# The Continuous Acceptance Sampling of Truncated Frechet Distribution Based on CUSUM Schemes 

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#### Abstract

Acceptance sampling plans were evaluated to draw valid conclusions regarding lots of finished products. Based on the assumption that the variable under consideration is distributed according to a Frechet distribution, CASP-CUSUM schemes are presented here for optimizing Type-C OC curves and ARLs. The Truncated Frechet distribution, its Type-C OC curve values, and ARL values at different shapes parameters were determined under this assumption. Finally, we determined an optimal CASP-CUSUM scheme that maximizes $P_{A}$.


Keywords: CASP - CUSUM Schemes, Type-C OC curves, ARL Values, Truncated Frechet distribution.

## I. INTRODUCTION

In a global business market, the quality of products has become one of the most important factors that distinguish different commodities. The statistical process control and the statistical product control, or acceptance sampling, are two important techniques for ensuring quality.
Acceptance sampling is a quality control method used to accept or reject lots after testing a random sample of a product. Instead of estimating the quality of the entire batch, acceptance sampling is used to determine whether to accept or reject a lot of the product. Sample acceptance is a very useful technique when a lot is so large or when testing is destructive. It is too time-consuming and too expensive to inspect every single item in a large lot. Moreover, checking every single product does not guarantee that it will comply with the specification.
Vardeman.S, Diou Ray (9) introduced CUSUM control charts with the restriction that the values refer to quality and that they are exponentially distributed. Furthermore, the events under study are the rate of rare events and the inter-arrival time for a homogenous poison process that are identically distributed exponential random variables.
Lonnie. C. Vance (7) considers Average Run Length of Cumulative Sum Control Charts when controlling for normal means and determining CUSUM Chart parameters. To develop the parameters of CUSUM Chart, the acceptable and rejectable quality levels are considered as well as the desired respective ARL's.
The CASP-CUSUM optimization scheme was implemented by Sarma and Akhtar(1)by solving the integral equation using GaussChebyshev integration using a computer program. Finally, the results of the analysis were compared at different values of the parameters.
Narayan Murthy(8) et. al examined CASP-CUSUM schemes on the basis of the truncated Rayleigh distribution to determine ARL values for CASP-CUSUM schemes. By evaluating integral equations using the Lobatto integration method, we obtained optimum continuous acceptance sampling plans cumulative sums. We compared the results obtained using different integration methods
According to Venkatesulu.G and Mohammed Akhtar.P.(2), they made Truncated Lomax Distributions and Optimised CASPCUSUM Schemes by changing parameters and finally making critical comparisons based on the numerical results.
In the present paper, the CASP-CUSUM chart is determined when the variable under study follows Truncated Frechet Distribution. Therefore, it would be worthwhile to examine some interesting characteristics of this distribution.

## A. Frechet Distribution

Extreme value theory plays a crucial role in statistical analysis. Generally, extreme data are described by generalized extreme value (GEV) distributions Gumbel, Weibull, and Fréchet distributions are all special cases of GEV distributions. According to mathematician Maurice René Fréchet, the Fréchet distribution was developed in the 1920s as a maximum value distribution (also known as the extreme value distribution of type II).

These distributions were discussed in depth by Kotz and Nadarajah [3], D. Kundu and H. Howlader, Bayesian [5], Pedro L. Ramos, Francisco [6], including applications for accelerated life testing, natural calamities, horse racing, rainfall, supermarket queues, sea currents, wind speeds, track race records, etc.
Definition: A random variable with a non-negative Frechet distribution is said to have the P.D.F is given by

$$
\begin{equation*}
\mathbf{f}(\mathbf{x} ; \boldsymbol{\alpha}, \boldsymbol{\beta})=\boldsymbol{\alpha} \boldsymbol{\beta} \mathbf{x}^{-(\boldsymbol{\alpha}+\mathbf{1})} \mathbf{e}^{-\boldsymbol{\alpha x}-\boldsymbol{\beta}} \quad \text { Where } \alpha, \beta, \gamma, \mathrm{x}>0 \tag{1.1}
\end{equation*}
$$

## B. Truncated Frechet Distribution

Basically, it is the ratio of the probability density function of the Frechet distribution to the corresponding cumulative distribution function at B.

$$
\begin{equation*}
f_{B}(x)=\frac{\alpha \beta \mathbf{x}^{-(\alpha+1)} \mathbf{e}^{-\alpha x}-\beta}{\left(1-\left[1+\mathbf{e}^{-\alpha \mathbf{B}-\beta}\right]\right)} \quad \alpha>0, \beta>0 \tag{1.2}
\end{equation*}
$$

Where' $\mathbf{B}$ ' is the upper truncated point of the Frechet Distribution.

## II. A DESCRIPTION OF THE PLAN AND THE CURVE OF TYPE-C

Beattie[4] proposed a method for the construction of continuous acceptance sampling plans. It consists of a chosen decision interval, "Return interval" with the length h ', above which a decision line is taken. Sum $S_{m}=\sum\left(X_{i}-k_{1}\right) X_{i}{ }^{\prime} s(i=1,2,3 \ldots \ldots .$. is distributed independently, and $\mathrm{k}_{1}$ is the reference value, plotted on the chart. If the sum lies in the area of the normal chart, the product is accepted and if it lies in the of the return chart, the product is rejected, subject to the following assumptions.

1) When the recently plotted point on the chart touches the decision line, then the next point is to be plotted at maximum, i.e., $\mathrm{h}+\mathrm{h}$ '.
2) Once the decision line is reached or crossed from above, the next point on the chart should be plotted from the baseline. A network or a change of specification may be used instead of outright rejection when the CUSUM falls in the return chart.

The procedure is summarized below.
a) Begin plotting the CUSUM at Zero.
b) It is accepted when $S_{m}=\sum\left(X_{i}-k\right)<h$; when $\mathrm{S}_{\mathrm{m}}<0$, return cumulative to zero.
c) When $\mathrm{h}<\mathrm{S}_{\mathrm{m}}<\mathrm{h}+\mathrm{h}$ ' the product is rejected: when $\mathrm{S}_{\mathrm{m}}$ crosses h , i.e., when $\mathrm{S}_{\mathrm{m}}>\mathrm{h}+\mathrm{h}$ ' and continue rejecting product until $\mathrm{S}_{\mathrm{m}}>\mathrm{h}+\mathrm{h}^{\prime}$ return cumulative to $\mathrm{h}+\mathrm{h}$ '.

The Type - C, OC function is determined by incoming item quality and the sampling rate in acceptance and rejection regions is equal. Therefore, the probability of acceptance $P_{A}$ can be calculated as follows:

$$
\begin{equation*}
\mathrm{P}_{\mathrm{A}}=\frac{L_{0}}{L_{0}+L_{0}^{\prime}} \tag{2.1}
\end{equation*}
$$

where, $L_{0}=$ Average Run Length in acceptance zone and
$L_{0}^{\prime}=$ Average Run Length in rejection zone.
Page E.S. ${ }^{10}$ has introduced the formulae for $L_{0}$ and $L_{0}^{\prime}$ as

$$
\begin{align*}
L_{0} & =\frac{N_{0}}{1-P_{0}}  \tag{2.2}\\
L_{0}^{\prime} & =\frac{N_{0}^{\prime}}{1-P_{0}^{\prime}} \tag{2.3}
\end{align*}
$$

Where, $\mathrm{P}_{0}=$ Probability for the test starting from zero on the normal chart,
$\mathrm{N}_{0}=$ ASN for the test starting from zero on the normal chart,
$P_{0}^{\prime}=$ Probability for the test on the return chart and
$N_{0}^{\prime}=\mathrm{ASN}$ for the test on the return chart

He further obtained integral equations for the quantities $\mathrm{P}_{0}, \mathrm{~N}_{0}$, and $N_{0}^{\prime}, P_{0}^{\prime}$ as follows:

$$
\begin{align*}
\mathrm{P}_{\mathrm{z}} & =\mathrm{F}\left(\mathrm{k}_{1}-\mathrm{z}\right)+\int_{0}^{h} P(\mathrm{y}) \mathrm{f}\left(\mathrm{y}+\mathrm{k}_{1}-\mathrm{z}\right) \mathrm{dy},  \tag{2.4}\\
\mathrm{~N}_{\mathrm{z}} & =1+\int_{0}^{h} N(\mathrm{y}) \mathrm{f}\left(\mathrm{y}+\mathrm{k}_{1}-\mathrm{z}\right) \mathrm{dy},  \tag{2.5}\\
P_{z}^{\prime} & =\int_{k 1+\mathrm{z}}^{B} f(y)+\int_{0}^{h} P^{\prime}(\mathrm{y}) \mathrm{f}\left(-\mathrm{y}+\mathrm{k}_{1}+\mathrm{z}\right) \mathrm{dy}  \tag{2.6}\\
N_{z}^{\prime} & =1+\int_{0}^{h} N^{\prime}(\mathrm{y}) \mathrm{f}\left(-\mathrm{y}+\mathrm{k}_{1}+\mathrm{z}\right) \mathrm{dy},  \tag{2.7}\\
\mathrm{~F}_{\mathrm{x}} & =1+\int_{A}^{h} f(x) d x \\
\mathrm{~F}\left(\mathrm{k}_{1}-\mathrm{z}\right) & =1+\int_{A}^{k 1-\mathrm{z}} f(y) d y
\end{align*}
$$

And z is the distance of the starting of the test in the normal chart from zero.

## III. DETERMINATION OF ARLS AND $P_{A}$

We developed computer programs to solve equations (2.4), (2.5), (2.6) and (2.7) and we obtained the following results in Tables (3.1) to (3.28).

Table 3.1

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 1}, \mathbf{h}^{\prime}=\mathbf{0 . 0 1}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 2.541672 | 1.065869 | 0.70454418 |
| 4.4 | 3.033483 | 1.073191 | 0.73867154 |
| 4.3 | 3.858639 | 1.081370 | 0.78109961 |
| 4.2 | 5.517222 | 1.090516 | 0.83496379 |
| 4.1 | 10.509679 | 1.100754 | 0.90519261 |

Table 3.3

| $\boldsymbol{\alpha} \boldsymbol{1} \boldsymbol{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 3}, \mathbf{h}^{\prime}=\mathbf{0 . 0 3}$ |  |  |  |
| B | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 2.542042 | 1.217929 | 0.67608022 |
| 4.4 | 3.033982 | 1.244409 | 0.70914083 |
| 4.3 | 3.859361 | 1.274466 | 0.75175118 |
| 4.2 | 5.518432 | 1.308655 | 0.80831426 |
| 4.1 | 10.512637 | 1.347626 | 0.88637465 |

Table 3.5

| $\begin{aligned} & \alpha=1 \beta=1 \quad \gamma=1 \quad k=4, h=0.05, h^{\prime}= \\ & 0.05 \end{aligned}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| B | $L_{0}$ | $L_{0}^{\prime}$ | $\boldsymbol{P}_{\text {A }}$ |
| 4.5 | 2.542424 | 1.397284 | 0.64533311 |
| 4.4 | 3.034497 | 1.449014 | 0.67681258 |
| 4.3 | 3.860113 | 1.508409 | 0.71902710 |
| 4.2 | 5.519692 | 1.576774 | 0.77780848 |
| 4.1 | 10.51572 | 1.655669 | 0.86397045 |

Table 3.2

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{0 . 0 2}$ | $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 2 ,} \mathbf{h}^{\prime}=$ |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 2.541856 | 1.138502 | 0.69065439 |
| 4.4 | 3.033730 | 1.154647 | 0.72432100 |
| 4.3 | 3.858997 | 1.172840 | 0.76691603 |
| 4.2 | 5.517822 | 1.193376 | 0.82218134 |
| 4.1 | 10.51114 | 1.216596 | 0.89626336 |

Table 3.4

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{B}$ | $\mathbf{\boldsymbol { L } _ { \mathbf { 0 } }}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}, \mathbf{h}=\mathbf{0 . 0 4}, \mathbf{h}=\mathbf{0 . 0 4}$ |  |
| 4.5 | 2.542232 | 1.304179 | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 3.034237 | 1.342517 | 0.6932596192 |
| 4.3 | 3.859733 | 1.386303 | 0.73574268 |
| 4.2 | 5.519056 | 1.436428 | 0.79348260 |
| 4.1 | 10.51416 | 1.493948 | 0.87558847 |

Table 3.6

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{2} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :--- | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 1}, \mathbf{h}^{\prime}=\mathbf{0 . 0 1}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.815961 | 0.843483 | 0.68283468 |
| 4.3 | 2.216361 | 0.820288 | 0.72987043 |
| 4.2 | 3.033228 | 0.797591 | 0.79179620 |
| 4.1 | 5.516596 | 0.778270 | 0.87636429 |
| 4.0 | 524286.1250 | 0.7672019 | 0.99999850 |

Table 3.7

| $=\mathbf{\beta}=\mathbf{2} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 2}, \mathbf{h}^{\prime}=\mathbf{0 . 0 2}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.815958 | 0.771691 | 0.70177900 |
| 4.3 | 2.216356 | 0.766948 | 0.74291980 |
| 4.2 | 3.033220 | 0.783666 | 0.79468435 |
| 4.1 | 5.516580 | 0.837654 | 0.86817389 |
| 4.0 | 524284.2812 | 0.953647 | 0.99999815 |

Table 3.9

| $\boldsymbol{\alpha}=\mathbf{\beta} \boldsymbol{\gamma}=\mathbf{1}$ |  |  | $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 4}, \mathbf{h}^{\prime}=\mathbf{0 . 0 4}$ |
| :--- | :---: | :---: | :---: |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.815952 | 0.882017 | 0.67308110 |
| 4.3 | 2.216348 | 1.038910 | 0.68085169 |
| 4.2 | 3.033206 | 1.320415 | 0.69670873 |
| 4.1 | 5.516548 | 1.798221 | 0.75416570 |
| 4.0 | 524280.656 | 2.581485 | 0.99999505 |

Table 3.11

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{2}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 1}, \mathbf{h}^{\prime}=\mathbf{0 . 0 1}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 2.541541 | 1.260030 | 0.66855013 |
| 4.4 | 3.033307 | 1.287440 | 0.70203286 |
| 4.3 | 3.858383 | 1.317748 | 0.74541836 |
| 4.2 | 5.516794 | 1.351260 | 0.80325424 |
| 4.1 | 10.508627 | 1.388318 | 0.88330459 |

Table 3.13

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{2}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 3}, \mathbf{h}^{\prime}=\mathbf{0 . 0 3}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 2.541640 | 1.783212 | 0.58768260 |
| 4.4 | 3.033440 | 1.866151 | 0.61912107 |
| 4.3 | 3.858576 | 1.957944 | 0.66338217 |
| 4.2 | 5.517115 | 2.059553 | 0.72817170 |
| 4.1 | 10.509411 | 2.172049 | 0.82872247 |

Table 3.15

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{2}$ |  |  | $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 5}, \mathbf{h}^{\prime}=\mathbf{0 . 0 5}$ |
| :--- | :---: | :---: | :---: |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 2.541741 | 2.310717 | 0.52380484 |
| 4.4 | 3.033576 | 2.450187 | 0.55319237 |
| 4.3 | 3.858773 | 2.604706 | 0.59701174 |
| 4.2 | 5.517444 | 2.775949 | 0.66528183 |
| 4.1 | 0.510208 | 2.965792 | 0.77992045 |

Table 3.8

| $\boldsymbol{\beta}=\mathbf{2} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 3}, \mathbf{h}^{\prime}=\mathbf{0 . 0 3}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.815955 | 0.784557 | 0.69830662 |
| 4.3 | 2.216352 | 0.839861 | 0.72519546 |
| 4.2 | 3.033213 | 0.958011 | 0.75997054 |
| 4.1 | 5.516564 | 1.177764 | 0.82406538 |
| 4.0 | 524282.437500 | 1.558634 | 0.99999701 |

Table 3.10

| $\boldsymbol{\alpha}=\mathbf{\beta} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{B}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 5}, \mathbf{h}^{\prime}=\mathbf{0 . 0 5}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.815949 | 1.064007 | 0.63054740 |
| 4.3 | 2.216344 | 1.363980 | 0.61903429 |
| 4.2 | 3.033199 | 1.870674 | 0.61853134 |
| 4.1 | 5.516533 | 2.698657 | 0.67150402 |
| 4.0 | 508391.656250 | 4.021540 | 0.99999207 |

Table 3.12

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{2}$ |  |  | $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 2 ,} \mathbf{h}^{\prime}=\mathbf{0 . 0 2}$ |
| :--- | :---: | :---: | :---: |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 2.541590 | 1.521093 | 0.62559401 |
| 4.4 | 3.033373 | 1.576147 | 0.65806698 |
| 4.3 | 3.858479 | 1.637048 | 0.70211267 |
| 4.2 | 5.516954 | 1.704424 | 0.76397514 |
| 4.1 | 10.50901 | 1.778974 | 0.85522657 |

Table 3.14

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{1} \gamma=\mathbf{2}$ |  |  | $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 4}, \mathbf{h}^{\prime}=\mathbf{0 . 0 4}$ |
| :--- | :---: | :---: | :---: |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 2.541690 | 2.046411 | 0.55397427 |
| 4.4 | 3.033508 | 2.157486 | 0.58437889 |
| 4.3 | 3.858674 | 2.280481 | 0.62853497 |
| 4.2 | 5.517279 | 2.416707 | 0.69539809 |
| 4.1 | 10.50980 | 2.567626 | 0.80365979 |

Table 3.16

| $\boldsymbol{\alpha}=\mathbf{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{1}$ |  |  | $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 1}, \mathbf{h}^{\prime}=\mathbf{0 . 0 1}$ |
| :--- | :---: | :---: | :---: |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 4.531469 | 1.602746 | 0.73872029 |
| 4.4 | 5.531766 | 1.642751 | 0.77102971 |
| 4.3 | 7.203088 | 1.685779 | 0.81034934 |
| 4.2 | 10.555097 | 1.732090 | 0.85903280 |
| 4.1 | 20.661203 | 1.781969 | 0.92060083 |

Table 3.17

| $\boldsymbol{\alpha} \boldsymbol{\mathbf { 2 }} \boldsymbol{=} \mathbf{1} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 2}, \mathbf{h}^{\prime}=\mathbf{0 . 0 2}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 4.544199 | 2.545325 | 0.64097380 |
| 4.4 | 5.549904 | 2.661849 | 0.67584890 |
| 4.3 | 7.231904 | 2.788398 | 0.72172510 |
| 4.2 | 10.611459 | 2.925940 | 0.78386247 |
| 4.1 | 20.850109 | 3.075539 | 0.87145429 |

Table 3.19

| $\boldsymbol{\alpha}=\mathbf{2} \boldsymbol{\beta}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\boldsymbol{\gamma}=\mathbf{1}$ | $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 4}, \mathbf{h}^{\prime}=\mathbf{0 . 0 4}$ |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 4.580497 | 5.505215 | 0.45415702 |
| 4.4 | 5.602752 | 5.894555 | 0.48730996 |
| 4.3 | 7.317762 | 6.321855 | 0.53650790 |
| 4.2 | 10.783370 | 6.791235 | 0.61357676 |
| 4.1 | 21.443075 | 7.307314 | 0.74583601 |

Table 3.21

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{1}$ |  |  |  |
| :--- | :--- | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 1}, \mathbf{h}^{\prime}=\mathbf{0 . 0 1}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.815961 | 0.843483 | 0.68283468 |
| 4.3 | 2.216361 | 0.820288 | 0.72987043 |
| 4.2 | 3.033228 | 0.797591 | 0.79179620 |
| 4.1 | 5.516596 | 0.778270 | 0.87636429 |
| 4.0 | 524286.12500 | 0.767201 | 0.99999850 |

Table 3.23

| $=\mathbf{\beta}=\mathbf{2} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 3}, \mathbf{h}^{\prime}=\mathbf{0 . 0 3}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.815955 | 0.784557 | 0.69830662 |
| 4.3 | 2.216352 | 0.839861 | 0.72519546 |
| 4.2 | 3.033213 | 0.958011 | 0.75997054 |
| 4.1 | 5.51656 | 1.177764 | 0.82406538 |
| 4.0 | 524282.437 | 1.558634 | 0.99999701 |

Table 3.25

| $\boldsymbol{\alpha}=\mathbf{1} \boldsymbol{\beta}=\mathbf{2} \boldsymbol{\gamma}=\mathbf{1}$ | $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 5}, \mathbf{h}^{\prime}=\mathbf{0 . 0 5}$ |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.815949 | 1.064007 | 0.63054740 |
| 4.3 | 2.216344 | 1.363980 | 0.61903429 |
| 4.2 | 3.033199 | 1.870674 | 0.61853134 |
| 4.1 | 5.516533 | 2.698657 | 0.67150402 |
| 4.0 | 508391.6562 | 4.021540 | 0.99999207 |

Table 3.18

| $\boldsymbol{\alpha}=\mathbf{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 3}, \mathbf{h}^{\prime}=\mathbf{0 . 0 3}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 4.560001 | 3.840910 | 0.54279839 |
| 4.4 | 5.57267 | 4.072864 | 0.57774621 |
| 4.3 | 7.268473 | 4.326282 | 0.62687593 |
| 4.2 | 10.683753 | 4.603369 | 0.69887280 |
| 4.1 | 21.095354 | 4.906580 | 0.81129944 |

Table 3.20

| $\boldsymbol{\alpha}=\mathbf{\beta}=\mathbf{1} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 5}, \mathbf{h}^{\prime}=\mathbf{0 . 0 5}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.5 | 4.608503 | 7.557050 | 0.37881577 |
| 4.4 | 5.644982 | 8.149601 | 0.40921729 |
| 4.3 | 7.389117 | 8.802523 | 0.45635384 |
| 4.2 | 10.932836 | 9.522729 | 0.53446757 |
| 4.1 | 21.990833 | 10.318037 | 0.68064385 |

Table 3.22

| $=\mathbf{\beta}=\mathbf{2} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{\alpha}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 2 ,} \mathbf{h}^{\prime}=\mathbf{0 . 0 2}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.815958 | 0.771691 | 0.70177900 |
| 4.3 | 2.216356 | 0.766948 | 0.74291980 |
| 4.2 | 3.033220 | 0.783666 | 0.79468435 |
| 4.1 | 5.516580 | 0.837654 | 0.86817389 |
| 4.0 | 524284.281 | 0.9536476 | 0.99999815 |

Table 3.24

| $\boldsymbol{\alpha}=\mathbf{\beta}=\mathbf{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 4}, \mathbf{h}^{\prime}=\mathbf{0 . 0 4}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.815952 | 0.882017 | 0.67308110 |
| 4.3 | 2.216348 | 1.038910 | 0.68085169 |
| 4.2 | 3.033206 | 1.320415 | 0.69670873 |
| 4.1 | 5.516548 | 1.798221 | 0.75416570 |
| 4.0 | 524280.6562 | 2.581485 | 0.99999505 |

Table 3.26

| $=\mathbf{\beta}=\mathbf{3} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{\alpha}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 1}, \mathbf{h}^{\prime}=\mathbf{0 . 0 1}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.431011 | 1.756738 | 0.44890955 |
| 4.3 | 1.685115 | 2.139418 | 0.44060680 |
| 4.2 | 2.216362 | 2.752937 | 0.44601106 |
| 4.1 | 3.858267 | 3.757776 | 0.50659734 |
| 4.0 | 349525.37500 | 5.436113 | 0.99998444 |

Table 3.27

| $\boldsymbol{\alpha}=\mathbf{\beta}=\mathbf{3} \boldsymbol{\gamma}=\mathbf{1}$ |  |  | $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 2}, \mathbf{h}^{\prime}=\mathbf{0 . 0 2}$ |
| :--- | :---: | :---: | :---: |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.431011 | 3.012907 | 0.32201567 |
| 4.3 | 1.685115 | 4.188890 | 0.28687670 |
| 4.2 | 2.216363 | 6.164184 | 0.26446521 |
| 4.1 | 3.858267 | 9.537398 | 0.28802359 |
| 4.0 | 349525.4062 | 15.378903 | 0.99995601 |

Table 3.28

| $\boldsymbol{\alpha}=\mathbf{\beta}=\mathbf{3} \boldsymbol{\gamma}=\mathbf{1}$ |  |  |  |
| :--- | :---: | :---: | :---: |
| $\mathbf{k}=\mathbf{4}, \mathbf{h}=\mathbf{0 . 0 3}, \mathbf{h}^{\prime}=\mathbf{0 . 0 3}$ |  |  |  |
| $\mathbf{B}$ | $\boldsymbol{L}_{\mathbf{0}}$ | $\boldsymbol{L}_{\mathbf{0}}^{\prime}$ | $\boldsymbol{P}_{\boldsymbol{A}}$ |
| 4.4 | 1.431011 | 4.768560 | 0.23082429 |
| 4.3 | 1.685115 | 7.148550 | 0.19076062 |
| 4.2 | 2.216363 | 11.23406 | 0.16478008 |
| 4.1 | 3.858268 | 18.339675 | 0.17381195 |
| 4.0 | 349525.406250 | 30.830390 | 0.99991178 |

## IV. RESULTS AND CONCLUSIONS

The hypothetical values of the parameters $\mathrm{k}, \mathrm{h}$, and h are given at the top of each table, By determining optimum truncated point B , we can determine at which value $\mathrm{P}_{\mathrm{A}}$ the probability of accepting an item is maximum, and by obtaining ARL's values, we can determine the acceptance zone and rejection zone. For random variable ' X ', we have the values for the truncated point B , , and the values for the Type - C OC Curve, that is, $\mathrm{P}_{\mathrm{A}}$ are given in columns I, II, III and IV respectively.

The following conclusions can be drawn from the above tables 3.1 to 3.28 .

1) By observing the above tables, we see that $h, h$ increases and the related value of $L_{0}$ decreases. Therefore, the size of accepted and rejected zones is inversely related to $\mathrm{L}_{0}$.
2) By observing the above tables, we notice that as h,h' increases, then related values of $P_{A}$ decrease. $P_{A}$ is inversely related to the sizes of accepted and rejected zones.
3) By observing the above table, we can see that as the value of parameter a of Frechet distribution changes, $\mathrm{P}_{\mathrm{A}}$ changes along with it.
4) It is observed that the Table - 4.1 values of Maximum Probabilities increased as the decreased values of $\mathbf{h} \boldsymbol{\&}$ h'as shown below the Figure-4.1

| TABLE 4.1 |  |
| :---: | :---: |
| $\mathbf{h \&} \mathbf{h}^{\prime}$ | $\mathbf{P}_{\mathbf{A}}$ |
| 0.01 | 0.9999985 |
| 0.02 | 0.9999981 |
| 0.03 | 0.999997 |
| 0.04 | 0.999995 |
| 0.05 | 0.999992 |


5) From the following Table No.4.2 we can observe the different relationships between the ARL's and Type-C OC Curves with the parameters of the CASP-CUSUM.

TABLE 4.2
CONSOLIDATED TABLE

| B | $\alpha$ | $\beta$ | $\gamma$ | h | $\mathrm{h}^{\prime}$ | $P_{A}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.1 | 1 | 1 | 1 | 0.01 | 0.01 | 0.90519261 |
| 4.1 | 1 | 1 | 1 | 0.02 | 0.02 | 0.89626336 |
| 4.1 | 1 | 1 | 1 | 0.03 | 0.03 | 0.88637465 |
| 4.1 | 1 | 1 | 1 | 0.04 | 0.04 | 0.87558847 |


| 4.1 | 1 | 1 | 1 | 0.05 | 0.05 | 0.86397045 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4.0 | 1 | 2 | 1 | 0.01 | 0.01 | 0.99999850 |
| 4.0 | 1 | 2 | 1 | 0.02 | 0.02 | 0.99999815 |
| 4.0 | 1 | 2 | 1 | 0.03 | 0.03 | 0.99999701 |
| 4.0 | 1 | 2 | 1 | 0.04 | 0.04 | 0.99999505 |
| 4.0 | 1 | 2 | 1 | 0.05 | 0.05 | 0.99999207 |
| 4.1 | 1 | 1 | 2 | 0.01 | 0.01 | 0.88330459 |
| 4.1 | 1 | 1 | 2 | 0.02 | 0.02 | 0.85522657 |
| 4.1 | 1 | 1 | 2 | 0.03 | 0.03 | 0.82872247 |
| 4.1 | 1 | 1 | 2 | 0.04 | 0.04 | 0.80365979 |
| 4.1 | 1 | 1 | 2 | 0.05 | 0.05 | 0.77992045 |
| 4.1 | 2 | 1 | 1 | 0.01 | 0.01 | 0.92060083 |
| 4.1 | 2 | 1 | 1 | 0.02 | 0.02 | 0.87145429 |
| 4.1 | 2 | 1 | 1 | 0.03 | 0.03 | 0.81129944 |
| 4.1 | 2 | 1 | 1 | 0.04 | 0.04 | 0.74583601 |
| 4.1 | 2 | 1 | 1 | 0.05 | 0.05 | 0.68064385 |
| 4.0 | 1 | 2 | 1 | 0.02 | 0.02 | 0.99999815 |
| 4.0 | 1 | 2 | 1 | 0.03 | 0.03 | 0.99999701 |
| 4.0 | 1 | 2 | 1 | 0.04 | 0.04 | 0.99999505 |
| 4.0 | 1 | 2 | 1 | 0.05 | 0.05 | 0.99999207 |
| 4.0 | 1 | 3 | 1 | 0.01 | 0.01 | 0.99998444 |
| 4.0 | 1 | 3 | 1 | 0.02 | 0.02 | 0.99995601 |
| 4.0 | 1 | 3 | 1 | 0.03 | 0.03 | 0.99991178 |

$$
\left[\begin{array}{l}
B=4.0 \\
\alpha=1 \\
\beta=2 \\
\gamma=1 \\
h=0.01 \\
h^{\prime}=0.01
\end{array}\right]
$$

## REFERENCES

[1] Akhtar, P. Md. and Sarma, K.L.A.P. (2004). "Optimization of CASP-CUSUM Schemes based on Truncated Gamma Distribution". Bulletin of Pure and applied sciences, Vol-23E (No.2), pp215-223.
[2] G.Venkatesulu, P.Mohammed Akhtar, B.Sainath and Narayana Murthy, B.R. (2018)"Continuous Acceptance Sampling Plans for Truncated Lomax Distribution Based on CUSUM Schemes ".International Journal Mathematics Trends and Technology, Vol-55, pp174-184.
[3] S. Kotz and S. Nadarajah, Extreme value distributions: theory and applications, World Scientific, (2000).
[4] Beattie, B.W. (1962). "A Continuous Acceptance Sampling procedure based upon a Cumulative Sums Chart for a number of defective". Applied Statistics, Vol. 11, No. 2, pp. 137-147.
[5] D. Kundu and H. Howlader, Bayesian inference and prediction of the inverse weibull distribution for type-II censored data, Computational Statistics \& Data Analysis 54, pp. 1547-1558 (2010).
[6] Pedro L. Ramos, Francisco Louzada, Eduardo Ramos \& Sanku Dey (2019): The Fréchet distribution: Estimation and application - An overview, Journal of Statistics and Management Systems.
[7] Lonnie, C. Vance. (1986). "Average Run Length of CUSUM Charts for Controlling Normal means". Journal of Quality Technology, Vol.18, pp189-193.
[8] G.Venkatesulu, P.Mohammed Akhtar, B.Sainath and Narayana Murthy, B.R. (2017)"Truncated Gompertz Distribution and its Optimization of CASPCUSUMSchemes".Journal of Research in Applied Mathematics, Vol3-Issue7, pp19-28.
[9] Vardeman, S. And Di-ou Ray. (1985). "Average Run Lenghts for CUSUM scheme Where observations are Exponentially Distributed", Technometrics, vol. 27 (No.2), pp145-150.
[10] Page, E.S., (1954) "Continuous Inspection Schemes", Biometrika, Vol. XLI, pp104-114.

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