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A New Direction Towards Plus weighted Grammar

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Abstract: The core of this paper is to establish plus weighted grammar and to illustrate the language accepted by the pwfa and pwg are equivalent.
Keywords: Plus weighted grammar, plus weighted automata, regular language.

I. INTRODUCTION

Weighted context free grammars and weighted finite automata were initially introduced in significant articles by Marcel-Paul Schutzenberger (1961) and Noam Chomsky (1963), respectively.Weighted finite automata are standard nondeterministic finite automata in which the transitions have weights.We consider the following scenarios to demonstrate the variation of weighted finite automata. We may determine the wide range of a word by counting the number of paths that can be used to represent it as follows: Let each transition have a weight of 1, and for a path that is taken again, the sum of the weights of its successful paths. The wide range of a word equals the sum of its successful paths' weights. The algebraic structures of a semiring involve the computations with weights in the previously mentioned illustration. Here the multiplication of semiring is utilised for estimating the weights of the paths and the weight of the word is successively predicted by the sum of the weights of its successful paths.Applications for weighted automata are numerous. Weighted automata and their accompanying algorithm are developed by contemporary spoken-dialog or handheld speech recognition systems to express their concepts and promote successful combination and search [1,7].

A plus weighted automata [8,9,10,11] is an automata that deals with plus weights up to infinity. Many algebraic structures of plus weighted automata has been discussed in [8,9,10]. A grammar related to this automata is proposed in this paper. This study is a generalisation of plus weighted multiset grammar [9].Plus weighted grammar (pwg) can also be extended further in right linear and left linear grammar. The plus weighted automata can be applied in max weighted automata cited as[2,3,4,5,6]. This work can be further motivated to work in field of graph theory [13,14,15,16].

In addition to this section, this paper comprises four more. Basic concepts and notions are discussed in Section 2 for usage in later parts. In Section 3, a new grammar called pwg is proposed which offers a fresh perspective on pwfa and it elaborates with illustration that for every plus weighted regular grammar there exists a pwfa. The final section, Section 4, concludes and describes the future extension of pwg.

II. PRELIMINARIES

In this section we review some basic notions and definitions about grammar and its types.

Definition 2.1 A phrase-structure grammar or grammar is a four tuple $G = < V_N, V_T, S, P >$ where, V_N is a set of non-terminal symbols, V_T is a set of terminal symbols called alphabets, S is a special element of V_N and is called the starting symbol, P is the production. Relation on $(V_T \cup V_N)^*$, the set of strings of elements of terminals and non-terminals.

Types of grammar

(i) Type 0 or unrestricted grammar:

A grammar in which there are no restrictions on its productions.(ii)Type 1 or context sensitive grammar:Grammar that contains only productions of the form $\alpha \to \beta$ where $|\alpha| \le |\beta|$ and $\alpha, \beta \in (V_T \cup V_N)^+$. (iii) Type 2 or context free grammar:Grammar that contains only productions of the form $\alpha \to \beta$ where $|\alpha| \le |\beta|$ and $\alpha \in V_N$. (iv) Type 3 or Regular grammar:Grammar that contains productions of the form $\alpha \to \beta$ where $|\alpha| \le \beta, \alpha \in V_N$ and β is of the form a or aB where $a \in V_T$ and $B \in V_N$.

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III. PLUS WEIGHTED REGULAR GRAMMAR AND THEIR FORMS

Definition 3.1 A plus weighted Automata (pwfa) is a sextuple $M_p = (Q, \Sigma, W, \alpha, \pi_I, \eta_F)$ where, Q is a finite non-empty set of states, Σ is a finite non-empty set of input symbols, W is a weighting space. That is, $W = ([0,\infty),+,\cdot)$, where '+' is usual additions and '$\cdot$' is usual multiplication, The weighted subset $\alpha : Q \times \Sigma \times Q \to [0,\infty)$ is a function called the weighted transition function, π_I is a weighted subset of Q. That is, $\pi_I : Q \to [0,\infty)$ called the weighted subset of initial states, η_F is a weighted subset of final states. That is, $\eta_F : Q \to [0,\infty)$ called the weighted subset of final states.

Definition 3.2 A plus weighted grammar (pwg) is a 5-tuple $G = (V_N, \Sigma_T, W, S, P_{pw})$ where, V_N is a non-empty finite set of variables, Σ_T is a non-empty set of terminals such that $V_N \cap \Sigma_T = \Phi$. W is a weighting space. i.e., $W = ([0,\infty),+,\cdot)$ where '+' is usual additions and '$\cdot$' is usual multiplication, S is a plus weighted set of V_N That is, $S : V_N \to [0,\infty)$ called the set of plus weighted initial variables, P_{pw} is a plus weighted set of $(V_N \cup \Sigma_T)^* \times (V_N \cup \Sigma_T)^*$ such that the following conditions holds.

If $P_{pw}(\alpha,\beta) = \delta > 0$, for $\alpha, \beta \in (V_N \cup \Sigma_T)^*$ then the string α contains atleast one variable from V_N.

Definition 3.3A plus weighted grammar $G = (V_N, \Sigma_T, W, S, P_{pw})$ is said to be regular if each of its plus weighted production is either of the form $A \xrightarrow{\delta} aB, A \xrightarrow{\delta} a,$ where

$A, B \in V_N, a \in \Sigma_T$ and $\delta > 0.$

Definition 3.4A string $x \in \Sigma^*$ is said to be generated by the plus weighted regular grammar G, if

$$\sum_{A_0 \in V_N} \{S(A_0) \cdot d(A_0 \Rightarrow x)\} > 0$$

Equivalently, there exists $A_0 \in V_N$ such that $S(A_0) > 0$ and $d(A_0 \Rightarrow x) > 0.$

Definition 3.5 The set of all strings $x \in \Sigma^*$ that are generated by a plus weighted regular grammar G is called the plus weighted regular language. We shall denote it by L(G).

Theorem 3.6

Given a plus weighted finite automata $M_p = (Q, \Sigma, W, \alpha, \pi_I, \eta_F)$, there is a plus weighted regular grammar G such that $L(G) = L(M_P) - \{\wedge\}.$

Proof

Let $M_p = (Q, \Sigma, W, \alpha, \pi_I, \eta_F)$ be a plus weighted finite automata. We construct a plus weighted grammar $G = (V_N, \Sigma_T, W, S, P_{pw})$ where,

$V_N = Q \cup \{S\} - \{q_f\}$ and $a \in \Sigma_T$.

For all $p, q, q_0 \in V_N, a, b \in V_T$.

Production P_{pw} is defined as,

(i) $q \xrightarrow{\delta} b = \sum_{r \in Q} \alpha(q,b,r) \cdot \eta_F(r)$ (ii) $S \xrightarrow{\delta} bq_0 = \sum_{q,r \in Q} \pi_I(q_0) \cdot \alpha(q_0,b,r)$

(iii) $S \xrightarrow{\delta} b = \sum_{q,r \in Q} \pi_I(q_0) \cdot \alpha(q_0,b,r) \cdot \eta_F(r)$

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(iii) $q \xrightarrow{\delta} ap = \alpha(q,a,p)$.

Suppose $x \in L(M_P) - \{\wedge\}$.

Suppose $\beta \neq \wedge, \beta = x_1 x_2 \ldots x_n$ where $x_i \in \Sigma_T$ for every $i = 1,2,\ldots,n$. So there is a sequence of transition steps caused by β from states $p(\pi_I(p) \neq 0)$ into $q(\eta_F(q) \neq 0)$ through a sequence of states. If the sequence of states is empty, we have $\alpha(p,\beta,q) = t$. Therefore there is a derivation of β of the form $S \overset{t'}{\Rightarrow} \beta$ where $S \xrightarrow{t'} \beta \in G$ and

$$t' = \sum_{q,r \in Q} \{\pi_I(q) \cdot \alpha(q,\beta,r) \cdot \eta_F /$$

$$q,r \in Q\}.$$

$$= \pi_I(p) \cdot t \cdot \eta_F(q) \forall p,q \in Q.$$

Otherwise we suppose the sequence of states is

$$r_1^1, r_2^1, \cdots r_{n_1}^1, q_1, r_1^2, \cdots q_{m-1}, r_1^m, \cdots r_{n_m}^m,$$

where $m > 1, n_1, n_2, \cdots n_m \geq 0$ or $r_1^1, r_2^1, \cdots r_{n_1}^1, q_1, r_1^2, \cdots q_{m-1}, r_1^m, \cdots r_{n_m}^m, q_m, r_1^{m+1}, \cdots r_{m+1}^{m+1}$,

where $m \geq 1, n_1, n_2, \cdots n_m, n_{m+1} \geq 0$. That is we have , $\alpha^*(p,\beta,q) = t'$

where $t' = \rho_1^1 \cdot \rho_2^1 \cdots \rho_{n_1}^1 \cdot t_1 \cdot \rho_1^2 \cdots t_{m-1} \cdot \rho_1^m \cdots \rho_{n_m}^m \cdot \rho_m$ or $t = \rho_1^1 \cdot \rho_2^1 \cdots \rho_{n_1}^1 \cdot t_1 \cdot \rho_1^2 \cdots t_{m-1} \rho_1^m \cdots \rho_{n_m}^m \cdot t_m \cdot \rho_1^{m+1} \cdot \rho_2^{m+1} \cdots \rho_{n_{m+1}}^{m+1} \cdot t_{m+1}$

Therefore there is a derivation of β with the form

$$S \overset{\rho_1^1}{\Rightarrow} r_1^1 \overset{\rho_2^1}{\Rightarrow} \cdots \overset{\rho_{n_1}^1}{\Rightarrow} r_{n_1}^1 \overset{t_1}{\Rightarrow} a_1 q_1 \overset{\rho_1^2}{\Rightarrow} \cdots \overset{t_{m-1}}{\Rightarrow} a_1 a_2 \cdots a_{m-1} q_{m-1} \overset{\rho_1^m}{\Rightarrow} a_1 a_2 \cdots a_{m-1} r_1^m \overset{\rho_2^m}{\Rightarrow} \cdots \overset{\rho_{n_m}^m}{\Rightarrow} a_1 a_2 \cdots a_{m-1} r_{n_m}^m$$

$$\overset{t_m}{\Rightarrow} a_1 a_2 \cdots a_m = x_1 x_2 \cdots x_n \text{ where}$$

$$\rho_1^1 = \sum_{q \in Q} \{\pi_I(q) \cdot \alpha(q,b,r_1^1)\}$$

$$\geq \pi_I(p) \cdot \alpha(q,b,r_1^1)$$

$$= \pi_I(p) \cdot \rho_1^1$$

and

$$t'_m = \sum_{r \in Q} \{\alpha(r_{n_m}^m, a_m, r) \cdot \eta_F(r)\}$$

$$\geq \alpha(r_{n_m}^m, a_m, q) \cdot \eta_F(q)$$

$$= t_m \cdot \eta_F(q)$$

or of the form

$$S \overset{\rho_1^1}{\Rightarrow} r_1^1 \overset{\rho_2^1}{\Rightarrow} \cdots \overset{\rho_{n_m}^m}{\Rightarrow} a_1 a_2 \cdots a_{m-1} r_{n_m}^m \overset{t_m}{\Rightarrow} a_1 a_2 \cdots a_m q_m \overset{\rho_1^{m+1}}{\Rightarrow} a_1 a_2 \cdots a_m r_1^{m+1} \overset{\rho_2^{m+1}}{\Rightarrow} \cdots$$

$$\overset{\rho_{n_{m+1}}^{m+1}}{\Rightarrow} a_1 a_2 \cdots a_m r_{m+1}^{m+1} \overset{t_{m+1}}{\Rightarrow} a_1 a_2 \cdots a_m = x_1 x_2 \cdots x_n \text{ where}$$

$$\rho_1'^1 = \sum_{q\in Q}\{\pi_I(q)\cdot\alpha(q,b,r_1^1)\}$$
$$\geq \pi_I(p)\cdot\alpha(p,b,r_1^1)$$
$$= \pi_I(p)\cdot\rho_1^1 \text{ and}$$
$$t'_{m+1} = \sum_{q\in Q}\{\alpha(r_{m+1}^{m+1},b,r)\cdot\eta_F(r)\}$$
$$\geq \alpha(r_{m+1}^{m+1},b,q)\cdot\eta_F(q)$$
$$= t_{m+1}\cdot\eta_F(q)$$

i.e., We have $S \overset{t'}{\Rightarrow} * \beta$ where

$$t' = \rho_1'^1\cdots\rho_{n_1}^1\cdot t_1\cdot\rho_1^2\cdots t_{m-1}\cdot\rho_1^m\cdots\rho_{n_m}^m\cdot t'_m. \geq \pi_I(p)\cdot\rho_1^1\cdots\rho_{n_1}^1\cdot t_1\cdot\rho_1^2\cdots t_{m-1}\cdot$$
$$\rho_1^m\cdots\rho_{nm}^m\cdot t_m\cdot\eta_F(q)$$
$$= \pi_I(p)\cdot t\cdot\eta_F(q) \text{ or}$$

$$t' = \rho_1'^1\cdots\rho_{n_1}^1\cdot\rho_1^2\cdots t_{m-1}\cdot\rho_1^m\cdots\rho_{n_m}^m\cdot t_m\cdot\rho_1^{m+1}\cdots\rho_{n_{m+1}}^{m+1}\cdot t'_{m+1}$$
$$\geq \pi_I(p)\cdot\rho_1^1\cdots\rho_{n_1}^1\cdot t_1\cdot\rho_1^2\cdots t_{m-1}\cdot \begin{matrix}\rho_1^m\cdots\rho_{n_m}^m\cdot t_m\cdot\rho_1^{m+1}\cdots\rho_{n_{m+1}}^{m+1}\cdot t_{m+1}\cdot\\ \eta_F(q).\end{matrix}$$
$$= \pi_I(p)\cdot t\cdot\eta_F(q)$$

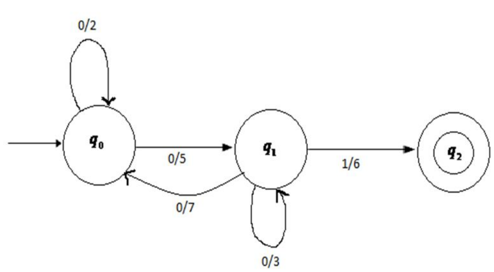
Because for every sequence of transition steps caused by β in P_{pw}, there exist a derivation generating β with greater or equal to value in G and furthermore, we have

$\alpha G(\beta) \geq \alpha p(\beta)$.Therefore we have $L(M_p)\subseteq L(G)$.Similarly suppose $(\beta,\alpha_G(\beta))\in L(G)$. We can prove for every derivation of β in G, there exist a sequence of transition steps accepting β with the same value in P_{pw} and furthermore $\alpha p(\beta)\geq\alpha_G(\beta)$ i.e., $L(G)\subseteq L(M_p)$. Thus $L(G)=L(M_P)$.

Example 3.7

Consider a plus weighted finite automata $M_p = (Q,\Sigma,W,\alpha,\pi_I,\eta_F)$ where,

$$Q=\{q_0,q_1,q_2\},\pi_I=\{2/q_0\},$$
$$\eta_F=\{3/q_2\},\Sigma_{in}=\{0,1\} \text{ and}$$
$$\alpha(q_0,0,q_0)=2,\alpha(q_0,0,q_1)=5,\alpha(q_1,0,q_1)=3,\alpha(q_1,1,q_2)=6,\alpha(q_1,0,q_0)=7$$



Plus Weighted Automata

$$L(001) = \pi_I(q_0)\cdot\alpha(q_0,0,q_0)\cdot\alpha(q_0,0,q_1)$$
$$\cdot\alpha(q_1,1,q_2)\cdot\eta_F(q_2) + \pi_I(q_0)\cdot\alpha(q_0,0,q_1)$$
$$\cdot\alpha(q_1,0,q_1)\cdot\alpha(q_1,1,q_2)\cdot\eta_F(q_2)$$

=2.2.5.6.3+2.5.3.6.3=360+540=900

By the use of above theorem one can construct an equivalent plus weighted regular grammar G as

$$G = (V_N, \Sigma_T, W, S, P_{pw})$$
$$G = (\{q_0, q_1, q_2\}, \{0,1\}, \{2/q_0\}, P_{pw})$$
$$P_{pw} = S \xrightarrow{4} 0q_0, S \xrightarrow{10} 0q_1,$$
$$q_1 \xrightarrow{3} 0q_1, q_1 \xrightarrow{18} 1,$$
$$q_1 \xrightarrow{14} 0q_0, q_0 \xrightarrow{2} 0q_0,$$
$$q_0 \xrightarrow{5} 0q_1$$

(i) $S \overset{4}{\Rightarrow} 0q_0 \overset{5}{\Rightarrow} 00q_1 \overset{18}{\Rightarrow} 001$

(ii) $S \overset{10}{\Rightarrow} 0q_1 \overset{3}{\Rightarrow} 0q_1 \overset{18}{\Rightarrow} 001$

L(X) =4.5.18+10.3.18=360+540

= 900

Theorem 3.8

Given a plus weighted regular grammar

$G = (V_N, \Sigma_T, W, S, P_{pw})$, there is a plus weighted finite automata M_P such that $L(M_P) = L(G)$.

Proof

Construction of the required plus weighted finite automata will be as follows:

$Q = V_N \cup \{q_f\}$, where q_f not in V_N, $\pi_I = S, \eta_F = \{1/q_f\}$,

and α is defined as ,

$\alpha(q,a,p) = \delta$ if and only if $q \xrightarrow{\delta} ap.$

$\alpha(q,a,q_f) = \delta$ if and only if $q \xrightarrow{\delta} a.$

IV. CONCLUSION

This paper introduces a different approach on plus weighted grammar, this proposal can be applied in numerous works done in fuzzy grammar and fuzzy multiset grammar. Further plus weighted algebra-related tasks can be extended in the future.

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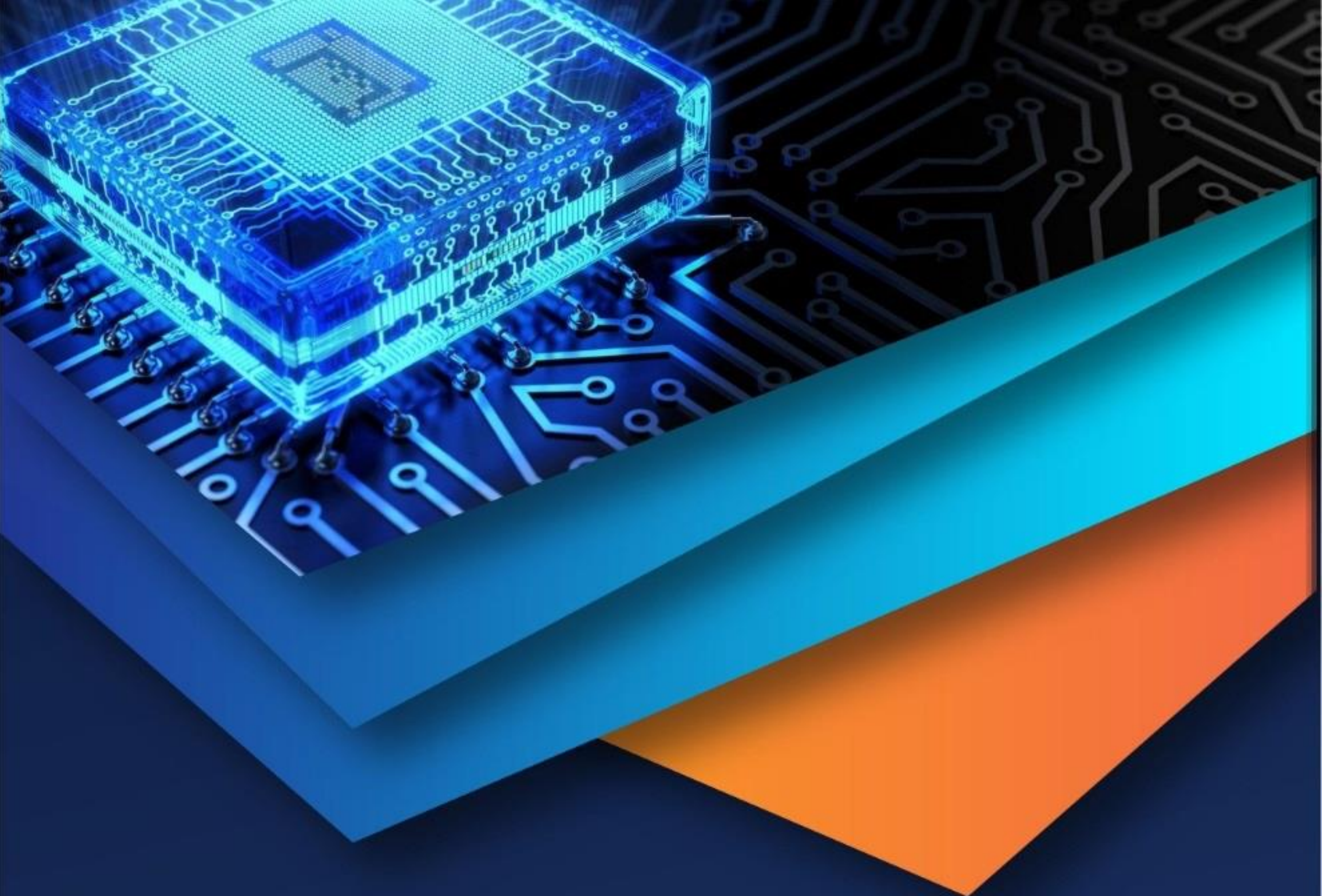

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