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A Novel Shape Adjustable Quartic Trigonometric Bézier Curve and its Application in Engineering Design

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Abstract: In this study, we introduce a novel class of quartic trigonometric Bézier curves characterized by two adjustable shape parameters, λand μ. Based on these curves, the associated quartic trigonometric Bézier surface formulation is also developed. The proposed curves retain most of the fundamental geometric and algebraic properties of the classical quartic Bézier curves defined with the Bernstein polynomial basis, while simultaneously offering additional flexibility that is particularly useful for shape modelling and geometric design. The inclusion of shape parameters provides greater freedom for designers, enabling intuitive and efficient control over the geometry of the curve. By modifying the values of these parameters, the form of the curve can be smoothly adjusted without altering the underlying control polygon. Consequently, a wide variety of smooth shapes can be generated using the same set of control points. Owing to these advantageous characteristics, the proposed curves and surfaces provide an effective and versatile modelling tool for applications in computer-aided geometric design (CAGD), computer graphics, and related fields of geometric modelling.
Keywords: Quartic Trigonometric Bézier basis function, Quartic Trigonometric Bézier curve, Quartic Trigonometric Bézier surface, Shape parameters.

I. INTRODUCTION

The complexity of real-world shapes necessitates advanced geometric modeling tools capable of creating highly adaptable and adjustable curves and surfaces. Bézier curves and surfaces serve as fundamental, indispensable components for creating and manipulating complex shapes in both Computer-Aided Geometric Design (CAGD) and computer graphics. Due to the computational simplicity and high numerical stability of Bézier curves , they are widely employed across science and engineering, to create smooth, precise curves in industrial design, particularly for automotive car bodies and manufacturing components, generating robot paths, animating camera movements, medical imaging (MRI/CT scans), optimize robot trajectories by calculating path length, curvature, and continuous velocity, ensuring smooth motion control. applied in modeling railway routes and highway design, where smooth, continuous curves are required, solving complex differential equations and for data interpolation, particularly in CAD systems, animation, robotic path planning, network design, and the modeling of transportation routes.

The classical Bézier curves have some limitations that their shape and position are fixed relative to their defining control polygon. Thus recent research has shifted toward investigating alternative solutions within non-polynomial function spaces. In recent years, the academic focus has shifted significantly toward employing trigonometric functions or combining them with polynomial forms to enhance approximation and modelling.

Trigonometric B-splines were first presented in [1]. In recent years, several new trigonometric splines have been studied in the literature; see [2], [3] and [4]. In [5] cubic trigonometric Bézier curve with two shape parameters were presented. In [6], a novel generalization of Bézier curve and surface with n shape parameters are presented. In [7], Quartic splines with C^2 continuity are presented for a non-uniform knot vectors which are C^2 and G^3 continuous under special case. Algebraic-Trigonometric blended spline curves are presented in [8] which can represent some transcendental curves. Recently in [9], a quadratic trigonometric Bézier curve with shape parameter is constructed which is G^1 continuous. The cubic trigonometric polynomial spline curve of G^1 continuity is constructed in [10], which can be G^3 continuity under special condition. In [11], the cubic trigonometric polynomial curve similar to the cubic Bézier curves is constructed. In [12], the shape features of the cubic trigonometric polynomial curves with a shape parameter are analysed. In [13], [14] and [15] Quartic and cubic trigonometric Bézier curve respectively with shape parameter is presented and the effect of shape parameter is studied.

In [16], a new kind of splines, called cubic trigonometric polynomial B-spline (cubic TP B-spline) curves with a shape parameter, are constructed. In [17], Quartic trigonometric Bézier Curves and surfaces with single shape parameter is presented. In these curves, if we change the value of shape parameter, the entire curve changes. This is not the desirable situation in shape designing. So, to overcome these shortcomings, we have introduced the same Quartic trigonometric Bézier Curve with two shape parameters. By using two shape parameters, we can modify the shape of the one segment of the curve in two different directions, without changing the control polygon. In [18]application of piecewise cubic functions for constructing a Bezier type curve of C1 smoothness are presented. In [19], Quartic trigonometric Bézier Curves and surfaces with two shape parameters are discussed. In [20], a class of quasi-quintic trigonometric Bezier curve with two shape parameters, based on newly constructed trigonometric basis functions, is presented. A new class of sextic trigonometric Bernstein basis functions with two shape parameters along with their geometric properties which are similar to the classical Bernstein basis functions is studied in [21]. In [22], a new recursive formula in explicit expression is constructed that produces the generalized blended trigonometric Bernstein (or GBT-Bernstein, for short) polynomial functions of degree *m*.

In this paper a new Quartic trigonometric Bézier Curve with two shape parameters is presented. The paper is organized as follows. In section 2, Quartic trigonometric Bézier basis functions with two shape parameters are given. In section 3, Quartic trigonometric Bézier curves are given. In section 4, the representation of Quartic trigonometric Bézier surface has been shown. Construction of various models by Quartic Trigonometric Bézier curves and surfaces are presented in Section 5. Composite Quartic Trigonometry Bézier curves and surfaces are given in Section 6. Conclusion is given in Section 7.

II. QUARTIC TRIGONOMETRIC BÉZIER BASIS FUNCTIONS WITH TWO SHAPE PARAMETERS

Firstly, the definition of quartic trigonometric Bézier basis functions is given as follows.

A. *The construction of the Quartic trigonometric Bézier basis functions*

Definition 1. For an arbitrarily selected real values of λ and μ where $\lambda, \mu \in$ [-1, 1], the following five functions of $t\left(t \in \left[0,\frac{\pi}{2}\right]\right)$ are defined as Quartic Trigonometric Bézier basis functions with two shape parameters λ and μ:

$$\begin{cases} b_0(t) = (1-\sin t)^3\,(1-\lambda\sin t) \\ b_1(t) = (3/2)\sin t\,(1-\sin t)^2\big(1+\lambda(1-\sin t)\big) \\ b_2(t) = -1+\frac{1}{2}\{(3-\lambda)\sin t+(3-\mu)\cos t\}+\frac{3}{2}(\lambda\, sin^2\, t+\mu\, cos^2\, t) \\ \qquad -\frac{1}{2}\{(1+3\lambda)\, sin^3\, t+(1+3\mu) cos^3\, t\}+\frac{1}{2}(\lambda\, sin^4\, t+\mu\, cos^4\, t) \\ b_3(t) = (3/2)\cos t\,(1-\cos t)^2\big(1+\mu(1-\cos t)\big) \\ b_4(t) = (1-\cos t)^3(1-\mu\cos t) \end{cases} \qquad (1)$$

B. *The properties of the Quartic trigonometric Bézier basis functions*

Theorem 1: The basis functions (1) have the following properties:

(a) Non-negativity: $b_i(t) \geq 0$ for i = 0, 1, 2, 3, 4.

(b) Partition of unity: $\sum_{i=0}^{4} b_i\,(t) = 1$

(c) Symmetry: $b_i(t;\ \lambda,\mu) = b_{4-i}\left(\frac{\pi}{2} - t;\ \mu,\lambda\right)$, for i = 0, 1, 2, 3, 4.

(d) Monotonicity: For a given parameter t, as the shape parameters λ and μ increases (or decreases),$b_0(t), b_2(t)$ and $b_4(t)$decreases (or increases) and $b_1(t), b_3\,(t)$ increases (or decreases).

(e) Terminal Property: $b_0(0) = 1$, $b_1(0) = 0$, $b_2(0) = 0$, $b_3(0) = 0$, $b_4(t) = 0$,
$b_0\left(\frac{\pi}{2}\right) = 0$, $b_1\left(\frac{\pi}{2}\right) = 0$, $b_2\left(\frac{\pi}{2}\right) = 0$, $b_3\left(\frac{\pi}{2}\right) = 0$, $b_4\left(\frac{\pi}{2}\right) = 1$

Proof :(a) For t ∈ [0, π/ 2] and $\lambda, \mu \in [-1, 1]$, then 0 ≤ (1 − sin t) ≤ 1, 0 ≤ (1 − cos t) ≤ 1, 0 ≤ sin t ≤ 1, 0 ≤ cos t ≤ 1. It is obvious that $b_i(t) \geq 0$ for i = 0, 1, 2, 3, 4.

(b) $\sum_{i=0}^{4} b_i\,(t) = (1-\sin t)^3\,(1-\lambda\sin t) + (3/2)\sin t\,(1-\sin t)^2\big(1+\lambda(1-\sin t)\big) + -1 + \frac{1}{2}\{(3-\lambda)\sin t + (3-\mu)\cos t\} + \frac{3}{2}(\lambda\, sin^2\, t + \mu\, cos^2\, t) - \frac{1}{2}\{(1+3\lambda)\, sin^3\, t + (1+3\mu)\, cos^3\, t\} + \frac{1}{2}(\lambda\, sin^4\, t + \mu\, cos^4\, t) + (3/2)\cos t\,(1-\cos t)^2\big(1 + \mu(1-\cos t)\big) + (1-\cos t)^3(1-\mu\cos t)$
$= 1$

The remaining cases follow obviously. The curves of the Quartic Trigonometric Bézier basis functions are shown in Figure 1 for different values of shape parameters λ and μ. In these figures, $b_0(t), b_1(t), b_2(t), b_3(t)$ and $b_4(t)$ are shown by cyan lines, red lines, green lines, blue lines and magenta lines respectively. Also, change in the values of shape parameter λ affects only $b_0(t), b_1(t)$ and $b_2(t)$ while change in the values of shape parameter μ affects only $b_2(t), b_3(t)$and $b_4(t)$. Figure 2 shows Quartic Trigonometric Bézier basis functions with $\lambda = -1,\ 0,\ 1$ and $\mu = -1$ in (a), $\mu = 0$ in (b) and $\mu = 1$ in (c). Figure 3 shows Quartic Trigonometric Bézier curve with $\mu = -1,\ 0,\ 1$ and $\lambda = -1$ in (a), $\lambda = 0$ in (b) and $\lambda = 1$ in (c).

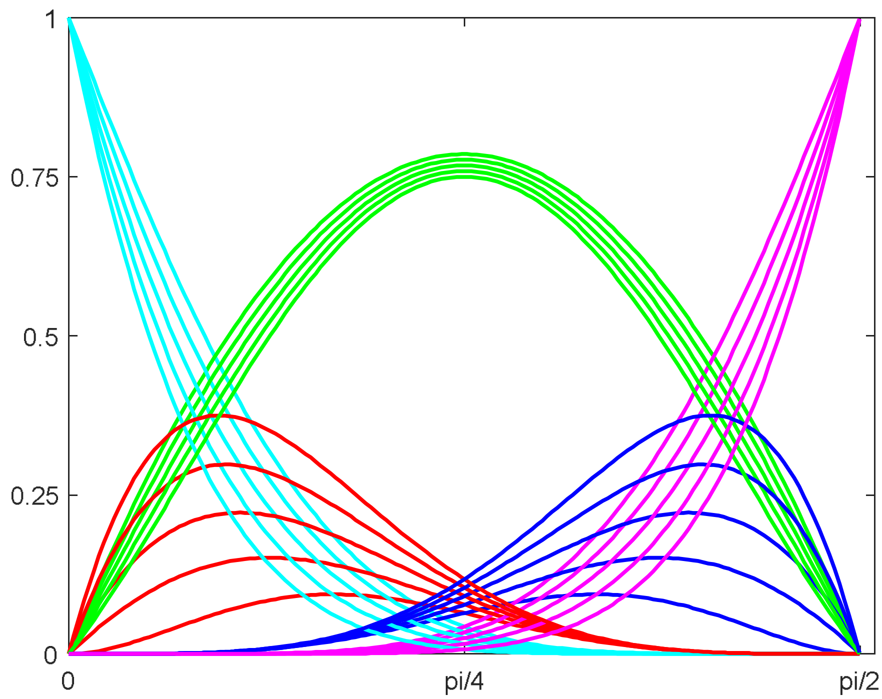
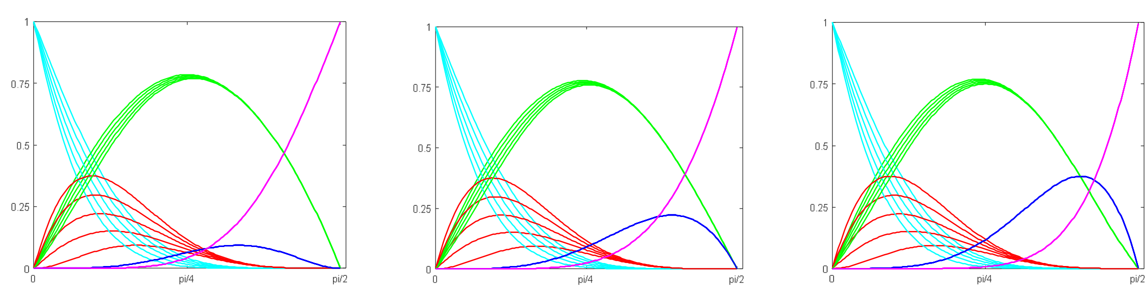


Fig. 1. Quartic Trigonometric Bézier basis functions for different values of λ and μ



(a) (b) (c)
Figure 2: Quartic Trigonometric Bézier Basis functions with $\lambda = -1,\ 0,\ 1$ and $\mu = -1$ in (a), $\mu = 0$ in (b) and $\mu = 1$ in (c)

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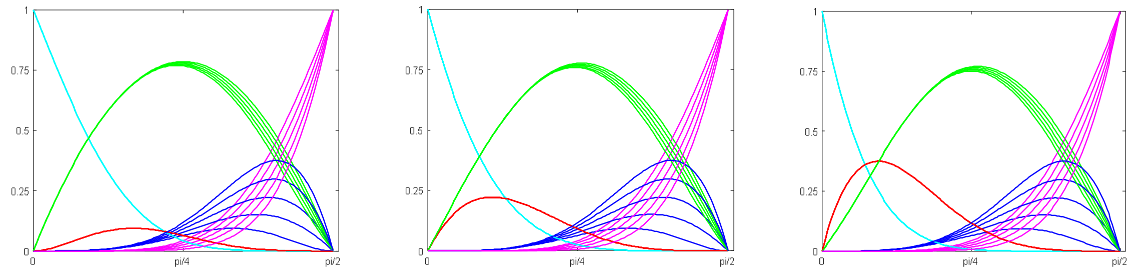

(a) (b) (c)

Figure 3: Quartic Trigonometric Bézier Basis functions with $\mu = -1, 0, 1$ and $\lambda = -1$ in (a), $\lambda = 0$ in (b) and $\lambda = 1$ in (c)

III. QUARTIC TRIGONOMETRIC BÉZIER CURVES WITH TWO SHAPE PARAMETERS

A. The Construction of the Quartic Trigonometric Bézier curve

Definition 2. Given control points $P_i = 0,1,2,3,4$ in R^2 or R^3, then

$$r(t) = \sum_{i=0}^{4} b_i(t)P_i;\ \ t \in \left[0, \frac{\pi}{2}\right]; \lambda, \mu \in [-1,1] \quad (2)$$

is called a Quartic Trigonometric Bézier curve with two shape parameters λ and μ.

From the definition of the basis function, some properties of the Quartic Trigonometric Bézier curve can be obtained as follows:

Theorem 2: The Quartic Trigonometric Bézier curves (2) have the following properties:

a) Terminal Properties: $r(0) = P_0,\ r(\pi/2) = P_4$

The First derivative value of curve $r(t)$ at t = 0 and t = $\frac{\pi}{2}$ is

$$r'(0) = -(3+a)P_0 + \frac{3}{2}(a+1)P_1 - \frac{1}{2}(a-3)P_2$$

$$r'\left(\frac{\pi}{2}\right) = \frac{1}{2}(b-3)P_2 - \frac{3}{2}(b+1)P_3 + (b+3)P_4$$

The Second derivative value of curve $r(t)$ at t = 0 and t = $\frac{\pi}{2}$ is:

$$r''(0) = 6(a+1)P_0 - 3(3a+2)P_1 + 3aP_2$$

$$r''(\pi/2) = 3bP_2 - 3(3b+2)P_3 + 6(b+1)P_4$$

The Third derivative value of curve $r(t)$ at t = 0 and t = $\frac{\pi}{2}$ is:

$$r'''(0) = -(17a+3)P_0 + \frac{1}{2}(51a+15)P_1 - \frac{1}{2}(17a+9)P_2$$

$$r'''(\pi/2) = \frac{1}{2}(17b+9)P_2 - \frac{1}{2}(51b+15)P_3 + (17b+3)P_4$$

b) Symmetry: P_0, P_1, P_2, P_3, P_4 and P_4, P_3, P_2, P_1, P_0 define the same Quartic Trigonometric Bézier curve in different parametrizations, i.e.,

$r(t; \lambda, \mu; P_0, P_1, P_2, P_3, P_4) = r(\pi/2 - t; \mu, \lambda; P_4, P_3, P_2, P_1, P_0)$; $t \in [0, \pi/2]$, $\lambda, \mu \in [-1, 1]$

c) Geometric invariance: The shape of a Quartic Trigonometric Bézier curve is independent of the choice of coordinates, i.e. (2) satisfies the following two equations:

$$r(t; \lambda, \mu; P_0 + q, P_1 + q, P_2 + q, P_3 + q, P_4 + q) = r(t; \lambda, \mu; P_0, P_1, P_2, P_3, P_4) + q$$

$$r(t; \lambda, \mu; P_0 * T, P_1 * T, P_2 * T, P_3 * T, P_4 * T) = r(t; \lambda, \mu; P_0, P_1, P_2, P_3, P_4) * T$$

$$t \in \left[0, \frac{\pi}{2}\right],\ \ \lambda, \mu \in [-1,1]$$

where q is arbitrary vector in R^2 or R^3, and T is an arbitrary $d \times d$ matrix, d = 2 or 3.

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d) Convex hull property: The entire Quartic Trigonometric Bézier curve segment lies inside its control polygon spanned by P_0, P_1, P_2, P_3, P_4.

B. Shape Control of the Quartic Trigonometric Bézier Curve

For $t \in \left[0, \frac{\pi}{2}\right]$, we rewrite (2) as follows:

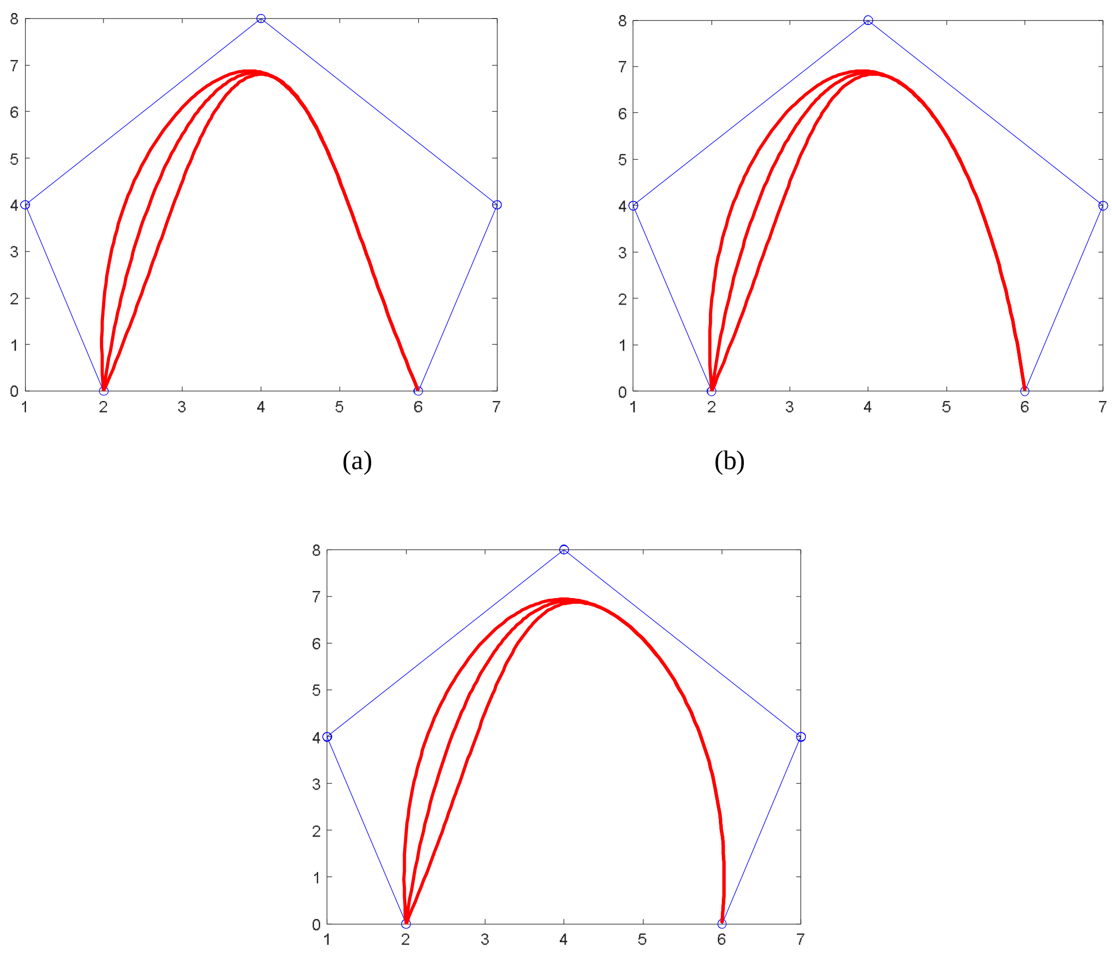
$$r(t) = \sum_{i=0}^{4} P_i c_i(t) + \lambda \sin t(1 - \sin t)^3 \frac{1}{2}[-2P_0 + 3P_1 - P_2) + \mu \cos t\,(1 - \cos t)^3 \frac{1}{2}[-P_2 + 3P_3 - 2P_4]$$

$$= \sum_{i=0}^{4} P_i c_i(t) + \lambda \sin t(1 - \sin t)^3 \left[(P_1 - P_0) - \frac{1}{2}(P_2 - P_1)\right] +$$

$$\mu \cos t\,(1 - \cos t)^3 \left[\frac{1}{2}(P_3 - P_2) - (P_4 - P_3)\right]$$

where $c_0(t) = (1 - \sin t)^3$, $c_1(t) = \frac{3}{2}\sin t(1 - \sin t)^2$, $c_2(t) = \frac{1}{2}(3\sin t + 3\cos t - sin^3 t - cos^3 t - 1)$, $c_3(t) = \frac{3}{2}\cos t(1 - \cos t)^2$, $c_4(t) = (1 - \cos t)^3$.

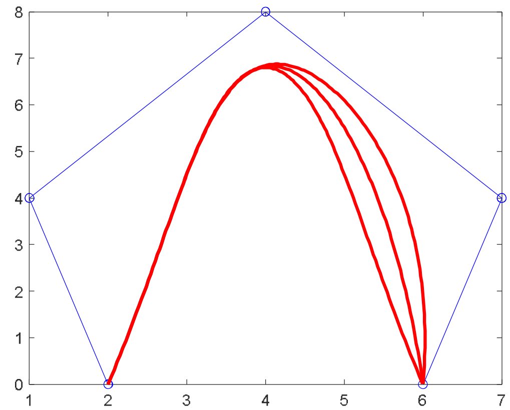
Obviously, shape parameter λ affects the curve on the control edges $(P_1 - P_0)$ and $(P_2 - P_1)$. Similarly shape parameter μ affects the curve on the control edges $(P_3 - P_2)$ and $(P_4 - P_3)$. Figure 4 shows a computed example of quartic trigonometric Bézier Curves with with $\lambda = -1, 0, 1$ and $\mu = -1$ in (a), $\mu = 0$ in (b) and $\mu = 1$ in (c). These curves are generated by setting $P_0 = (2, 0)$, $P_1 = (1, 4)$, $P_2 = (4, 8)$, $P_3 = (7, 4)$, $P_4 = (8, 0)$. Figure 5 shows same computed example of Quartic trigonometric Bézier Curves with $\mu = -1, 0, 1$ and $\lambda = -1$ in (a), $\lambda = 0$ in (b) and $\lambda = 1$ in (c).



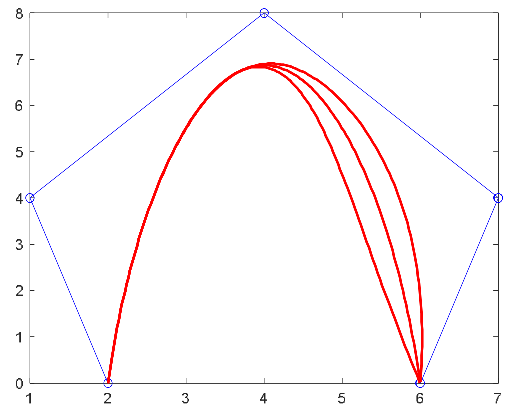
(a) (b)

(c)

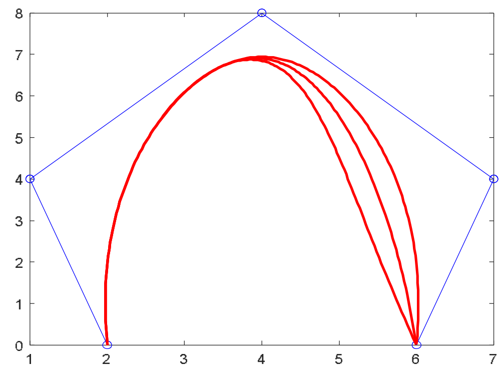
Figure 4: Quartic Trigonometric Bézier curve with $\lambda = -1, 0, 1$ and $\mu = -1$ in (a), $\mu = 0$ in (b) and $\mu = 1$ in (c)



(a)



(b)



(c)

Figure 5: Quartic Trigonometric Bézier curve with $\mu = -1, \ 0, \ 1$ and $\lambda = -1$ in (a), $\lambda = 0$ in (b) and $\lambda = 1$ in (c)

IV. THE QUARTIC TRIGONOMETRIC BÉZIER SURFACES

Given control points $P_{rs}(r = 0, 1, 2, 3, 4; \ s = 0, 1, 2, 3, 4)$, using the tensor product method, we can construct the Quartic Trigonometric Bézier surface

$$T(u, v) = \sum_{r=0}^{4} \sum_{s=0}^{4} b_{r,4}(\lambda_1, \mu_1; u) b_{s,4}(\lambda_2, \mu_2; v) \, P_{rs}; \quad (u, v) \in \left[0, \tfrac{\pi}{2}\right] \times \left[0, \tfrac{\pi}{2}\right]$$

$\lambda_1, \lambda_2; \ \mu_1, \mu_2 \in [-1, 1]$

where $b_{r,4}(\lambda_1, \mu_1; u)$ and $b_{s,4}(\lambda_2, \mu_2; v)$ are the Quartic trigonometric polynomial basis functions with two shape parameters λ_1, λ_2 and μ_1, μ_2.

Construction of various models by Quartic Trigonometric Bézier curves and surfaces are presented in Section 5. Conclusion is given in Section 7.

V. CONSTRUCTION OF VAROIUS MODELS BY QUARTIC TRIGONOMETRIC BÉZIER CURVES AND SURFACES

By choosing different values of shape parameters, we can construct various Quartic Trigonometric Bézier curves and surfaces with one segment only. Figure 1, 2, 3 and 4 shows computed example of Quartic trigonometric Bézier Curves with one segment for different values of shape parameters λ and μ.

Figure 6 shows computed example of Quartic trigonometric Bézier surface for different values of shape parameters λ and μ. This surface is generated by taking control net P(1,1) = (0 , 0 , 0.2779), P(1,2) = (0, 1, 0.75), P(1,3) = (0, 2 , 1.05), P(1,4) = (0, 3, 0.75), P(1,5) = (0, 4, 0.27), P(2,1) = (1, 0, 0.75), P(2,2) = (1, 1, 2.05), P(2,3) = (1, 2, 2.86), P(2,4) = (1, 3, 2.05), P(2,5) = (1, 4, 0.75), P(3,1) = (2, 0, 1.05), P(3,2) = (2, 1, 2.86), P(3,3) = (2, 2, 4), P(3,4) = (2, 3, 2.86), P(3,5) = (2, 4, 1.05), P(4,1) = (3, 0, 0.75), P(4,2) = (3, 1, 2.05), P(4,3) = (3, 2, 2.86), P(4,4) = (3, 3, 2.05), P(4,5) = (3, 4, 0.75), P(5,1) = (4, 0, 0.27), P(5,2) = (4, 1, 0.75), P(5,3) = (4, 2, 1.05), P(5,4) = (4, 3, 0.75), P(5,5) = (4, 4, 0.27) and shape parameters $\lambda = -1, \mu = -1$ in Figure 6 (a), $\lambda = 0, \mu = 0$ in Figure 6 (b) and $\lambda = 1, \mu = 1$ in Figure 6 (c). The corresponding wireframe model is shown in Figure 6 (d).

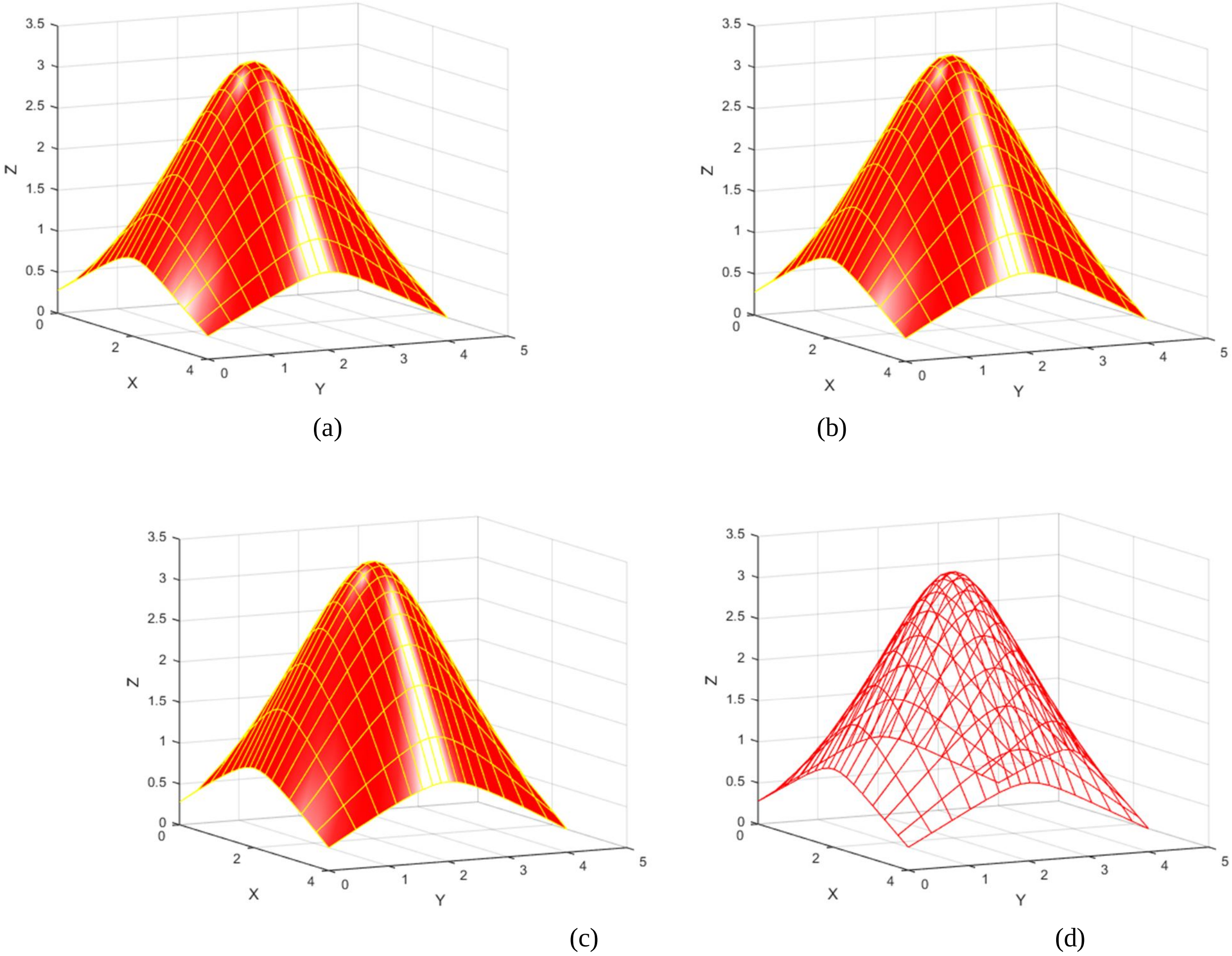
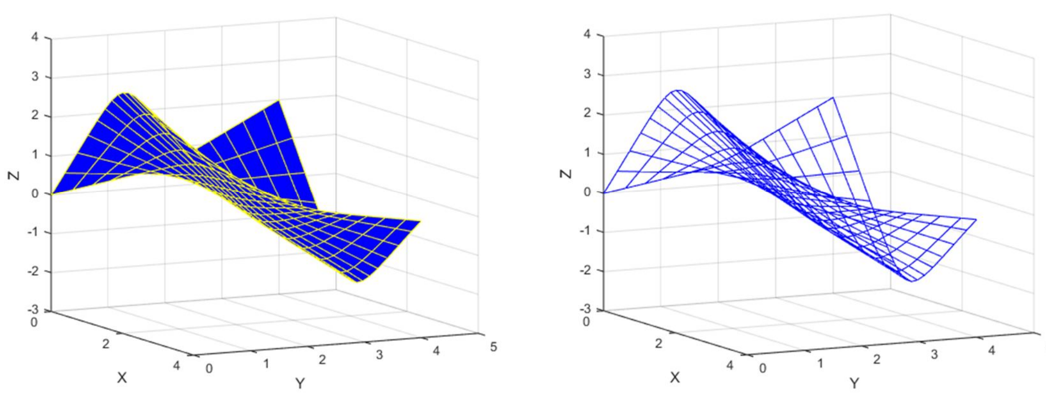


Figure 6: Quartic Trigonometric Bézier surface with $\lambda = \mu = -1$ in (a), $\lambda = \mu = 0$ in (b), $\lambda = \mu = 1$ in (c) and corresponding wireframe model in (d).

Another computed example of Quartic trigonometric Bézier surface with different values of shape parameters λ and μ in Figure 7(a) and its correspomging wireframe model is shown in Figure 7(b).



(a) (b)

Figure 7: Quartic Trigonometric Bézier surface with different values of shape parameters λ and μ

VI. COMPOSITE QUARTIC TRIGONOMETRY BÉZIER CURVES AND SURFACES

The condition of C^1 continuity between two Quartic trigonometric Bézier curves is discussed as follows:

Let a Quartic trigonometric Bézier curve $r(t;\ \lambda,\mu)$ with control points P_i (i = 0, 1, 2, 3, 4) in R^2 or R^3 be given as (2) and a second curve $r_1(t;\ \lambda_1,\mu_1)$ with control points Q_i (i = 0, 1, 2, 3, 4) in R^2 or R^3 by

$$r_1(t) = \sum_{i=0}^{4} b_i(t)\, Q_i,\ \ t \in [0, \pi/2\,];\ \lambda_1,\mu_1 \ \in\ [-1,1]$$

For the composite curve to be C^1 continuous, it is necessary and sufficient that

$$r\,(\,\pi/2\,) = r_1\,(0)$$
$$r'(\,\pi/2) = r_1'(0)$$

According to the terminal properties of Quartic trigonometric Bézier curves, the theorem below shows the condition of continuity of the composite Quartic trigonometric Bézier curves.

Theorem 3: Given two segments of Quartic trigonometric Bézier curves with the control points P_i (i = 0,1, 2, 3, 4) and Q_i (i = 0,1, 2, 3, 4) in R^2 or R^3, then the necessary and sufficient condition of continuity is

i. For C^0 continuity, $P_4 = Q_0$

ii. For C^1 continuity,

$P_4 = Q_0$,

$$\frac{1}{2}(b-3)P_2 - \frac{3}{2}(b+1)P_3 + (b+3)P_4 = -(3+a)Q_0 + \frac{3}{2}(a+1)Q_1 - \frac{1}{2}(a-3)Q_2$$

In Figure 8, an example of shape modelling of composite Quartic trigonometric Bézier surfaces is presented with C^0 continuity. The corresponding Quartic trigonometric Bézier curves and their control polygons are also shown in this figure.

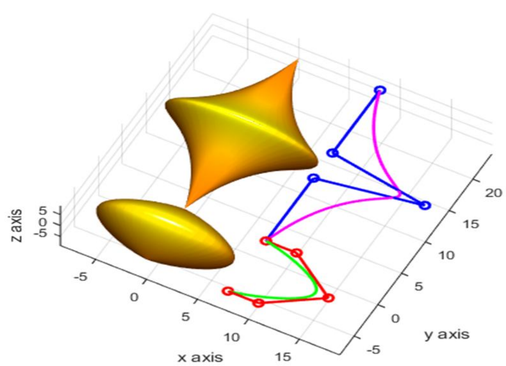


Figure 8: Composite Quartic Trigonometric Bézier surface with C^0 continuity

In Figure 9, another example of shape modelling of composite Quartic trigonometric Bézier surfaces is presented with C^0 continuity.

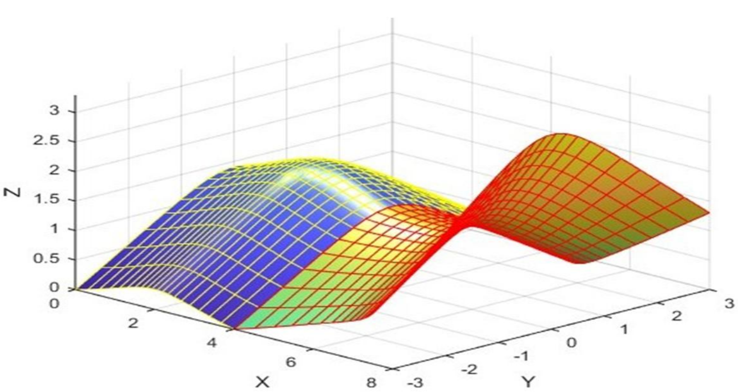


Figure 9: Composite Quartic Trigonometric Bézier surface with C^0 continuity

Figure 10 shows Composite Quartic Trigonometric Bézier surface with C^1 continuity using shape parameters $\lambda = \mu = 1$.

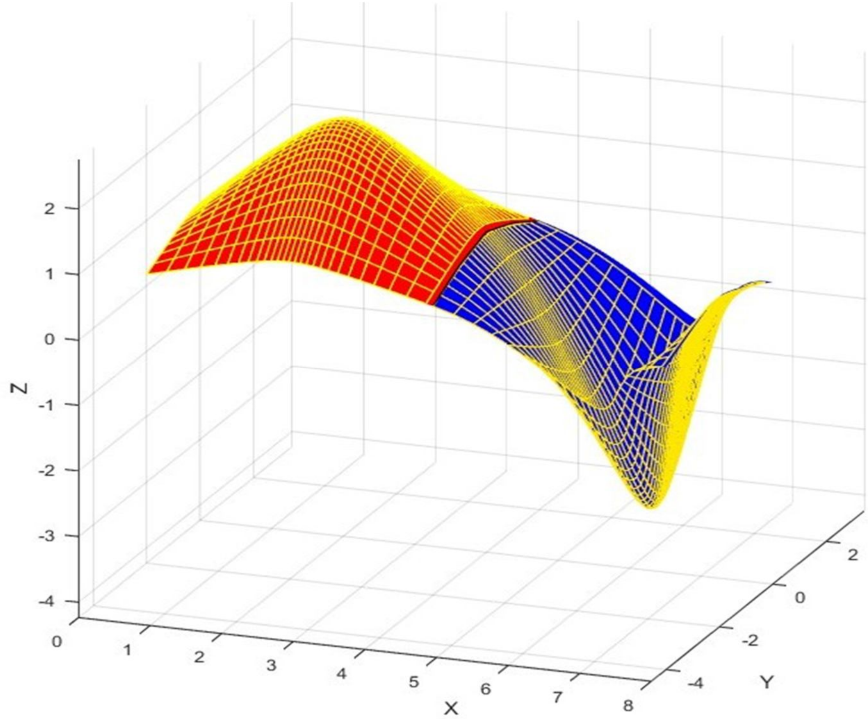


Figure 10: Composite Quartic Trigonometric Bézier surface with C^1 continuity.

VII. CONCLUSION

This paper proposes Quartic Trigonometric Bézier curves with two shape parameters. As mentioned above Quartic Trigonometric Bézier curve can be easily applied for drawing various curves in CAGD (Computer Aided Geometric Designing). The shape of the curve can be flexibly controlled by the shape parameter without altering the control points. Moreover, the constructed curves are governed by only two shape parameters, which allows the shape modification process to be handled with greater ease. Any variation in the curve geometry can be readily controlled and predicted by adjusting these parameters. Consequently, the proposed technique is both meaningful and practical, as it aids in simplifying the development and computational implementation of complex curves. The Quartic Trigonometric Bézier curves presented in this paper is still a polynomial model of the degree four. Hence, it has simpler structure. Some other important results of these curves will be presented in the following study.

VIII. ACKNOWLEDGEMENTS

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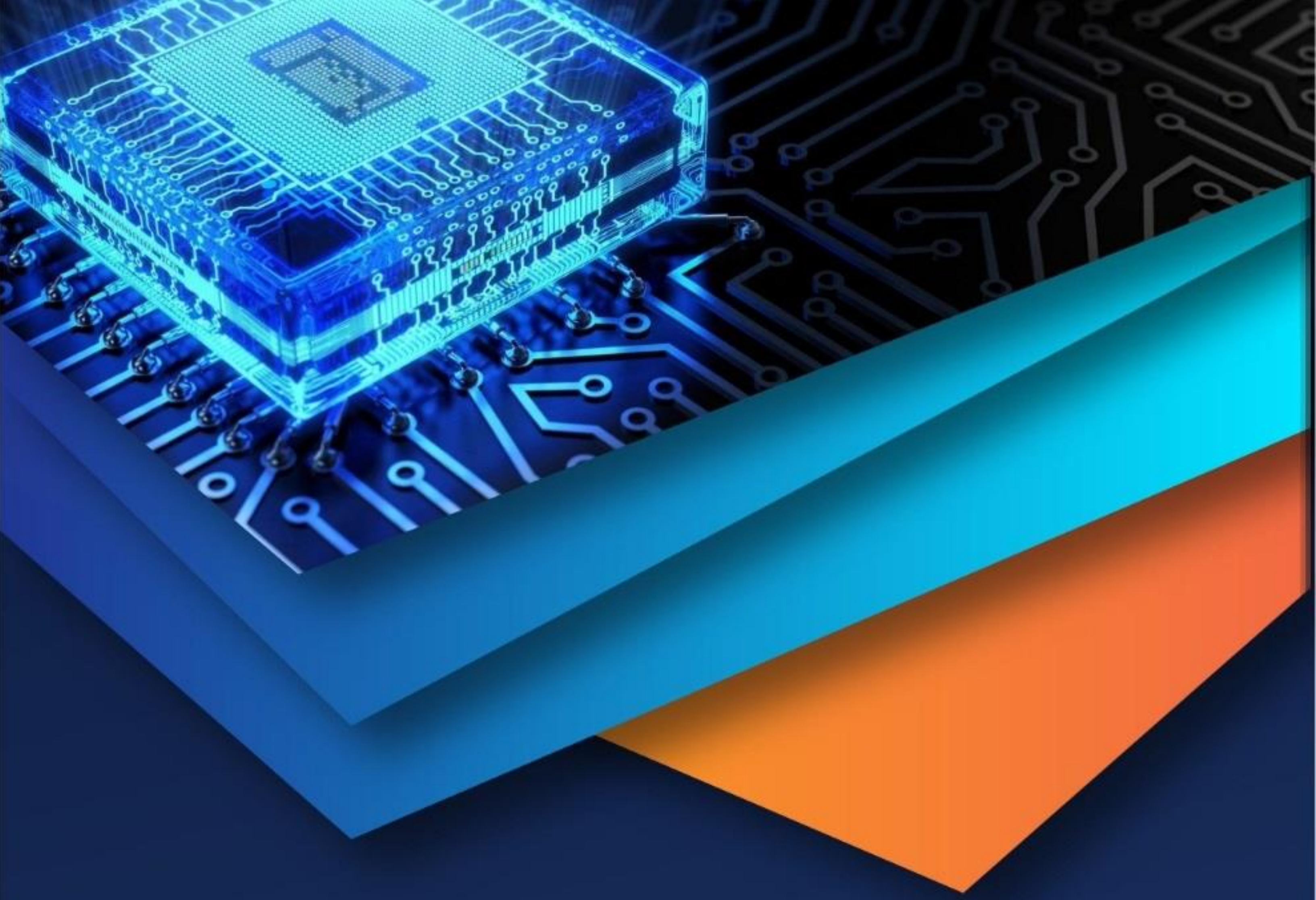

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