\right)-1+\frac{2D}{R_p}\right]}}$$

Fuzzy Green Inventory Model under above two discussed cases:

In the considered green supply chain model under two cases, several cost parameters and emission parameters are taken as trapezoidal fuzzy numbers to remove vagueness among them.

Let the parameters

$\tilde{D} = (d_1, d_2, d_3, d_4)$

$\tilde{R}_p = (R_{p_1}, R_{p_2}, R_{p_3}, R_{p_4})$

$\tilde{P}_s = (P_{s_1}, P_{s_2}, P_{s_3}, P_{s_4})$

$\tilde{C}_o = \left(C_{o_1}, C_{o_2}, C_{o_3}, C_{o_4}\right)$

$\tilde{C}_\alpha = \left(C_{\alpha_1}, C_{\alpha_2}, C_{\alpha_3}, C_{\alpha_4}\right)$

$\tilde{C}_v = \left(C_{v_1}, C_{v_2}, C_{v_3}, C_{v_4}\right)$ be taken as trapezoidal fuzzy numbers.

- Case 1: With Carbon tax policy:

After converting the above parameters as trapezoidal fuzzy numbers we get the fuzzified total cost as,

$$\begin{aligned} T\tilde{C}_{ct}(Q_{ct}) = {} & \frac{\tilde{P}_s \tilde{D}}{NQ_{ct}} + I_p \tilde{D} + H_V . \frac{Q_{ct}}{2}\left[N\left(1-\frac{\tilde{D}}{\tilde{R}_p}\right) - 1 + \frac{2\tilde{D}}{\tilde{R}_p}\right] + \frac{\tilde{C}_o \tilde{D}}{Q_{ct}} + H\left[\frac{Q_{ct}}{2} + r_p - \tilde{D}T + (1-B)D(s)\right] + \\ & \frac{b_s \tilde{D}}{Q_{ct}} D(s) + \frac{P(1-B)\tilde{D}}{Q_{ct}} D(s) + u_t \left[\begin{aligned} & e\tilde{D} + \tilde{C}_\alpha \left[\frac{Q_{ct}}{2} + r_p - \tilde{D}T + (1-B)D(s)\right] + \\ & \tilde{C}_v . \frac{Q_{ct}}{2}\left[N\left(1-\frac{\tilde{D}}{\tilde{R}_p}\right) - 1 + \frac{2\tilde{D}}{\tilde{R}_p}\right] \end{aligned}\right] \end{aligned}$$

Now applying magnitude ranking method for the fuzzifiedtotal cost with carbon tax policy we obtain the total cost under fuzzy as,

$$P\left(T\tilde{C}_{ct}(Q_{ct})\right) = \frac{1}{12}\left[\begin{aligned} & \left[\begin{aligned} & \frac{P_{s_1} d_1}{NQ_{ct}} + I_p d_1 + H_V . \frac{Q_{ct}}{2}\left[N\left(1-\frac{d_1}{R_{p_4}}\right) - 1 + \frac{2d_1}{R_{p_4}}\right] + \frac{C_{o_1} d_1}{Q_{ct}} + H\left[\frac{Q_{ct}}{2} + r_p - d_1 T + (1-B)D(s)\right] + \\ & \frac{b_s d_1}{Q_{ct}} D(s) + \frac{P(1-B)d_1}{Q_{ct}} D(s) + u_t \left[\begin{aligned} & ed_1 + C_{\alpha_1}\left[\frac{Q_{ct}}{2} + r_p - d_1 T + (1-B)D(s)\right] + \\ & C_{v_1} . \frac{Q_{ct}}{2}\left[N\left(1-\frac{d_1}{R_{p_4}}\right) - 1 + \frac{2d_1}{R_{p_4}}\right] \end{aligned}\right] \end{aligned}\right] + \\ & 5\left[\begin{aligned} & \frac{P_{s_2} d_2}{NQ_{ct}} + I_p d_2 + H_V . \frac{Q_{ct}}{2}\left[N\left(1-\frac{d_2}{R_{p_3}}\right) - 1 + \frac{2d_2}{R_{p_3}}\right] + \frac{C_{o_2} d_2}{Q_{ct}} + H\left[\frac{Q_{ct}}{2} + r_p - d_2 T + (1-B)D(s)\right] + \\ & \frac{b_s d_2}{Q_{ct}} D(s) + \frac{P(1-B)d_2}{Q_{ct}} D(s) + u_t \left[\begin{aligned} & ed_2 + C_{\alpha_2}\left[\frac{Q_{ct}}{2} + r_p - d_2 T + (1-B)D(s)\right] + \\ & C_{v_2} . \frac{Q_{ct}}{2}\left[N\left(1-\frac{d_2}{R_{p_3}}\right) - 1 + \frac{2d_2}{R_{p_3}}\right] \end{aligned}\right] \end{aligned}\right] + \\ & 5\left[\begin{aligned} & \frac{P_{s_3} d_3}{NQ_{ct}} + I_p d_3 + H_V . \frac{Q_{ct}}{2}\left[N\left(1-\frac{d_3}{R_{p_2}}\right) - 1 + \frac{2d_3}{R_{p_2}}\right] + \frac{C_{o_3} d_3}{Q_{ct}} + H\left[\frac{Q_{ct}}{2} + r_p - d_3 T + (1-B)D(s)\right] + \\ & \frac{b_s d_3}{Q_{ct}} D(s) + \frac{P(1-B)d_3}{Q_{ct}} D(s) + u_t \left[\begin{aligned} & ed_3 + C_{\alpha_3}\left[\frac{Q_{ct}}{2} + r_p - d_3 T + (1-B)D(s)\right] + \\ & C_{v_3} . \frac{Q_{ct}}{2}\left[N\left(1-\frac{d_3}{R_{p_2}}\right) - 1 + \frac{2d_3}{R_{p_2}}\right] \end{aligned}\right] \end{aligned}\right] + \\ & \left[\begin{aligned} & \frac{P_{s_4} d_4}{NQ_{ct}} + I_p d_4 + H_V . \frac{Q_{ct}}{2}\left[N\left(1-\frac{d_4}{R_{p_1}}\right) - 1 + \frac{2d_4}{R_{p_1}}\right] + \frac{C_{o_4} d_4}{Q_{ct}} + H\left[\frac{Q_{ct}}{2} + r_p - d_4 T + (1-B)D(s)\right] + \\ & \frac{b_s d_4}{Q_{ct}} D(s) + \frac{P(1-B)d_4}{Q_{ct}} D(s) + u_t \left[\begin{aligned} & ed_4 + C_{\alpha_4}\left[\frac{Q_{ct}}{2} + r_p - d_4 T + (1-B)D(s)\right] + \\ & C_{v_4} . \frac{Q_{ct}}{2}\left[N\left(1-\frac{d_4}{R_{p_1}}\right) - 1 + \frac{2d_4}{R_{p_1}}\right] \end{aligned}\right] \end{aligned}\right] \end{aligned}\right]$$

--------------------(3)

Differentiating eqn (3) with respect to Q_{ct} andequating to zero gives the fuzzy optimum order quantity with carbon tax policy as,

$$\tilde{Q}_{ct}^{*}=\sqrt{\frac{2\left[\begin{array}{l}\left(C_{o_1}d_1+5C_{o_2}d_2+5C_{o_3}d_3+C_{o_4}d_4\right)+\left(\frac{P_{s_1}d_1}{N}+5\frac{P_{s_2}d_2}{N}+5\frac{P_{s_3}d_3}{N}+\frac{P_{s_4}d_4}{N}\right)\\ \left(b_sd_1D(s)+5b_sd_2D(s)+5b_sd_3D(s)+b_sd_4D(s)\right)+\\ \left(P(1-B)d_1D(s)+5P(1-B)d_2D(s)+5P(1-B)d_3D(s)+P(1-B)d_4D(s)\right)\end{array}\right]}{\begin{array}{l}12H+\left(u_tC_{\alpha_1}+5u_tC_{\alpha_2}+5u_tC_{\alpha_3}+u_tC_{\alpha_4}\right)+\\ H_V\left[\begin{array}{l}\left(N\left(1-\frac{d_1}{R_{p_4}}\right)-1+\frac{2d_1}{R_{p_4}}\right)+5\left(N\left(1-\frac{d_2}{R_{p_3}}\right)-1+\frac{2d_2}{R_{p_3}}\right)\\ +5\left(N\left(1-\frac{d_3}{R_{p_2}}\right)-1+\frac{2d_3}{R_{p_2}}\right)+\left(N\left(1-\frac{d_4}{R_{p_1}}\right)-1+\frac{2d_4}{R_{p_1}}\right)\end{array}\right]\\ +u_tC_{v_1}\left(N\left(1-\frac{d_1}{R_{p_4}}\right)-1+\frac{2d_1}{R_{p_4}}\right)+5u_tC_{v_2}\left(N\left(1-\frac{d_2}{R_{p_3}}\right)-1+\frac{2d_2}{R_{p_3}}\right)\\ +5u_tC_{v_3}\left(N\left(1-\frac{d_3}{R_{p_2}}\right)-1+\frac{2d_3}{R_{p_2}}\right)+u_tC_{v_4}\left(N\left(1-\frac{d_4}{R_{p_1}}\right)-1+\frac{2d_4}{R_{p_1}}\right)\end{array}}}$$

- Case 2: With Carbon cap-and-trade policy:

After converting the six mentioned parameters as trapezoidal fuzzy numbers, we get the fuzzified total cost as,

$$TC_{cct}(Q_{cct})=\frac{\tilde{P}_s\tilde{D}}{NQ_{cct}}+I_p\tilde{D}+H_V.\frac{Q_{cct}}{2}\left[N\left(1-\frac{\tilde{D}}{\tilde{R}_p}\right)-1+\frac{2\tilde{D}}{\tilde{R}_p}\right]+\frac{\tilde{C}_o\tilde{D}}{Q_{cct}}+H\left[\frac{Q_{cct}}{2}+r_p-\tilde{D}T+(1-B)D(s)\right]+$$
$$\frac{b_s\tilde{D}}{Q_{cct}}D(s)+\frac{P(1-B)\tilde{D}}{Q_{cct}}D(s)+u_{pt}\left[\begin{array}{l}e\tilde{D}+\tilde{C}_\alpha\left[\frac{Q_{cct}}{2}+r_p-\tilde{D}T+(1-B)D(s)\right]+\\ \tilde{C}_v.\frac{Q_{cct}}{2}\left[N\left(1-\frac{\tilde{D}}{\tilde{R}_p}\right)-1+\frac{2\tilde{D}}{\tilde{R}_p}\right]-\hat{C}_e\end{array}\right]$$

Now applying magnitude ranking method for the fuzzified total cost with carbon cap-and-trade policy we obtain the total cost under fuzzy as,

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$$P\left(\tilde{TC}_{cct}(Q_{cct})\right)=\frac{1}{12}\left[\begin{array}{l}
\left[\begin{array}{l}\frac{P_{s_1}d_1}{NQ_{cct}}+I_pd_1+H_V.\frac{Q_{cct}}{2}\left[N\left(1-\frac{d_1}{R_{p_4}}\right)-1+\frac{2d_1}{R_{p_4}}\right]+\frac{C_{o_1}d_1}{Q_{cct}}+H\left[\frac{Q_{cct}}{2}+r_p-d_1T+(1-B)D(s)\right]+\\
\frac{b_sd_1}{Q_{cct}}D(s)+\frac{P(1-B)d_1}{Q_{cct}}D(s)+u_{pt}\left[\begin{array}{l}ed_1+C_{\alpha_1}\left[\frac{Q_{cct}}{2}+r_p-d_1T+(1-B)D(s)\right]+\\ C_{v_1}.\frac{Q_{cct}}{2}\left[N\left(1-\frac{d_1}{R_{p_4}}\right)-1+\frac{2d_1}{R_{p_4}}\right]-\hat{C}_e\end{array}\right]\end{array}\right]+\\
5\left[\begin{array}{l}\frac{P_{s_2}d_2}{NQ_{cct}}+I_pd_2+H_V.\frac{Q_{cct}}{2}\left[N\left(1-\frac{d_2}{R_{p_3}}\right)-1+\frac{2d_2}{R_{p_3}}\right]+\frac{C_{o_2}d_2}{Q_{cct}}+H\left[\frac{Q_{cct}}{2}+r_p-d_2T+(1-B)D(s)\right]+\\
\frac{b_sd_2}{Q_{cct}}D(s)+\frac{P(1-B)d_2}{Q_{cct}}D(s)+u_{pt}\left[\begin{array}{l}ed_2+C_{\alpha_2}\left[\frac{Q_{cct}}{2}+r_p-d_2T+(1-B)D(s)\right]+\\ C_{v_2}.\frac{Q_{cct}}{2}\left[N\left(1-\frac{d_2}{R_{p_3}}\right)-1+\frac{2d_2}{R_{p_3}}\right]-\hat{C}_e\end{array}\right]\end{array}\right]+\\
5\left[\begin{array}{l}\frac{P_{s_3}d_3}{NQ_{cct}}+I_pd_3+H_V.\frac{Q_{cct}}{2}\left[N\left(1-\frac{d_3}{R_{p_2}}\right)-1+\frac{2d_3}{R_{p_2}}\right]+\frac{C_{o_3}d_3}{Q_{cct}}+H\left[\frac{Q_{cct}}{2}+r_p-d_3T+(1-B)D(s)\right]+\\
\frac{b_sd_3}{Q_{cct}}D(s)+\frac{P(1-B)d_3}{Q_{cct}}D(s)+u_{pt}\left[\begin{array}{l}ed_3+C_{\alpha_3}\left[\frac{Q_{cct}}{2}+r_p-d_3T+(1-B)D(s)\right]+\\ C_{v_3}.\frac{Q_{cct}}{2}\left[N\left(1-\frac{d_3}{R_{p_2}}\right)-1+\frac{2d_3}{R_{p_2}}\right]-\hat{C}_e\end{array}\right]\end{array}\right]+\\
\left[\begin{array}{l}\frac{P_{s_4}d_4}{NQ_{cct}}+I_pd_4+H_V.\frac{Q_{cct}}{2}\left[N\left(1-\frac{d_4}{R_{p_1}}\right)-1+\frac{2d_4}{R_{p_1}}\right]+\frac{C_{o_4}d_4}{Q_{cct}}+H\left[\frac{Q_{cct}}{2}+r_p-d_4T+(1-B)D(s)\right]+\\
\frac{b_sd_4}{Q_{cct}}D(s)+\frac{P(1-B)d_4}{Q_{cct}}D(s)+u_{pt}\left[\begin{array}{l}ed_4+C_{\alpha_4}\left[\frac{Q_{cct}}{2}+r_p-d_4T+(1-B)D(s)\right]+\\ C_{v_4}.\frac{Q_{cct}}{2}\left[N\left(1-\frac{d_4}{R_{p_1}}\right)-1+\frac{2d_4}{R_{p_1}}\right]-\hat{C}_e\end{array}\right]\end{array}\right]
\end{array}\right]$$

--------------------(4)

Differentiating eqn (4) with respect to Q_{cct} andequating to zero gives the fuzzy optimum order quantity with carbon cap-and-trade policyas,

$$\tilde{Q}^{*}_{cct}=\sqrt{\frac{2\left[\begin{array}{l}\left(C_{o_1}d_1+5C_{o_2}d_2+5C_{o_3}d_3+C_{o_4}d_4\right)+\left(\frac{P_{s_1}d_1}{N}+5\frac{P_{s_2}d_2}{N}+5\frac{P_{s_3}d_3}{N}+\frac{P_{s_4}d_4}{N}\right)\\ \left(b_sd_1D(s)+5b_sd_2D(s)+5b_sd_3D(s)+b_sd_4D(s)\right)+\\ \left(P(1-B)d_1D(s)+5P(1-B)d_2D(s)+5P(1-B)d_3D(s)+P(1-B)d_4D(s)\right)\end{array}\right]}{\begin{array}{l}12H+\left(u_{pt}C_{\alpha_1}+5u_{pt}C_{\alpha_2}+5u_{pt}C_{\alpha_3}+u_{pt}C_{\alpha_4}\right)+\\ H_V\left[\begin{array}{l}\left(N\left(1-\frac{d_1}{R_{p_4}}\right)-1+\frac{2d_1}{R_{p_4}}\right)+5\left(N\left(1-\frac{d_2}{R_{p_3}}\right)-1+\frac{2d_2}{R_{p_3}}\right)\\ +5\left(N\left(1-\frac{d_3}{R_{p_2}}\right)-1+\frac{2d_3}{R_{p_2}}\right)+\left(N\left(1-\frac{d_4}{R_{p_1}}\right)-1+\frac{2d_4}{R_{p_1}}\right)\end{array}\right]\\ +u_{pt}C_{v_1}\left(N\left(1-\frac{d_1}{R_{p_4}}\right)-1+\frac{2d_1}{R_{p_4}}\right)+5u_{pt}C_{v_2}\left(N\left(1-\frac{d_2}{R_{p_3}}\right)-1+\frac{2d_2}{R_{p_3}}\right)\\ +5u_{pt}C_{v_3}\left(N\left(1-\frac{d_3}{R_{p_2}}\right)-1+\frac{2d_3}{R_{p_2}}\right)+u_{pt}C_{v_4}\left(N\left(1-\frac{d_4}{R_{p_1}}\right)-1+\frac{2d_4}{R_{p_1}}\right)\end{array}}}$$

Numerical Example:

Crisp Model:

$\hat{C}_e=160$, $u_t=34$, $u_{pt}=34$, $D=400$, $C_o=100$, $P_s=1300$, $R_p=1000$, $I_p=200$, $T=5.5$, $B=0.2$, $D(s)=2$, $b_s=35$, $P=55$, $H=6$, $H_V=2$, $e=4$, $C_\alpha=1$, $C_v=1.2$, $r_p=15.37$, $N=3$.

Case 1: With Carbon tax policy:

The order quantity, $Q_{ct}=71.4025799$.

The Total cost, $TC_{ct}=54824.55187$.

Case 2: With Carbon cap-and-trade policy:

The order quantity, $Q_{cct}=71.4025799$.

The total cost, $TC_{cct}=49384.55187$.

Fuzzy Model:

$\tilde{D}=(150,350,450,500)$, $\tilde{P}_s=(1100,1250,1350,1500)$, $\tilde{C}_o=(50,75,125,150)$, $\tilde{C}_\alpha=(0.8,0.9,1.1,1.2)$, $\tilde{C}_v=(0.9,1.1,1.3,1.5)$, $\tilde{R}_p=(850,950,1050,1150)$.

Case 1: With Carbon tax policy:

The fuzzyoptimum order quantity, $\tilde{Q}^{*}_{ct}=71.0876622$.

The fuzzy optimum total cost, $P\left(\tilde{TC}_{ct}(Q_{ct})\right)=51440.31736$.

Case 2: With Carbon cap-and-trade policy:

The fuzzy optimum order quantity, $\tilde{Q}^{*}_{cct}=71.0876622$

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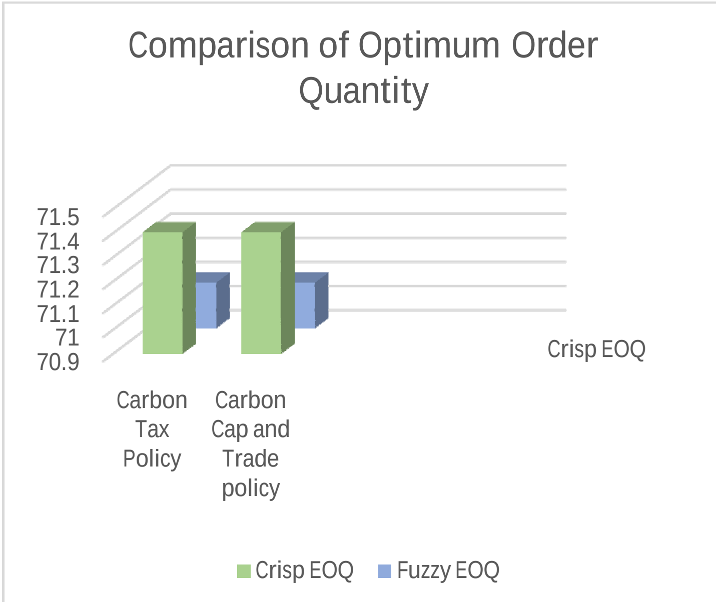
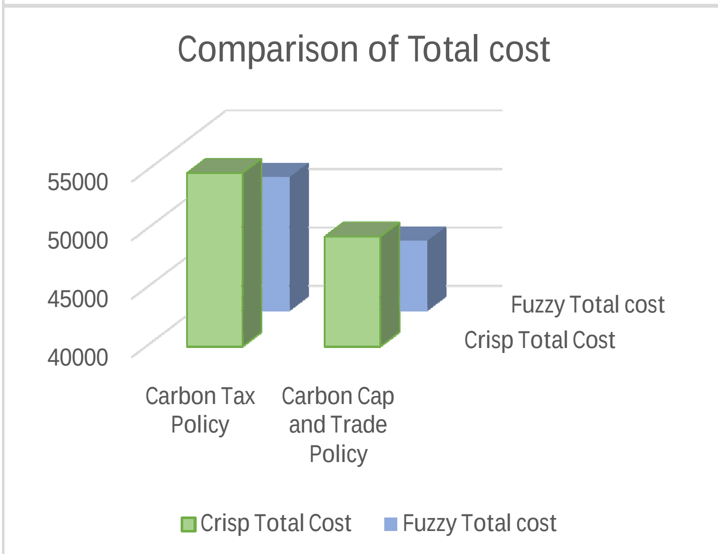
The fuzzy optimum total cost , $P\left(T\tilde{C}_{cct}(Q_{cct})\right) = 46000.31736.$





II. CONCLUSION

In the global environment, uncertainty exists everywhere making or creating a path to rectify it. This study considered a green inventory model under two policies and implemented magnitude ranking defuzzification approach by considering certain parameters with ambiguous nature as trapezoidal fuzzy numbers. Comparison of crisp and fuzzy perspective solutions is performed for both cases which helps to identify the effectiveness of fuzzy idea in green inventory system. Here it is observed that the fuzzy optimum order quantity under two cases provides same result but slightlythat is 0.32 size lower than crisp order quantity. And fuzzy optimum total cost in two cases differs but most importantly compared to crisp results it gets reduced and thus resulting in minimized total expenses. Hence this research work proves the advantage of fuzzy frame work in supply chain especially with sustainable features.

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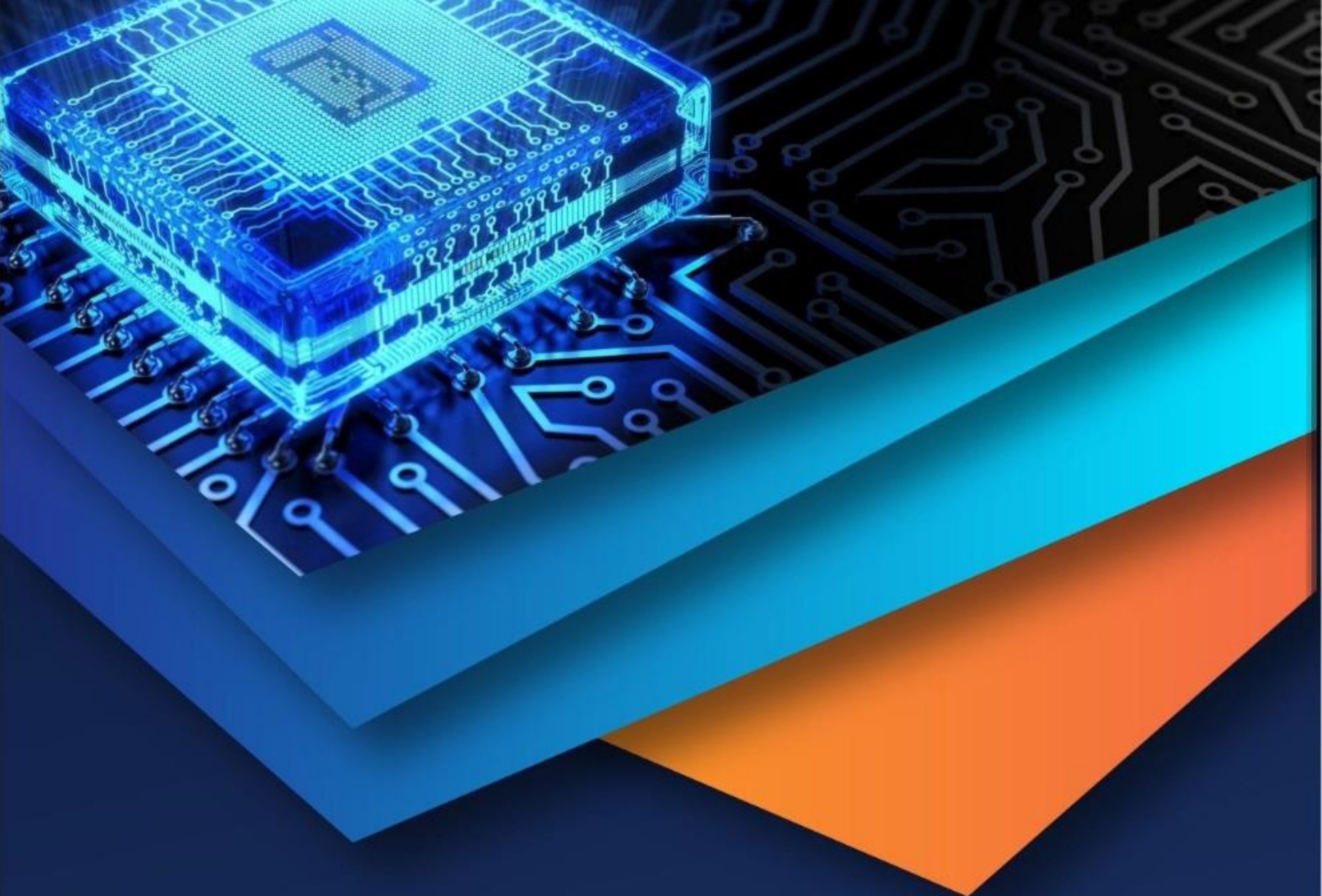

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