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Adaptive Fuzzy Wavelet Automata with Context-Dependent t-Norm Transition Dynamics

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Abstract: This paper proposes a fuzzy wavelet automaton with context-dependent t-norm transition dynamics (cwt) for modelling temporal-symbolic systems whose uncertainty varies across scale, regime and input context. Classical fuzzy automata extend finite-state computation by allowing graded state membership and fuzzy transitions, yet most formulations use fixed transition memberships and a single aggregation rule throughout the computation. The situation is restrictive for systems in which local signal behaviour, noise intensity, and contextual reliability vary as the sequence is processed. The suggested framework consists of multiresolution features based on discrete wavelet, fuzzy automata and dynamically selected t-norm operator. A formal tuple is constructed in which every transition is reliant on the current input symbol, the wavelet coefficient vector, a context variable, and an adaptive transition matrix. The model combines a hard t-norm selector for formal consistency with a soft-gated computational variant for differentiable training when associativity is unimportant. The authors of this manuscript offer you the mathematical specification, transition equations, acceptance mechanism, adaptive update rule, algorithm, and simulation protocol. Due to lack of evidence, here the results section is limited to analytical interpretation and clear reporting templates, rather than alleged numerical performance. The proposed model provides a logical bridge between fuzzy automata and wavelet representation for dynamic uncertain systems and context-sensitive aggregation. It is placed for validation in future pattern detection, bio-signal understanding, smart control, computational linguistics, and time-series modelling in finance.
Keywords: fuzzy automata; wavelet transform; t-norm; adaptive transition systems; uncertainty modelling; computational intelligence; formal methods.

I. INTRODUCTION

Formal language theory heavily relies on finite state models as they provide a compact narration of actions such as sequence recognition, state transition, and acceptance. However, their classical form assumes crisp state occupancy and binary transition viability. Often, those assumptions are too restrictive to temporal processes where the observations are noisy, partially interpretable (so that not all features mean much), or expressed in a graded way. To counter such limitations, fuzzy automata assign degrees of membership to states, transitions, and recognized strings, generalizing the automata-theoretic reasoning into the interval-valued framework created by Zadeh (1965). The first formulation of a sequential machine by Santos and Wee (1968) and Wee and Fu (1969) with fuzzy transition structure allows for approximate recognition. Moreover, it can be interpreted as a device that can learn.

Over the years, several algebraic forms have been developed. These include deterministic and nondeterministic fuzzy automata, lattice-valued fuzzy automata, fuzzy regular expressions, and comparison methods based on bisimulation (Bělohlávek, 2002; Doostfatemeh & Kremer, 2005; Li & Pedrycz, 2005; Li et al., 2006). These models typically define the degree of a transition through a membership function, while any successive degrees are combined using an aggregation operator. Often in implementations, the aggregation operator is fixed prior to computation. A rule that is fixed, minimum, or Lukasiewicz may be mathematically convenient but poorly matched to processes for which uncertainty varies over time, input scale and local context.

Wavelet analysis provides a tool for analyzing dynamic signals. It achieves this objective by breaking down observations into multiresolution steps. The multiresolution signal decomposition was first formalized through wavelet representations by Mallat (1989), whereas Daubechies (1992) provided a rigorous account of compactly supported wavelets. Subsequent work on scattering showed that cascaded wavelet features can be stable under deformations and useful for classification-oriented representation (Bruna & Mallat, 2013; Mallat, 2012). These properties are important for automata processing since many symbolic streams come from signals with discriminative structure at multiple temporal scales.

In this paper, we suggest an Adaptive Fuzzy Wavelet Automaton with Context-Dependent t-Norm, abbreviated AFWA-CDT. The model unifies three components that are commonly treated separately, namely fuzzy transition structures, wavelet based multi-scale representation and context dependent selection of t-norm aggregation. AFWA-CDT allows the transition function to vary depending on the current symbol, the wavelet coefficient vector, and a context variable that summarizes local reliability, noise and scale distribution, rather than applying the same transition intensity and same t-norm throughout the sequence. The aim is not to assert empirical supremacy but rather to develop a framework that is mathematically consistent and offers the potential for implementation and testing in dynamic systems that are uncertain.

II. RESEARCH GAP AND MOTIVATION.

The fuzzy automata theory that already exists is sufficient in terms of graded acceptance and recognition of fuzzy languages. But there are some gaps when your object is a changing temporal-symbolic process. Initially, many fuzzy automata use transition memberships which do not change after model specification. Fixed memberships are appropriate for static fuzzy languages, but they are less appropriate for applications in which the signal-to-symbol relation is variable in local scale, noise, or regime.

Secondly, the aggregate operator employed to merge state membership and transition degree is generally chosen globally. Assumptions on conjunction and attenuation for the minimum t-norm, product t-norm and Lukasiewicz t-norm are different (Klement et al., 2000; Schweizer & Sklar, 1983).

It is not possible for a single rule to represent effectively all uncertainty regimes. The product t-norm, for example, may be excessively conservative in long noisy sequences as membership values may decay quite rapidly, while on the other hand the minimum t-norm may be too preserving of membership. Consequently, a context-dependent operator is not just a computational preference; it encodes a modelling assumption about the local semantics of uncertainty.

Additionally, fuzzy automata are often constructed over symbolic inputs. Often the representation step is external to the automaton when symbols are obtained from observed signals. To hide the underlying multiscale information would cause a transition to be reliable in one regime and unreliable in another regime. The temporal-scale features are incorporated directly into the transition dynamics using wavelet coefficients, making the automaton sensitive to local smoothness, sharp transitions, and high-frequency noise.

The behaviour of fuzzy automata has been further elaborated in a recent work (Micić et al., 2022; Micić et al., 2024; Nguyen, 2023; Nguyen et al., 2023; Stanimirović et al., 2022, 2025), about fuzzy simulations, bisimulations, approximate reduction, and state comparison. While these studies enhance the formal comparison and the reduction of fuzzy automata, they by themselves do not create such transition mechanism. The AFWA-CDT initiative aims to fill this gap by proposing a modelling layer where multiscale observations and context sensitive t-norm selection are embedded into transition computation.

Table 1. Key gaps addressed by AFWA-CDT

Gap in common formulation	Consequence	AFWA-CDT response
Fixed transition memberships	Poor adaptation to changing signal regimes	Transitions are modulated by wavelet coefficients and context variables
Single t-norm throughout sequence	Uniform conjunction semantics under heterogeneous uncertainty	A context-dependent selector chooses the local t-norm
External signal-to-symbol representation	Loss of scale information before automata processing	Wavelet coefficients enter the transition function
Limited adaptive update	Model parameters remain insensitive to sequence feedback	Projected update adjusts transition weights under supervised or self-supervised loss
Weak transparency in validation	Risk of ungrounded numerical claims	Simulation protocol and reporting templates are separated from empirical results

III. AIM AND OBJECTIVES

The main aim of this paper is to formulate a mathematically explicit adaptive fuzzy wavelet automaton in which transition dynamics are jointly shaped by wavelet-based multiresolution features and context-dependent t-norm selection.

1) To define AFWA-CDT as a formal automaton tuple with state space, alphabet, membership function, wavelet feature map, context variable, transition operator, learning rate, and final-state membership.
2) To specify how wavelet coefficients modify fuzzy transition degrees for temporal-symbolic sequences.
3) To develop a context-dependent t-norm selection mechanism that preserves formal t-norm behaviour under hard selection and allows a differentiable soft-gated variant for computation.
4) To design an adaptive update rule for transition parameters without claiming convergence beyond the assumptions stated.
5) To provide pseudocode for sequence processing, transition computation, adaptation, and acceptance scoring.
6) To outline a transparent simulation framework with baselines, metrics, and reporting templates without fabricating empirical performance.

IV. LITERATURE REVIEW

A. *Classical finite automata and fuzzy automata*

Classical finite automata are an abstract model for recognizing regular languages through finitely many states and symbol-indexed transitions. Fuzzy automata generalise this scheme by replacing the crisp membership with a more general graded membership taking values in the unit interval or, more generally, in an ordered algebraic structure. Zadeh (1965) provided the conceptual basis for set membership as a degree rather than as a yes-no judgment. Santos and Wee (1968) as well as Wee and Fu (1969) took over the idea of secondary machine and learning-system model. Subsequent monographic research presented a systematic analysis of fuzzy automata, fuzzy languages, minimization, recognition, and application (Mordeson & Malik, 2002).

1) *Fuzzy transition systems and fuzzy languages.*

Fuzzy language theory relies on further combination of low membership values along paths and comparison of accepted strings. In 2002, Bělohlávek examined determinism in fuzzy automata. In 2005, Li and Pedrycz linked fuzzy finite automata with fuzzy regular expressions over lattice-ordered monoids. Li et al. (2006) conducted an analysis of deterministic and nondeterministic variants. Cao and Ezawa made a re-examination of non determinism and clarified equivalences among the deterministic fuzzy automata, the non-deterministic fuzzy automata, and the non-deterministic fuzzy automata with epsilon moves. The contributions of the fuzzy automata show that they possess a fairly mature formal base, but most of them are still concerned with fixed algebraic transition structures. Most of them ignore the opportunity for feature-conditioned adaptation.

2) *Triangular norms and aggregation operators 4.3*

The functions known as triangular norms represent conjunction-like aggregation in structures such as fuzzy logic or probabilistic metrics (or both). The minimum, product, Lukasiewicz t-norms are widely employed operations. Klement et al. (2000) offer a thorough mathematical study of t-norm families, while Schweizer and Sklar (1983) locate them in probabilistic metric spaces. A t-norm can be utilized in fuzzy automata to combine the membership of current state along with transition membership. All t-norms encode a different attenuation rule, which means a reasoning about which t-norm to use influences the accepted degree of the language. For instance, it influences the dominance of the path as well as the sensitivity to uncertainty. The paper treats the t-norm not as a universal constant but as a context-indexed element of the transition dynamics.

3) *Wavelet transforms for pattern representation and signal-symbol mapping*

Wavelet representations decompose signals into approximation and detail components on different scales. According to Mallat (1989), the algorithm for computing multiresolution analysis through a pyramid was shown. The result of this approach was seen at Daubechies, 1992, which established the theory of compactly supported wavelets. Wavelet features can be useful in pattern recognition because they can separate local changes from low-frequency trends and high-frequency variation. Extension of this notion occurs with scattering transforms via cascaded wavelet-modulus operators, the deformation stability and classification representation of which have been analysed. AFWA-CDT utilizes wavelet coefficients in a more established automata framework where they serve as transition modulators rather than merely classification features.

4) Adaptive and learning-based automata models.

The use of Weighted automata algorithms are more general than acceptability ones, as they typically apply to sequences in which transitions have weights rather than the one of binary feasibility (Mohri 2009). Fuzzy automata possess this weighted feature, but interpret the values through fuzzy membership and aggregation. Through projected updates over transition parameters, one can formulate adaptive transition learning. However, it is necessary that the updates remain inside the unit interval as well as respect the membership semantics. Accordingly, the current model utilizes bounded projection and divides supervised from self-supervised adaptation. This doesn't make the learning a blanket guarantee and pinning down adaptation necessarily as a constrained mechanism whose empirical behaviour needs later validation.

5) Recent behavioural comparison and reduction of fuzzy automata.

Recent works (Micić et al., 2022; Micić et al., 2024; Nguyen, 2023; Nguyen et al., 2023; Stanimirović et al., 2022, 2025) have progressed in the behavioural comparison of fuzzy automata using approximate simulation, bisimulation, depth-bounded bisimulation, weak bisimulation and state reduction. These studies matter because they look into ways to compare or reduce fuzzy automata without sharp crisp equivalence. The AFWA-CDT model is not a reduction methodology per se but rather a modelling architecture within which context, multiscale representation, and transition aggregation together determine fuzzy configurations' evolution.

V. THEORETICAL FRAMEWORK

Let $Q = \{q1, q2, ..., qm\}$ be a finite state space and Sigma be a finite input alphabet. A temporal-symbolic sequence is denoted by $x = (x1, x2, ..., xn)$, where xt belongs to Sigma. In applications where symbols are derived from a signal, let $s = (s1, s2, ..., sn)$ be an observed real-valued or vector-valued signal associated with x. The model permits x to be observed directly, derived from s through discretization, or paired with s as a signal-symbol stream.

A fuzzy configuration at time t is a vector α_t in $[0,1]^m$. The component $\alpha_t(i)$ gives the degree to which the automaton occupies state qi after processing the first t inputs. The initial configuration α_0 may be crisp, such as $\alpha_0(i0) = 1$ for an initial state qi0, or fuzzy when multiple initial states are plausible.

Wavelet information enters through a coefficient vector w_t. For a discrete wavelet transform with decomposition depth J, an observation window around time t is decomposed into approximation and detail coefficients. The local coefficient vector is written as follows:

$$s(t) = \sum_k a_{J,k} \phi_{J,k}(t) + \sum_{j=1}^{J} \sum_k d_{j,k} \psi_{j,k}(t),$$

$$w_t = [a_{J,k(t)}, d_{J,k(t)}, d_{J-1,k(t)}, ..., d_{1,k(t)}]^T$$

Here phi denotes a scaling function, psi denotes a wavelet function, $a_{J,k}$ is a scale-J approximation coefficient, and $d_{j,k}$ is a detail coefficient at scale j. For a symbolic-only stream, the symbols may first be embedded into vectors e(xt), after which the wavelet transform is applied to the embedding path. This preserves the automata-theoretic alphabet while allowing multiscale variation to influence transitions.

A context variable c_t belongs to a context space C. It may include a noise estimate, local scale entropy, symbol reliability, prior state confidence, and regime indicator. A simple context vector is:

$$c_t = [\nu_t, H_t, r_t, u_t]^T$$

where ν_t denotes local noise intensity, H_t is scale entropy computed from normalized wavelet energies, r_t is a regime feature, and u_t is transition uncertainty. These components are examples rather than mandatory variables. The framework only requires that c_t be available before the t-th transition is evaluated.

The context-dependent t-norm operator $T_{c_t}: [0,1] \times [0,1] \to [0,1]$ combines the previous state degree and the transition degree. The acceptance degree of a processed sequence is obtained by combining the final fuzzy configuration with fuzzy final-state memberships.

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Table 2. Mathematical elements of the theoretical framework

Symbol	Meaning	Domain
Q	Finite state set {q1, ..., qm}	finite set
Sigma	Input alphabet	finite set
alpha_t	Fuzzy state configuration after t symbols	[0,1]^m
w_t	Wavelet coefficient vector at time t	R^p
c_t	Context variable controlling transition semantics	C subset R^r
Theta_a(t)	Adaptive transition matrix for symbol a at time t	[0,1]^{m x m}
T_{c_t}	Context-dependent t-norm selected at time t	family of t-norms
F	Fuzzy final-state membership vector	[0,1]^m

VI. PROPOSED MODEL: ADAPTIVE FUZZY WAVELET AUTOMATA

The proposed model is denoted by AFWA-CDT and defined as the tuple:

AFWA-CDT = (Q, Sigma, mu_0, W, C, T_c, delta, eta, F)

In this tuple, Q is the finite state set, Sigma is the input alphabet, mu_0 is the initial fuzzy state distribution, W is a wavelet feature map, C is the context space, T_c is the context-dependent t-norm rule, delta is the adaptive transition function, eta is the learning-rate parameter, and F is the fuzzy final-state membership vector.

The wavelet feature map W takes a local signal or embedded symbolic window and returns a coefficient vector. Let z_t denote the observation window used at time t. Then:

w_t = W_J(z_t; psi, phi)

where J is the decomposition depth and psi and phi are the selected wavelet and scaling functions. For computational use, the coefficient vector can be normalized to avoid scale dominance:

bar{w}_t = (w_t - mean(w_t)) / (std(w_t) + epsilon)

The adaptive fuzzy transition degree from qi to qj under input a at time t is defined as:

delta_t(q_i, a, q_j | bar{w}_t, c_t) = Pi_[0,1] { sigma(theta_{ij,a}^T [1, bar{w}_t^T, c_t^T]^T) }

Here sigma(u) = 1/(1 + exp(-u)) is a logistic squashing function, theta_{ij,a} is a parameter vector for the transition qi -> qj under symbol a, and Pi_[0,1] denotes projection onto the unit interval. This form ensures that transition membership remains a valid fuzzy degree. Other bounded membership functions may be used when the application requires interpretability or sparsity.

The fuzzy configuration update is:

alpha_{t+1}(j) = max_{1 <= i <= m} T_{c_t}(alpha_t(i), delta_t(q_i, x_t, q_j | bar{w}_t, c_t))

The max operator acts as the join over alternative predecessor states. In a complete lattice-valued version, max can be replaced by the lattice join. This update preserves the automata interpretation: each new state degree is determined by the strongest fuzzy path that reaches it under the local t-norm semantics.

The final acceptance score is calculated by combining the terminal state configuration with final-state memberships:

A(x, s) = max_{1 <= j <= m} T_{c_n}(alpha_n(j), F(j))

When labels y in [0,1] are available for training sequences, a supervised loss can be specified as:

L(Theta) = sum_r (y^{(r)} - A(x^{(r)}, s^{(r)}))^2 + lambda ||Theta||_2^2

A projected gradient update for a transition parameter vector is:

theta_{ij,a}^{tau+1} = Pi_B(theta_{ij,a}^{tau} - eta * partial L / partial theta_{ij,a}^{tau})

where Pi_B projects parameters onto a bounded feasible set B. When labels are absent, the loss may be replaced by a self-supervised consistency objective, such as agreement between adjacent-scale transition predictions or reconstruction of held-out symbols. The model does not claim convergence in this paper; convergence depends on the choice of loss, optimizer, data distribution, and regularization.

VII. CONTEXT-DEPENDENT T-NORM TRANSITION DYNAMICS

A t-norm T is a binary operation on [0,1] satisfying commutativity, associativity, monotonicity, and the boundary condition T(x,1) = x. The present model uses a finite candidate family containing the minimum, product, and Lukasiewicz t-norms:

T_min(x,y) = min(x,y)

T_prod(x,y) = x y

T_Luk(x,y) = max(0, x + y - 1)

The minimum t-norm is weakly attenuating because it preserves the smaller of two degrees. The product t-norm is multiplicative and becomes conservative over long paths. The Lukasiewicz t-norm imposes a threshold-like penalty because the output becomes zero when x + y <= 1. These distinctions matter in fuzzy transition dynamics because the operator changes how quickly path membership decays.

For formal consistency, AFWA-CDT uses hard context-dependent selection. Let g(c_t) be a gating score vector in R^3:

g(c_t) = G c_t + b

The selected operator is:

k_t = argmax_{k in {min, prod, Luk}} g_k(c_t),

T_{c_t} = T_{k_t}

Because a genuine t-norm is selected at each transition, the local conjunction operation remains a t-norm. The computation is context-dependent across time, but each individual transition uses a valid t-norm. If a single global algebra over the entire sequence is required, further restrictions are needed because changing operators across time may affect algebraic properties such as global associativity over mixed contexts.

A differentiable soft-gated computational variant may be used during training:

lambda_k(c_t) = exp(g_k(c_t)) / sum_l exp(g_l(c_t)),

T_soft(x,y | c_t) = sum_k lambda_k(c_t) T_k(x,y)

This soft aggregation is useful for gradient-based optimization, but it should not automatically be called a t-norm because convex combinations of t-norms do not generally preserve all t-norm axioms under arbitrary conditions. For this reason, the paper distinguishes the formal hard-selection model from the computational soft-gated variant.

Table 3. Context-dependent interpretation of candidate t-norms

Context condition	Preferred operator	Interpretation in transition dynamics
High noise but weak conflict	Minimum	Avoids excessive decay when uncertainty may be caused by measurement noise
Moderate uncertainty with quasi-independent evidence	Product	Represents multiplicative attenuation of state and transition degrees
High conflict or strong negative evidence	Lukasiewicz	Suppresses transitions unless combined evidence exceeds a threshold
Stable low-noise regime	Product or minimum	Selected according to validation criterion and membership calibration
Abrupt regime change indicated by wavelet detail energy	Lukasiewicz or product	Increases selectivity when the local pattern is unstable

VIII. ALGORITHM DESIGN

The algorithm below describes the operational form of AFWA-CDT. It is written for a labelled or unlabelled sequence depending on whether an adaptive loss is available.

Algorithm 1: AFWA-CDT sequence processing and adaptive transition update
Input: sequence x = (x1, ..., xn), signal or embedded stream s, state set Q, initial configuration mu_0, wavelet map W_J, context map C_map, transition parameters Theta, final vector F, learning rate eta.
Output: acceptance degree A(x,s) and updated transition parameters Theta.
1. Set alpha_0 = mu_0.
2. For each time t = 1, ..., n:
   a. Extract local window z_t from s or from the embedding path of x.
   b. Compute wavelet coefficient vector w_t = W_J(z_t).
   c. Normalize w_t to obtain bar{w}_t.
   d. Compute context vector c_t = C_map(bar{w}_t, alpha_{t-1}, x_t).
   e. Select t-norm T_{c_t} by hard context gating.
   f. For each candidate transition qi -> qj, compute delta_t(qi, x_t, qj | bar{w}_t, c_t).
   g. Update alpha_t(j) = max_i T_{c_t}(alpha_{t-1}(i), delta_t(qi, x_t, qj | bar{w}_t, c_t)).
3. Compute A(x,s) = max_j T_{c_n}(alpha_n(j), F(j)).
4. If a training objective is supplied, compute L and update Theta through projected gradient or another bounded optimizer.
5. Return A(x,s) and Theta.

The computational cost for one sequence is dominated by wavelet decomposition and transition evaluation. If m is the number of states, n is sequence length, p is the number of wavelet coefficients, and |Sigma| is the alphabet size, the transition evaluation is O(n m^2 p) for dense state-to-state transitions. Sparse transition matrices reduce this cost substantially. Wavelet computation depends on the selected transform and windowing strategy; for standard fast discrete wavelet transforms it is typically linear in the window length for each processed window.

IX. SIMULATION DESIGN / EXPERIMENTAL FRAMEWORK

The proposed simulation framework for future validation has not been put forth as finished empirical work. We will see if context-dependent t-norm selection and wavelet-conditioned transitions lead to greater robustness under controlled noise and regime change. The data can be synthetic temporal-symbolic sequences generated through a hidden fuzzy automaton with transition matrices that differ across regimes. Both the symbolic sequence generated and the corresponding real-valued signal are used in the generation of the streams.

The benchmark should consist of at least three regimes: a smooth low-noise dynamic, high-frequency disturbance, and abrupt transition shift. Sigma, the alphabet, may be {a, b, c}. Sequence lengths may vary from 50 to 200, while ‘noise level’ may be preset as a protocol parameter. The train-validation-test split has to be fixed beforehand. Any introduction of real data later on should be regarded as a controlled diagnostic test and the synthetic benchmark should remain the main evidence of practical performance.

Table 4. Proposed simulation protocol

Component	Specification
Dataset type	Synthetic benchmark sequences with paired signal-symbol representation
Alphabet	Sigma = {a, b, c} or application-specific symbol set
Regimes	Smooth, noisy high-frequency, abrupt shift
Noise levels	Pre-specified additive noise levels; values to be declared before execution
Wavelet family	Daubechies or other compactly supported wavelet selected by validation
Baselines	Standard fuzzy automaton; fixed-min t-norm fuzzy automaton; fixed-product t-norm fuzzy automaton; non-wavelet adaptive fuzzy automaton
Metrics	Recognition accuracy, macro-F1, acceptance error, calibration error, robustness under noise, computation time
Validation method	Repeated train-validation-test runs with seeds reported and code archived

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The baseline models are defined to isolate the contribution of each component. A standard fuzzy automaton tests the value of adaptation. Fixed t-norm baselines test the value of context-dependent aggregation. A non-wavelet adaptive fuzzy automaton tests whether multiresolution coefficients contribute beyond adaptive transition learning alone. A fair comparison requires identical state cardinality, alphabet, training split, and stopping criteria across all models.

X. RESULTS AND ANALYSIS

No numerical experiment was executed for this manuscript. Therefore, this section does not report measured accuracy, F1 score, p-values, confidence intervals, or runtime. Instead, it provides analytical interpretation and reporting templates to be completed only after the simulation protocol is implemented. This distinction is necessary to avoid presenting illustrative values as empirical evidence.

The expected analytical behaviour of AFWA-CDT can be described at the level of model mechanics. When detail coefficients indicate abrupt local variation, the context variable can shift transition aggregation from a permissive operator to a stricter one. When noise is high but not accompanied by structural conflict, the minimum t-norm can prevent unnecessary path extinction. When evidence is moderately reliable, product aggregation can provide gradual attenuation. These are modelling consequences of the chosen operators, not observed performance results.

Table 5. Results reporting template for future simulation

Model	Accuracy	Macro-F1	Acceptance error	Calibration error	Runtime
Standard fuzzy automaton	To be computed	To be computed	To be computed	To be computed	To be computed
Fixed minimum t-norm fuzzy automaton	To be computed	To be computed	To be computed	To be computed	To be computed
Fixed product t-norm fuzzy automaton	To be computed	To be computed	To be computed	To be computed	To be computed
Non-wavelet adaptive fuzzy automaton	To be computed	To be computed	To be computed	To be computed	To be computed
AFWA-CDT	To be computed	To be computed	To be computed	To be computed	To be computed

Table 6. Sensitivity analysis template

Experimental factor	Levels to test	Main diagnostic question
Noise intensity	Low, medium, high	Does context gating prevent excessive decay or over-acceptance?
Wavelet decomposition depth	J = 1, 2, 3, ...	Which scale range contributes most to transition stability?
State cardinality	Small, moderate, large	Does the model overfit as state capacity increases?
t-norm selection rule	Hard selector, soft-gated variant, fixed baselines	Does adaptive aggregation add value beyond fixed semantics?
Learning rate	Grid or adaptive schedule	How sensitive are transition parameters to update magnitude?

Recommended graph outputs include: (a) accuracy or acceptance-error trend across noise levels, (b) acceptance-degree trajectory for representative sequences, (c) distribution of selected t-norms across regimes, (d) wavelet-scale contribution plot based on transition sensitivity, and (e) runtime as a function of state cardinality. These graphs should be generated only from executed simulations or real datasets, with code and random seeds reported.

XI. DISCUSSION

The AFWA-CDT provides a modeling architecture, not an empirical result. The main merit of our approach is its explicit coupling of three mathematical layers: fuzzy state evolution, a wavelet-based multiscale representation and context-sensitive conjunction semantics. Fuzzy automata may be used for graded recognition, but are unable to distinguish between uncertainty due to high-frequency noise, low-confidence symbolisation and ambiguity of transition. The model proposed provides the automaton with information about the local structure before the membership is propagated by inserting wavelet coefficients and context variables into the transition function.

The t-norm component depending on the context is important. The minimum, product and Lukasiewicz t-norms are modelling choices that cannot be treated interchangeably. Their views offer divergent assumptions about how state confidence and transition confidence react. A hard selector uses a genuine t-norm at each transition, while the soft-gated version allows for differentiable learning but is difficult to interpret as it does not guarantee every t-norm axiom is retained. The differentiation we make here veers away from a mistake commonly made in computational fuzzy systems, which is to treat a convenient differentiable aggregation as if it automatically preserves the algebraic semantics of the weakly associating operator family to which it belongs.

Recent research on fuzzy automata is compatible with the model. The approximate bisimulation and state reduction allow for comparison and simplification of fuzzy automata (Micić et al., 2022; Nguyen, 2023; Stanimirović et al., 2025). After training, AFWA-CDT could use these methods to remove duplicate states while maintaining approximate behavior over sequences of bounded length. In the same vein, weighted automata algorithms provide algorithmic insight for efficient path computation and parameter management (Mohri, 2009).

The main hurdle is validation. A model with wavelet features, context gating and adaptive transitions has greater flexibility due to higher design degrees of freedom than a standard fuzzy automaton If benchmarking is not disciplined, any improvement reported could merely be due to over-parameterization rather than a real gain in the representation of uncertainty. Consequently, the design of the simulations must compare against ablated baselines and report computation cost, calibration, and sensitivity not just classification accuracy.

XII. SOFTWARE PROGRAMS.

The AFWA-CDT framework works well in domains with sequential, uncertain, and partially symbolic observations. The most immediate use is uncertain pattern recognition, i.e. where the symbols may correspond to discretized signal events, and the wavelet coefficient may capture local temporal structure. In temporal signal classification, the model should be able to integrate coarse symbolic regimes and fine-scale signal variation. In intelligent control, partial system modes are represented by fuzzy state membership, while t-norms that depend on context, modify the strictness of transition.

In NLP, the model may be applied to a low-resource noisy symbolic stream in which the reliability of the token is context-dependent. The automaton won't replace neural sequence models, but can impose interpretable fuzzy transition constraints. In the analysis of bio-signals, wavelet features of nonstationary signals are already informative. AFWA-CDT could map these features into graded state transitions for event recognition. One likely application area is cyber-physical systems. Another plausible area is financial time-series modelling. This is because both have regime shifts, heterogeneous noise, and sequential dependencies.

XIII. CONSTRAINTS

1) The methodology proposed by the paper is a theoretical and simulation-ready framework that does not provide empirical executed results.
2) The adaptive updating procedure has a mathematical specifications but no convergence theorem is offered.
3) The stability or interpretability of t-norm selection can be compromised through incorrect construction of context variables.
4) The hyperparameters of wavelet decomposition are the wavelet family, the depth of decomposition and window length.
5) Dense transition matrices scale as O(m^2) per symbol. For large state spaces, this may be costly.
6) The softed-gated aggregating variant, despite being computationally appealing, should not be considered a formal t-norm without further proof or conditions.
7) Before the model can be recommended for operational deployment, it requires real-world validation.

XIV. POTENTIAL OF FUTURE

The proposed simulation framework along with the code, seeds and synthetic data generators should be used in future research. The model should then be tested on real temporal datasets where the coupling of signal and symbol is meaningful. For example, datasets include bio-signals, sensor streams, human activities, and financial events.

Another avenue is the formal analysis of stability and convergence with respect to particular loss functions and bounded update rules. Merging neuro-fuzzy learning is another form in which the neural modules estimate context variables while the automaton preserves the interpretable transition structure. Choosing a t-norm with reinforcement learning is another possible approach. An agent might learn which aggregation semantics minimizes long-run recognition error or maximizes calibrated acceptance, rather than specifying the context gate by hand. Nevertheless, such learning must maintain a clear distinction between formal t-norm selection and merely differentiable aggregation. The use of sparse transitions and compact wavelet filter implementations is a subject for future work dedicated to edge-cyberphysical systems.

XV. CONCLUSIONS

The introduction of AFWA-CDT was made for a fuzzy wavelet automaton whose transition dynamics is t-norm. The model extends fuzzy automata, wherein the transition degrees depend on wavelet-induced multiresolution information and contextual reliability. The proposal modifies the aggregation meaning through a formal hard selector over the established t-norms, and comes with a soft-gated computational variant which is carefully qualified. The mathematical specification progressively builds up an inference engine towards the final acceptance score according to internal-theory usage. The paper intentionally does not claim any empirical performance as no simulation nor real dataset is run. It contributes a rigorous theoretical and methodological framework that can inform implementation and validation. The AFWA-CDT links fuzzy automata, wavelet analysis and context-sensitive t-norm dynamics to create a framework for modelling dynamic uncertain systems with joint consideration of symbolic sequence processing and multi-scale signal behaviour.

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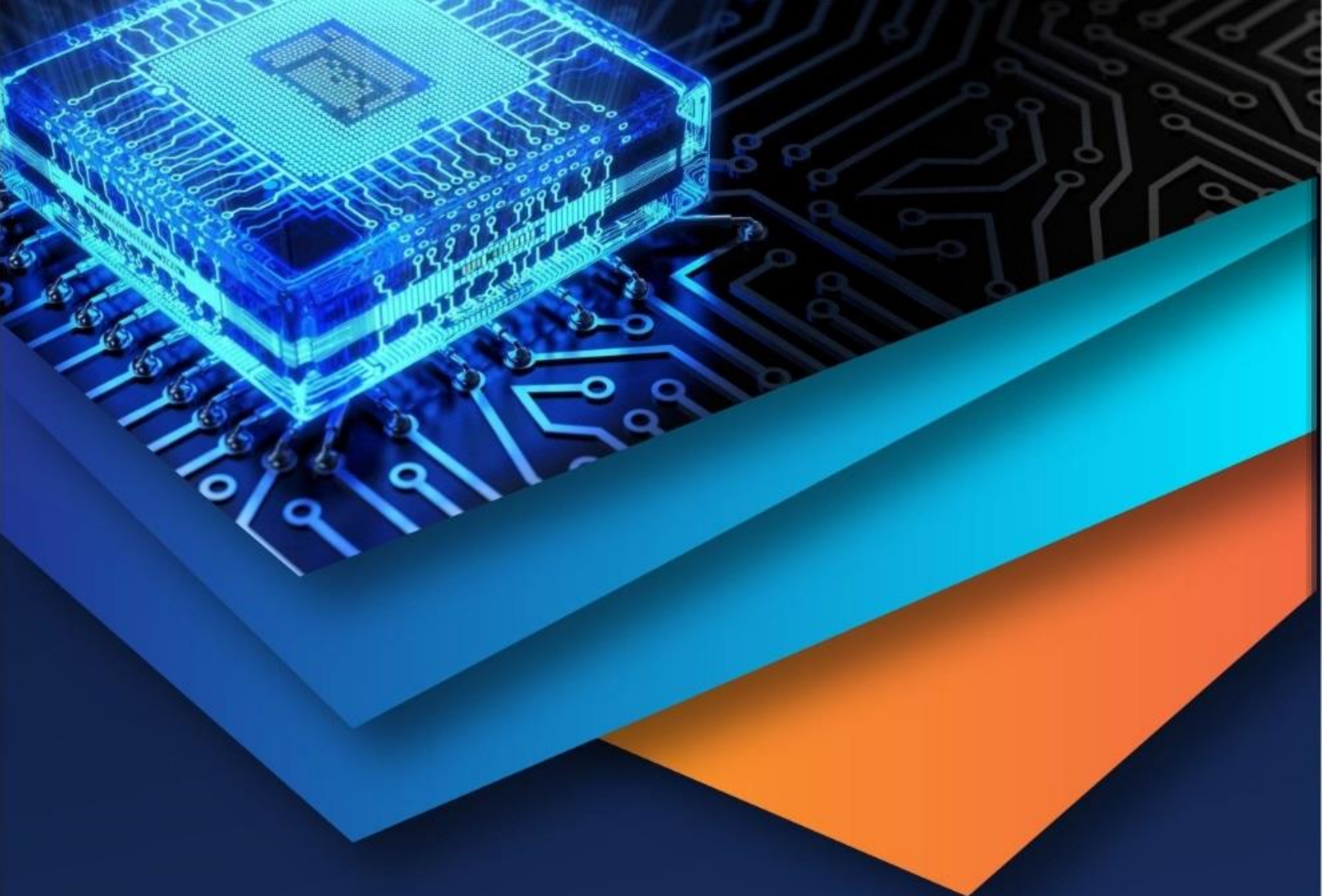

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