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A Study on Chromatic Number of Sierpinski's Cycle Graph with Order 'n'

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Abstract: Graph coloring is a major area in Discrete Mathematics because it helps solve real-world problems like setting up network connections or assigning wireless radio frequencies. This dissertation focuses on finding the exact vertex chromatic number, which we write as $\chi(G)$, for a special class of recursive networks called Sierpinski Cycle Graphs, or S(k, G). These graphs are built step-by-step using a repeating, fractal-like pattern based on a simple starting cycle graph G.The way these graphs are made is detailed through a step-by-step construction method. For example, to make a level-2 graph like $S(2, C_4)$, we take four separate copies of a level-1 graph $S(1, C_4)$ and link them together at specific corner points called extreme vertices. To figure out the coloring limits as the graphs grow larger, we used mathematical induction as our main tool to track how the points and lines change at each level (k).For the graphs built on a 4-cycle base, $S(k, C_4)$, the math shows that connecting the separate blocks together never creates an odd-cycle conflict. Because of this, the network always stays bipartite, meaning we only ever need exactly 2 colors.

$$\chi\, S(k, G) = 2$$

For the graphs built on a 5-cycle base, $S(k, C_5)$, things change. Because a 5-cycle has an odd number of corners, you cannot color it with just 2 colors without touching parts clashing. This odd-cycle conflict carries over into every new level of the graph. The proofs show that we can cleanly solve this using exactly 3 colors.

$$\chi\, S(k, G) = 3$$

Keywords: Graph Theory, Vertex Coloring, Chromatic Number, Sierpinski Cycle Graph, Self-similarity

I. INTRODUCTION

In this chapter, the basic definitionson graphs which are needed for thesubsequent chapters are given. Graph theory is the branch of Discrete Mathematics where Graphs are studied. Graphs are the mathematical structures consisting of a set of vertices and edges. It was first introduced by Leonhard Euler while solving the famous problems of mathematics namely Seven Bridges of Konigsberg through graph. Graphs have many applications in real world. For example, the vertex can be represented for people and edges for friends; or the vertex might be communication centres, with edges representing communication links. Graph theory is applicable in many branches, such as: mathematics, biology, chemistry, computer science, economics, finance, geography, sociology, physics, psychology, weather forecast and many more.

Colouring of a Graph:-A colouring of a graph $G$is an assignment of colours to the vertices of Gsuch that no two adjacent vertices receive the same colour. An k– colouring of a graph G uses k colours. The chromatic number χ is defined to be the minimum n for which G has an k- colouring.

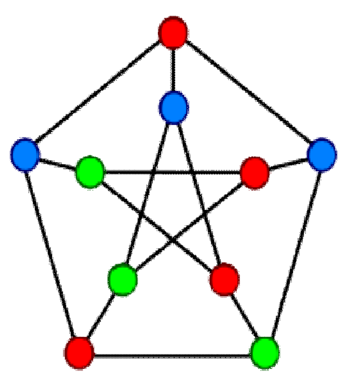

fig. 1.29

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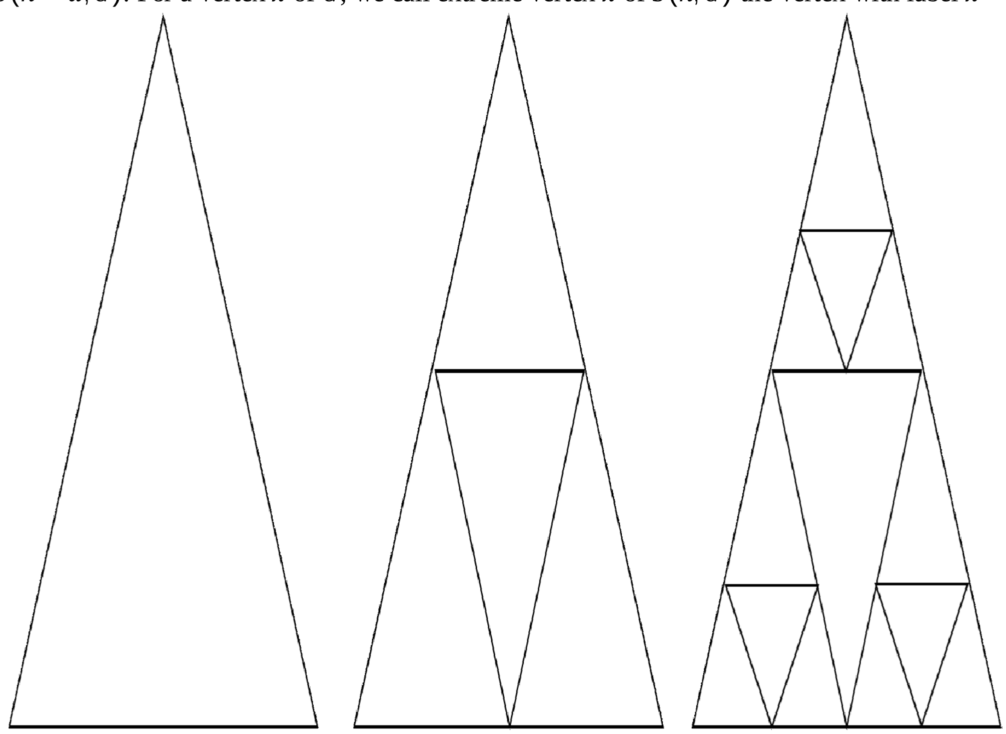
A. Generalized Sierpinski Graph

Sierpinski graphs are never ending pattern that are self similar across different scales.The introduction of Sierpinski graphs were first motivated by topological studies of Lipscomb's space[70,73]. In 1997 Klavzar and Miltunovic[60] introduced Sierpinski Graphs$S(n,G)$which are isomorphic to the graphs of Tower of Hanoi with n disk.They gave the method of construction of Sierpinski Graph of Complete Graph K_k of order 'n' which is obtained after finite number of iteration and denoted it $S(n,K_k)$. In the stage one, they took simply complete Graph and denoted it $S(1,K_k)$,In the stage two, they copied $S(1,K_k)$ Graph k times and add one edge between each pair of $S(1,K_k)$ for forming Graph $S(2,K_k)$. Repeating this process they defined $S(3,K_k)$,$S(4,K_k)$, $S(5,K_k)\cdots S(n,K_k)$. In 2011 Gravier, Kovse and Aline[37] introduced new Graph known as Generalised Sierpinski Graphs. They replaced Complete Graph K_k by any Graph. The Generalised Sierpinski graph of G of dimension "n" denoted by $S(n,G)$ is the graph with vertex set $\{1,2,3,\ldots\ldots,n\}^n$ and edge set defined by $\{u,v\}$is an edge if and only if there exists $i \in \{1,2,3,\ldots\ldots,n\}$ such that :

- $u_j = v_j if j < i$
- $u_i \neq v_i and\ (u_i,v_i) \in E(G)$
- $u_j = v_i and v_j = u_i if j >$

In other words, if $\{u,v\}$ is an edge of $S(n,G)$, there is an edge $\{x,y\}$ of G and a word "$w$such that $u = wxy\ldots\ldots\ldots yandv = wyx\ldots\ldots\ldots x$. We say that edge $\{u,v\}$ is using edge $\{x,y\}$ of G. Graphs $S(n,G)$ is can be constructed recursively from G with the following process: $S(1,G)$ is isomorphic to G. To construct $S(n,G)$ for $n > 1$, copy k times $S(n-1,G)$ and add to labels of vertices in copy x of $S(n-1,G)$ the letter x at the beginning. Then for any edge $\{x,y\}$ of G, add an edge between vertex $xy\cdots y$ and vertex $yx\cdots x$. For any word u of length d, with $1 \leq d \leq n$, the subgraph of $S(n,G)$ induced by vertices with label beginning by u, is isomorphic to $S(n-d,G)$. For a vertex x of G, we call extreme vertex x of $S(n,G)$ the vertex with label $x\cdots x$.



Stage -1 Stage -2 Stage -3

B. Vertex Coloring (or k-Coloring)

A vertex coloring (often simply called a coloring) of a graph G = (V, E) is an assignment of colors to the vertices of G such that no two adjacent vertices receive the same color.

C. Chromatic Number

The chromatic number of a graph G, denoted by χ(G) , is the minimum number of colors needed to properly color the vertices of G.

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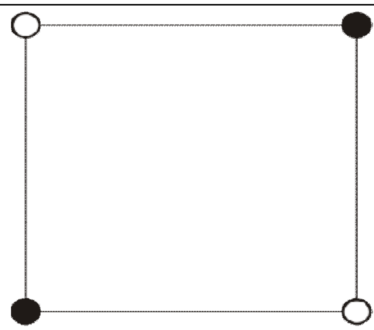
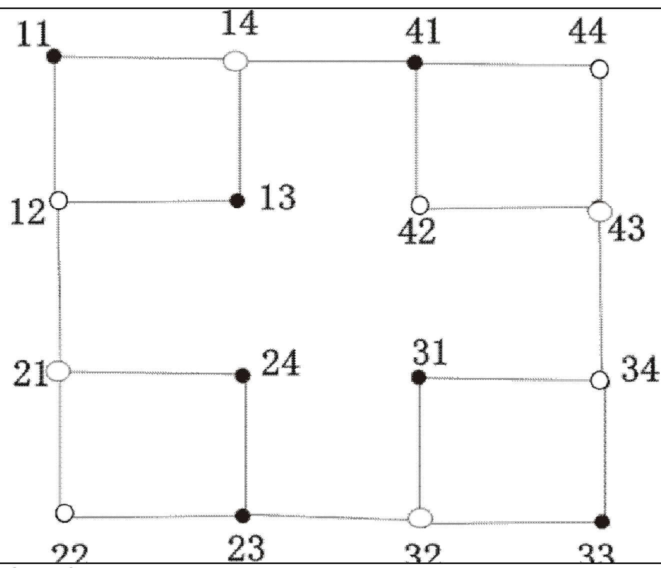
II. METHODOLOGY

Sierpinski Cycle graphs have been constructed using the basic theoryof Generalised Sierpinski graph and Chromatic number has been found using simple definition of coloring. Below the recursive construction of generalised Sierpinski's graph on vertices at different stage:

<u>Chromatic Number of Sierpinski Cycle Graph of order $'n'$</u>

Chromatic number of Sierpinski Cycle Graph of order '4' i.e. $\chi(S(k, C_4))$

Figure 4(A).1

<table>
<tr><td>Notation</td><td>Graph</td></tr>
<tr><td>$\mathrm{S}(1, \mathrm{C}_4)$</td><td></td></tr>
<tr><td colspan="2">In $\mathrm{S}(1, \mathrm{C}_4)$, the vertex Set is $V = \{1, 2, 3, 4, \}$. Here we have taken a minimum 2 colors$\{whitecolouredvertex\ and\ Blackcolouredvertex\}$ of vertex set $\mathrm{V}\{\mathrm{S}(1, \mathrm{C}_4)\}$. 2 is the minimum numbercolors to color the graph. Therefore the Chromatic number of $\mathrm{S}(1, \mathrm{C}_4)$ will be 2. i.e. $\chi\left(\mathrm{S}(1, \mathrm{C}_4)\right) = 2$.</td></tr>
<tr><td>$\mathrm{S}(2, \mathrm{C}_4)$</td><td>
</td></tr>
<tr><td colspan="2">Similarly, in $\mathrm{S}(2, \mathrm{C}_4)$, in the vertex set we have taken a minimum 2 colors{black colored vertices and white colored vertices} of vertex set $\mathrm{V}\{\mathrm{S}(2,\mathrm{C}_4)\}$. 2 is the minimum numbercolors to color the graph.Therefore the Chromaticnumber of$\mathrm{S}(2,\mathrm{C}_4)$ will be 2. i.e.
$\chi\left(\mathrm{S}(2, \mathrm{C}_4)\right) = 2$.</td></tr>
<tr><td>$\mathrm{S}(3, \mathrm{C}_4)$</td><td></td></tr>
</table>

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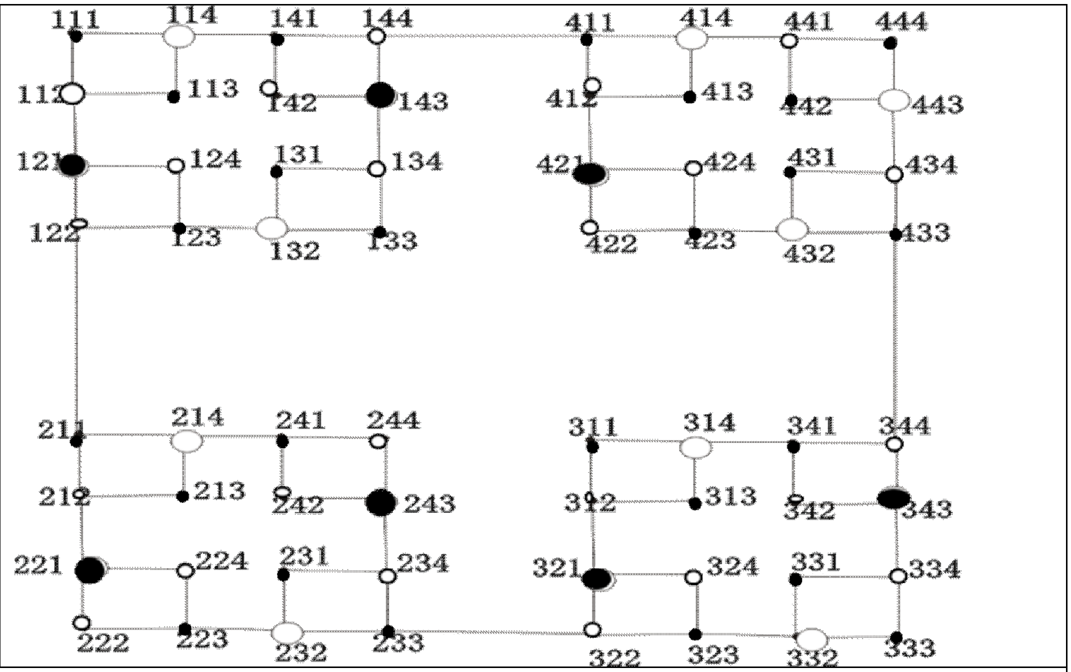
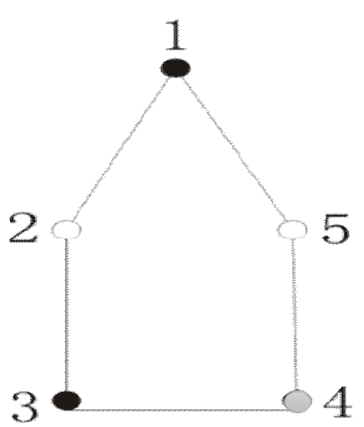
	
Again, in $S(3, C_4)$, here we are taking a proper vertex coloring of the vertex set $V\{S(3, C_4)\}$. Since each $S(2, C_4)$ contains copies of the cycle graph C_4, which is a bipartite graph, it requires a minimum of 2 colors to ensure that no two adjacent vertices share the same color. Therefore, the minimum number of colors needed to color all vertices of S(3, C_4) properly will be 2. Therefore, the Chromatic number of $S(3, C_4)$ will be 2 i.e. $\chi(S(3, C_4)) = 2$.	

Chromatic Number of Sierpinski Cycle Graph of order '5' i.e. $\chi\ (S(k, C_5))$.

otation	Graph
$S(1, C_5)$	
In $S(1, C_5)$, the vertex Set is $V = \{1, 2, 3, 4, 5\}$. Here we have taken a proper vertex coloringof the vertex set $V\{S(1, C_5)\}$. Since $S(1, C_5)$ is an odd cycle graph of length 5, it requires a minimum number of 3 distinct colors to ensure that no two adjacent vertices share the same color. Therefore, the chromatic number of $S(1, C_5)$ will be 3 i.e. $\chi(S(1, C_5) = 3$.	
$S(2, C_5)$	

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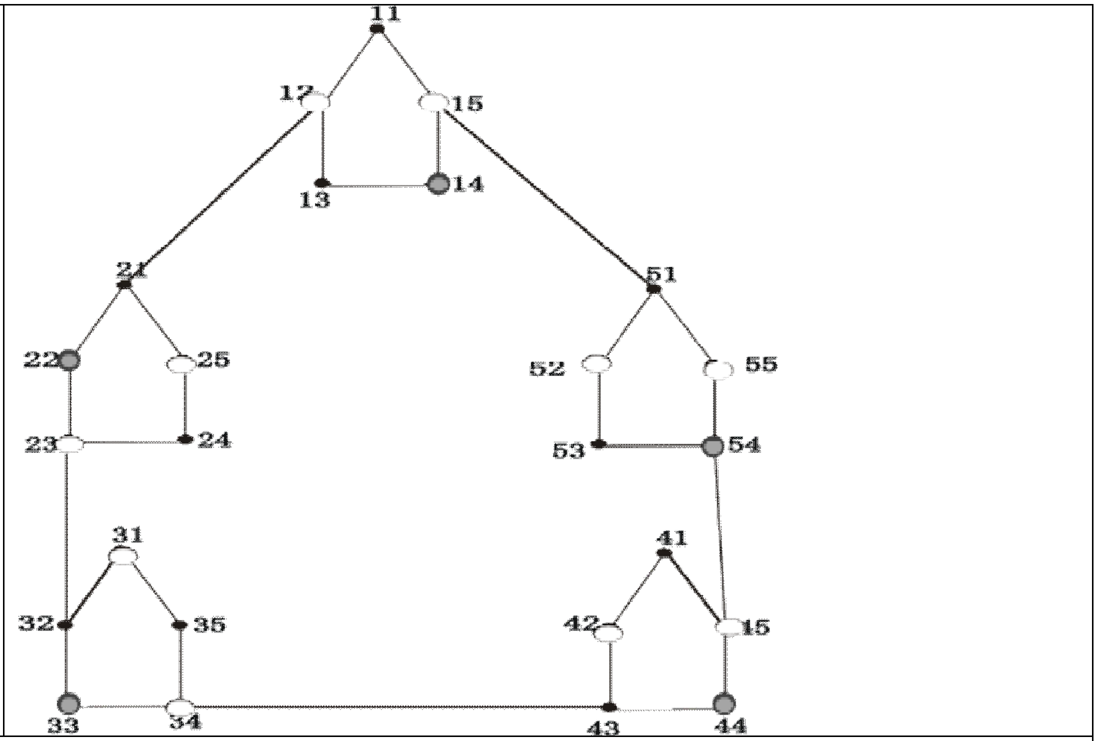

Similarly, in S(2, C_5), here we have taken a proper vertex coloring of the vertex set V{S(2, C_5)}. Since each subgraph block is isomorphic to S(1, C_5), which requires a minimum of 3 colors due to its odd cycle structure, the entire component requires at least 3 distinct colors. By matching colors at the boundary vertices, the graph can be completely and properly colored without needing any additional colors. Therefore, the total number of colors required for S(2, C_5) will be 3, which is the chromatic number. Therefore, χ(S(2, C_5) = 3.

III. CONCLUSION

Chromatic Number for Even Base Graph S(k, C_4): When we use a 4-cycle graph (C_4) as the starting base, the mathematical proofs show that connecting the different blocks together during expansion never creates an odd-cycle conflict. Because of this, the graph always stays bipartite no matter how many layers (k) we add. Therefore, we only need exactly two colors to properly color its vertices:

$$\chi\left(S\ (k, C_4)\right) = 2$$

where k = 1,2,3…,n

Chromatic Number for Odd Base Graph S(k, C_5): When the starting base graph is changed to a 5-cycle (C_5), things become different. Because a 5-cycle has an odd number of vertices, it is impossible to use just two colors without adjacent corners clashing. This odd-cycle conflict carries over into every new level of the graph. The tables prove that we can cleanly solve this using exactly three colors:

$$\chi\left(\boldsymbol{S}\ (\boldsymbol{k}, \boldsymbol{C_4})\right) = \mathbf{3}$$

where k = 1,2,3…,n.

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