iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 Issue: V Month of publication: May 2023

DOI: https://doi.org/10.22214/ijraset.2023.52661

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue V May 2023- Available at www.ijraset.com

A Review on Second Degree Homogeneous Diophantine Equation with ThreeUnknowns $x^2+y^2=122z^2$

K Hema[1] , Ammineni Deepak Teja[2] , Giri Kollati[3]
[1]*Assistant Professor, Department of Mathematics, School of Engineering and Technology, Dhanalakshmi SrinivasanUniversity, Trichy, Tamil Nadu, India.*
[2, 3]*B.Tech Student, Department of Computer Science Engineering, Dhanalakshmi Srinivasan University, Trichy – 621 112 , TN , India.*

Abstract: The homogeneous ternary second degree equation given by $x^2+y^2=122z^2$ is analysed for its non-zero distinctintegral points on that. Completely various patterns of the equation into consideration are obtained by using python.
Keywords: Ternary, quadratic, Integer solutions, Homogeneous, Diophantine, python.

I. INTRODUCTION

It is acknowledge that the quadratic Diophantine equations with 3 unknowns (homogeneous or non-homogeneous) are made in selection[1,2,]. Significantly, one might refer [3-17] for homogeneous or non-homogeneous ternary second degree Diophantine equations that are analysed for getting their corresponding non-zero distinct integer solutions . During this communication, one more attention-grabbing homogeneous ternary quadratic Diophantine equation given by $x^2 + y^2 = 122z^2$ is analysed for its non-zero distinct integer results through fully different strategies with simple python programs. One may gain different values for the input of their programs.

II. STYLES OF ANALYSIS

The ternary second degree equation to be answered for its integer results is

$$x^2 + y^2 = 122z^2 \tag{1}$$

A. *Pattern I*

Write 122 as

$$122=(11+i)(11-i) \tag{2}$$

Assume

$$z = a^2 + b^2 \tag{3}$$

Thus we tend to get,

$$x = 11a^2 - 11b^2 - 2ab$$
$$y = a^2 - b^2 + 22ab$$
$$z = a^2 + b^2$$

We are going to see this by simple python code as follows:

```
import math
a=int(input("enter the value of a"))
b=int(input("enter the value of b"))
x=(11*a**2)-(11*b**2)-2*a*b;
y=(a**2)-(b**2)+22*a*b;
z=(a**2)+(b**2);
print("the value of x is",x);
print("the value of y is",y);
print("the value of z is",z);
```

```
ouput :
enter the value of a 5
5
enter the value of b 5
5
the value of x is -50
the value of y is 550
the value of z is 50
```

B. Pattern 2

Equation (1) can also be wriiten as

$x^2 \square\ y^2 \ \square 121z^2 \ \square\ z^2$

$\square\ x^2 \ \square 121z^2 \ \square\ z^2 \ \square\ y^2$ (4)

Applying the tactic of cross-multiplication to the on-top system of equations, note that

$x \ \square 11\square^2 \ \square 11\square^2 \ \square\ 2\square\square$

$y \ \square\ \square\ \square^2 \ \square\ \square^2 \ \square\ 22\square\square z \ \square\square^2 \ \square\ \square^2$

Applying python codes and considering alpha and beta as a and b we get ,

```
import math
a=int(input("enter the value of a"))
b=int(input("enter the value of b"))
x=(11*a**2)-(11*b**2)+2*a*b;
y=(-a**2)+(b**2)+22*a*b;
z=(a**2)+(b**2);
print("the value of x is",x);
print("the value of y is",y);
print("the value of z is",z);
```

Output :

```
enter the value of a 5
5
enter the value of b 5
5
the value of x is 50
the value of y is 550
the value of z is 50
```

C. Pattern III

One can also be written as

$x^2 \ \square\ y^2 \ \square 122z^2 \ \square 1$

Write 1 as

$$1 \square \frac{\square 3 \ \square\ 4i\square\square 3 \ \square\ 4i\square}{25} \quad (5)$$

(6)

As our interest is on finding integer solutions replacing a by 5A & b by 5B , we get

$$\left.\begin{array}{l} x \ \square\ 29A^2 \ \square\ 29B^2 \ \square\ 94AB \\ y \ \square\ 47A^2 \ \square\ 47B^2 \ \square\ 58AB \\ z \ \square 5A^2 \ \square\ 5B^2 \end{array}\right\} \quad (7)$$

Considering A,B as a,b we are following results:

```
import math
a=int(input("enter the value of a"))
b=int(input("enter the value of b"))
x=(29*a**2)-(29*b**2)-94*a*b;
y=(47*a**2)-(47*b**2)+58*a*b;
z=(5*a**2)+(5*b**2);
print("the value of x is",x);
print("the value of y is",y);
print("the value of z is",z);
```

Output:

```
 enter the value of a 5
5
enter the value of b 5
5
the value of x is -2350
the value of y is 1450
the value of z is 250
```

D. Pattern IV

Introduction of the direct metamorphoses

x=u+v,y=u-v,z=2w (8)

in (1) leads to

$$u^2 + v^2 = 244w^2$$

Assume

$$w = c^2 + d^2$$

Thereby we are getting these following results

(10)

(11)

we are getting these following results :

$$x = 22c^2 - 22d^2 - 4cd$$
$$y = -2c^2 + 2d^2 - 44cd$$
$$z = 2c^2 + 2d^2$$

```
import math
c=int(input("enter the value of c"))
d=int(input("enter the value of d"))
x=(22*c**2)-(22*d**2)-4*c*d;
y=(-2*c**2)+(2*d**2)-44*c*d;
z=(2*c**2)+(2*d**2);
print("the value of x is",x);
print("the value of y is",y);
print("the value of z is",z);
```

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue V May 2023- Available at www.ijraset.com

Output :

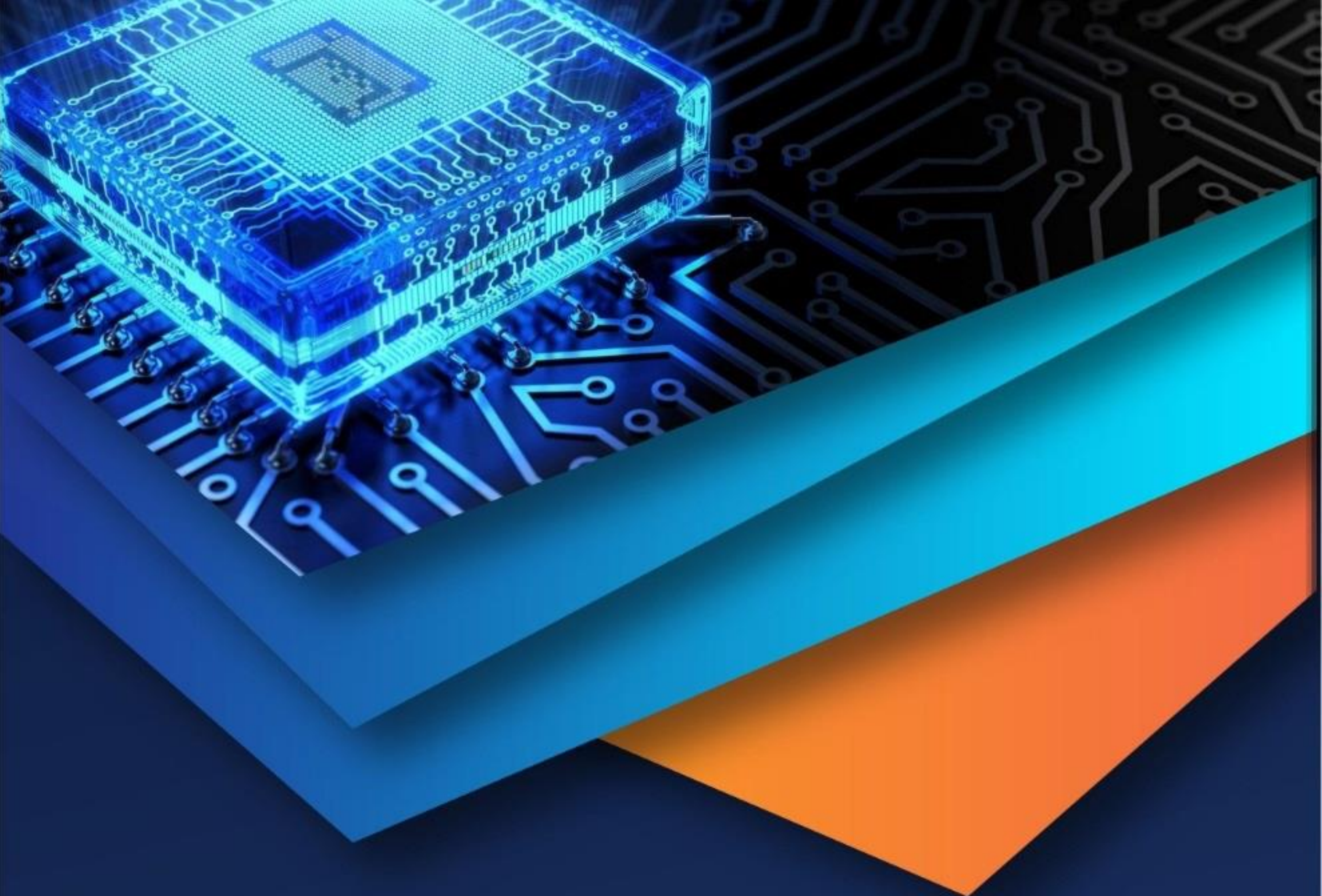
```
enter the value of c 5
5
enter the value of d 5
5
the value of x is -100
the value of y is -1100
the value of z is 100
```

III. CONCLUSION

In this paper, an bid has been created to get non-zero distinct integer results to the ternary quadratic Diophantineequation $x^2 \square y^2 \square 122 z^2$ representing homogeneous cone with python canons. As there are kinds of cones, the compendiums might rummage around for indispensable kinds of cones to get integer results for the corresponding cones with python codes.

REFERENCES

[1] L.E. Dickson, History of theory of Numbers, Vol. 2, Chelsea publishing Company, Newyork, 1952.
[2] Mordel , Diophantine Equations, Academic press, Newyork, 1969.
[3] Gopalan M.A., Geetha D,Lattice points on the hyperbola of two sheets $x^2 \square 6xy \square y^2 \square 6x \square 2y \square 5 \square z^2 \square 4$, Impact J Sci Tech:4:23-32,2010.
[4] Gopalan M.A., Vidhyalakshmi S,Kavitha A, Integral points on the homogeneous coneDiophantine J Math; 1(2):127-136, 2012. $z^2 \square 2x^2 \square 7y^2$, The
[5] Gopalan M.A., Vidhyalakshmi S, Sumathi G,Lattice points on the hyperboloid of one sheet $4z^2 \square 2x^2 \square 3y^2 \square 4$,Diophantine J Math; 1(2): 109-115, 2012.
[6] Gopalan M.A., Vidhyalakshmi S, Lakshmi K, Integral points on the hyprerboloid of two sheets $3y^2 \square 7x^2 \square z^2 \square 21$, Diophantine J Math; 1(2):99-107, 2012.
[7] Gopalan M.A., Vidhyalakshmi S, Mallika S, Observations on hyperboloid of one sheetJMath; 2(3):221-226,2012. $x^2 \square 2y^2 \square z^2 \square 2$, Bessel
[8] Gopalan M.A., Vidhyalakshmi S, Usha Rani T.R., Mallika S, Integral points on the homogeneous cone $6z^2 \square 3y^2 \square 2x^2 \square 0$, Impact J Sci Tech; 6(1):7-13, 2012.
[9] Gopalan M.A., Vidhyalakshmi S, Sumathi G, Lattice points on the elliptic paraboloidApplied Mathematics;7(4): 379-385, 2012. $z \square 9x^2 \square 4y^2$, Advances in
[10] Gopalan M.A., Vidhyalakshmi S, Usha Rani T.R.,Integral points on the non-homogeneous cone $2z^2 \square 4xy \square 8x \square 4z \square 0$, Global Journal of Mathematics and Mathematics sciences 2012;2(1):61-67.
[11] Gopalan M.A., Vidhyalakshmi S, Lakshmi K, Lattice points on the elliptic paraboloid $16 y^2 \square 9z^2 \square 4x$, Bessel JMath;
[12] Python Object-Oriented Programming by Steven F.Lott and Dusty Phillips ,4th edition.
[13] Effective Python by Brett Slatkin , 1st edition



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY