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Alpha Logarithm Transformed Half Logistic Distribution: Properties and Applications

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Abstract: In this paper, we introduce a new two parameter Alpha Logarithm Transformed Half Logistic (ALTHL) distribution. A new method has been proposed to introduce an extra parameter to a family of distributions for more flexibility .The proposed family may be named as Alpha Logarithm Transformed Half Logistic. This can be obtained via reparameterizing the exponentiated Kumaraswamy G-logarithmic family and the alpha logarithmic family of distributions. The recommended distribution reveals increasing, decreasing and bath-shaped probability Density, Distribution and Hazard rate function and Survival function. Some distributional properties of new model are investigated which include the Density Function, Distribution Function (DF), Quantile Function(QF), Moments, Moment Generating Function (MGF), Cumulative Generating Function (CGF).

Keywords: ALTHL, Probability density function, Cumulative distribution function, Hazard function and Survival function, Properties.

I. INTRODUCTION

In statistical literature a good measure of work has been dedicated to Half Logistic distribution. Several authors and the references cited therein have carried out extensive studies as relate to the estimation, prediction and several other inferences with respect to Half Logistic distribution. The reliability function of Half Logistic distribution decreases at a large amount higher rate than the reliability function of exponential distribution.

To pick up the characteristics of these traditional distributions, researchers have been raising various extensions and modified forms of these distributions. However, in the recent literature, researchers have shown a deep interest in proposing new families of distributions. The literature is packed with such methods that are quite prosperous and still growing quickly.

A α Logarithmic Transformed Family of Distributions with Application introduced Sunku Dey *et al* (2017). A Flexible Reduced Logarithmic-X Family of Distributions with Biomedical Analysis. A Flexible Reduced Logarithmic-X Family of Distributions with Biomedical analysis introduced YinglinLiu *et al* (2020). Vijaya lakshmi and Anjaneyulu (2018) studied The Odd Generalized Exponential Type-I Generalized Half Logistic Distribution: Properties and Application. Vijaya Lakshmi and Anjaneyulu (2019) studied Quadratic Rank Transmuted Half Logistic Lomax Distribution: Properties and Application. Vijaya Lakshmi and Anjaneyulu (2019) studied Half Logistic Pareto Distribution: Properties and Application. Laba Handique *et al* (2020) studied the New Extended Burr-III Distribution: Its Properties and Applications. Mi Zichun *et al* (2020) studied A New Extended X-family of distribution: Properties and Applications.

Major inspiration behind the proposed family is to find an expansion of the Half Logistic distribution with one addit ional parameters to bring in more flexibility with respect to skewness, kurtosis, tail weight and length.

This encompasses number known distributions as special and connected cases, also to ensure that it provides better alternative in the data modeling not only to its sub models including the Half Logistic distribution, but to other recent extensions.

The chapter is organized as follows. The new distribution Alpha Logarithm Transformed Half Logistic (ALTHL) is developed in the above. A comprehensive account of statistical properties of the new distribution is provided in Section 2. In section 3.1, 3.2 and 3.3 we discuss the Moments, Moment Generating Function and Cumulative Generating Function for ALTHL distribution. Sequentially, Section 4 applied the real life data sets the time-to-event data from different basketball matches and these matches were played during the period 1986– 2021 and the survival times of the patients affected the Leukemia at the final stage on ALTHL for checking the behavior of new distribution.

A random variable X ~ ALTHL $(m; \theta, \sigma^2, \alpha)$ has probability density function and is in the form

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$$f_{ALTHL}(m;\theta,\sigma^2,\alpha) = \frac{2(\alpha-1)e^m}{[\sigma(1+e^m)^2]\left[\log\left\{\alpha-(\alpha-1)\left[\frac{1-e^{-m}}{1+e^{-m}}\right]\right\}\right]}, if\alpha > 0, \alpha \neq 1, m > 0 \quad \ldots(1.1)$$

(θ,σ^2,α) are location scale and shape parameters

A random variable X ~ ALTHL $(m;\theta,\sigma^2,\alpha)$ has cumulative distribution function and is in the form

$$F_{ALTHL}(m;\theta,\sigma^2,\alpha) = 1-\frac{log\left\{\alpha-(\alpha-1)\left[\frac{1-e^{-m}}{1+e^{-m}}\right]\right\}}{\log\alpha}, if\alpha > 0, \alpha \neq 1, m > 0 \quad \ldots(1.2)$$

(θ,σ^2,α) are location scale and shape parameters

II. STATISTICAL PROPERTIES OF ALTHL

QUANTILES and Random Number Generation of ALTHL Distribution

1) QUANTILS

A random variable X ~ ALTHL $(m;\theta,\sigma^2,\alpha)$ has Quantile function and is in the form

The p^{th} quantile x_p of ALTHL distribution is the root of the equation

$$x_p = \sigma\sqrt{2log\left[\frac{\alpha-\alpha^{(1-p)}}{(\alpha-1)}-1\right]} \quad \ldots (2.1.1)$$

2) Random Number Generation

Let U~ U(0,1), then equation (2.1.1) can be used to simulate a random sample of size n from the ALTHL distribution as follows

$$x_i = \sigma\sqrt{2log\left[\frac{\alpha-\alpha^{(1-u_i)}}{(\alpha-1)}-1\right]}, \text{ I} = 1, 2, \ldots, n. \quad \ldots (2.1.2)$$

III. MOMENTS

A. Moments OF ALTHL

The follows theorem gives the moments of ALTHL

Theorem 1: The r^{th} moment about the origin of X ~ ALTHL $(m;\theta,\sigma^2,\alpha)$ is given by

$$\mu_r^1 = \frac{2}{\sigma}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\sum_{s=0}^{k+2}(-1)^{k+q+s+2}\binom{k}{q}\binom{k+2}{s}(1-\alpha)^{l+k+3}\frac{\gamma(r+1)}{(q+s)^{r+1}}$$

Proof: We have

$$\mu_r^1 = \mathrm{E}(m^r)$$
$$= \int_{-\infty}^{\infty} m^r f_{ALTHL}(m;\theta,\sigma^2,\alpha)dm$$
$$= \int_0^{\infty} m^r \frac{2(\alpha-1)e^m}{[\sigma(1+e^m)^2]\left[log\left\{\alpha-(\alpha-1)\left[\frac{1-e^{-m}}{1+e^{-m}}\right]\right\}\right]}dm \quad \ldots (3.1.1)$$

Byapplying logarithm series and binomial expansions on the equation (3.1.1), we get

$$=\frac{2}{\sigma}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\sum_{s=0}^{k+2}(-1)^{k+q+s+2}\binom{k}{q}\binom{k+2}{s}(1-\alpha)^{l+k+3}\int_0^{\infty} e^{-m(q+s)}m^r \quad \ldots (3.1.2)$$

$$=\frac{2}{\sigma}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\sum_{s=0}^{k+2}(-1)^{k+q+s+2}\binom{k}{q}\binom{k+2}{s}(1-\alpha)^{l+k+3}\frac{[\gamma(r+1)]}{(q+s)^{r+1}} \quad \ldots (3.1.3)$$

Hence proved.

B. MOMENT Generating Function of ALTHL

The follows theorem gives the moment generating function of ALTHL

Theorem 2: The moment generating function of X ~ ALTHL $(m;\theta,\sigma^2,\alpha)$ is given by

$$M_m(t) = \frac{2}{\sigma}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\sum_{s=0}^{k+2}\sum_{m=0}^{\infty}\frac{(-1)^{k+q+s+2}}{m!}\binom{k}{q}\binom{k+2}{s}(1-\alpha)^{l+k+3}(t-(q+s))^m$$

Proof: We have

M_m^t = E(e^{tm})

$= \int_{-\infty}^{\infty} e^{tm} f_{ALTHL}(m; \theta, \sigma^2, \alpha) dm$

$$= \int_0^{\infty} e^{tm} \frac{2(\alpha-1)e^m}{[\sigma(1+e^m)^2]\left[log\left\{\alpha-(\alpha-1)\left[\frac{1-e^{-m}}{1+e^{-m}}\right]\right\}\right]} dm \quad \ldots (3.2.1)$$

By using Logarithm series and binomial expansions on the equation (3.2.1), we obtain the following result

$$= \frac{2}{\sigma} \sum_{l=0}^{\infty} \sum_{k=0}^{\infty} \sum_{q=0}^{k} \sum_{s=0}^{k+2} (-1)^{k+q+s+2} \binom{k}{q} \binom{k+2}{s} (1-\alpha)^{l+k+3} \int_0^{\infty} e^{tm} e^{-m(q+s)} dm \quad \ldots (3.2.2)$$

=

$$= \frac{2}{\sigma} \sum_{l=0}^{\infty} \sum_{k=0}^{\infty} \sum_{q=0}^{k} \sum_{s=0}^{k+2} \sum_{m=0}^{\infty} \frac{(-1)^{k+q+s+2}}{m!} \binom{k}{q} \binom{k+2}{s} (1-\alpha)^{l+k+3} (t-(q+s))^m$$

Hence proved

C. Cumulative Generating Function of ALTHL Distribution

A random variable X ~ ALTHL $(m; \theta, \sigma^2, \alpha)$ has function and is in the form

$K_X(t) = \log(M_x(t))$

$$= \text{Log}\left[\frac{2}{\sigma} \sum_{l=0}^{\infty} \sum_{k=0}^{\infty} \sum_{q=0}^{k} \sum_{s=0}^{k+2} \sum_{m=0}^{\infty} \frac{(-1)^{k+q+s+2}}{m!} \binom{k}{q} \binom{k+2}{s} (1-\alpha)^{l+k+3} (t-(q+s))^m\right]$$

=

$$\prod_{l=0}^{\infty} \prod_{k=0}^{\infty} \prod_{q=0}^{k} \prod_{s=0}^{k+2} \prod_{m=0}^{\infty} \left[\frac{(-1)^{k+q+s+2}}{m!} \binom{k}{q} \binom{k+2}{s} + (l+k+3)\log(1-\alpha) + m\log(t-(q+s))\right. \quad \ldots (3.3.1)$$

IV. CONCLUSION

1) The ALTHL Distribution failure rate function can have the following forms depending on its shape parameters:
a) Decreasing
b) Upside down bathtub and reversed J-shaped shaped.
c) Therefore, it can be used quite electively in analyzing lifetime data.
2) The ALTHL new distribution gives Incomplete moments.

V. OBSERVATION FOR THE SIMULATION

1) When sample size increases variance decreases. So we conclude that fitted new ALTHL distribution is good.
2) It is observe that, Mean and Median and more are different in ALTHL distribution.

VI. APPLICATIONS

1) Sources of Real data set 1

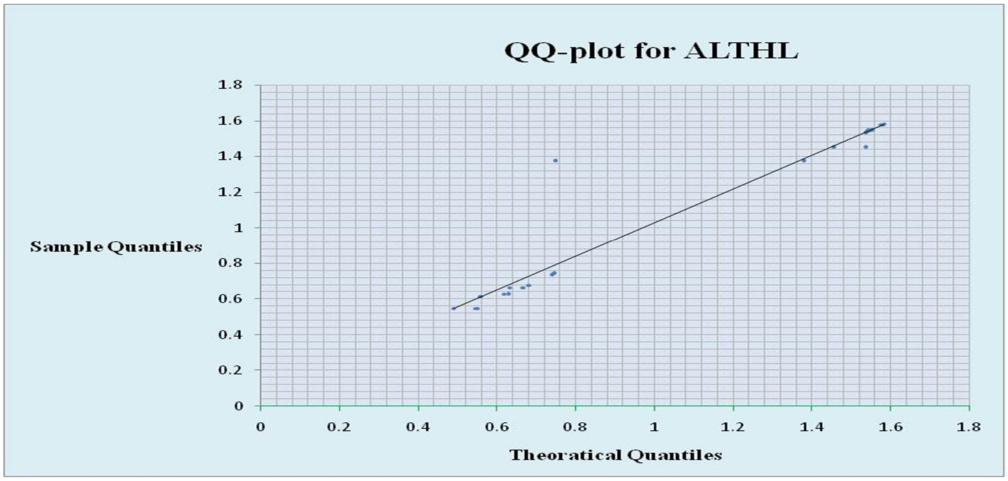
Data analysis in sport: Time-to-event data:

This subsection illustrates the ALTHL distribution by taking the time-to-event data from different basketball matches. These matche s were played during the period 1986– 2021. The observations of this data set represent the time waiting till the first goal is scored. The summary measures (SMs) of the time-to-event data are: 0.23, 0.261, .87, .210, 0.23, 0.47, 0.52, 0.25, 0.47, 5, 12, 0, 0.89, 0.51, 0 .63, 3, 0.90, 0.68, 16, 9.52, 15.6, 11.2, 5.4, 8, 6.3, 8.4, 8, 5, 3.4 and 9.8. The statistical summary of dataset is presented in the followi ng table and QQ-Plot for ALTHL distribution is presented in the below graph-4.1.

TABLE 4.1 Statistical summary dataset 1

Data set	Min.	Q_1	Median	Mean	Q_3	Max.	Skewness	Kurtosis	Variance
basketball	0	0.48	1.95	4.458	8	16	0.9718	-0.1213	0.8065

Graph 4.1



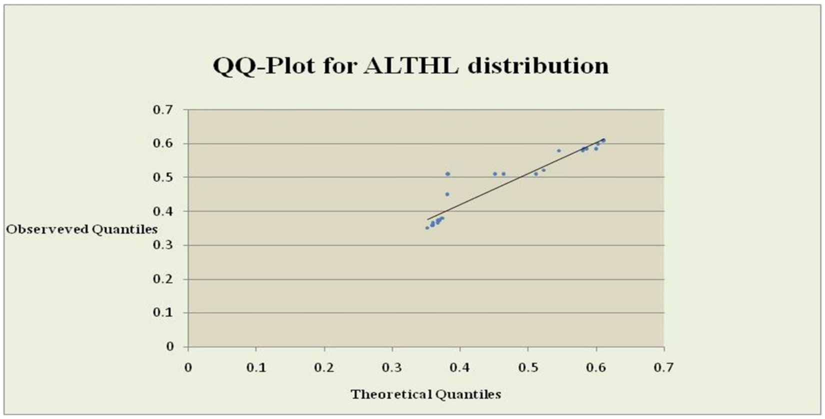
2) Sources of Real data set2

The survival times of the patients affected the Leukemia at the final stage 0.2, 0.3, 0.3, 6, 8, 14, 1.8, 2.1, 2.5, 3.4, 4.0, 4.3, 4.8, 6.5, 5, 8, 9, 1, 3, 2, 2.1, 2.6, 3.2, 4.2, 8, 6, 10, 13, 1.8 and 4.6. The statistical summary of dataset is presented in the following table QQ-Plot for ALTHL distribution is presented in the below graph-4.2.

TABLE 4.2 Statistical summary dataset 2

Data set2	Min.	Q_1	Median	Mean	Q_3	Max.	Skewness	Kurtosis	Variance
Leukemia	0.2	1.0972	4.1	4.7233	6.345	14	1.0191	0.6807	0.4236

Graph 4.2



3) Generalised data set 3

The data set consists of 150 observations from the generalized data. The data are:
1.3914, 4.5913, 15.1147, 10.9434, 15.463, 5.7787, 1.0211, 10.7795, 5.3855, 1.4225, 1.753, 3.9041, 15.1165, 4.7053, 15.0931, 12.4751, 14.1707, 6.632, 1.8808, 14.905, 12.0031, 2.2511, 4.2237, 5.6158, 15.6274,7.1827, 7.7907, 12.5181, 9.056, 8.6614, 15.5509, 5.1786, 5.6652, 12.8267, 12.7864, 9.1549, 7.0706, 12.466, 5.7288, 14.4591, 9.6099, 6.9415, 8.665,8.6211, 12.0302, 3.5082, 11.8498, 7.3196, 15.4653, 11.7495, 15.0739, 8.0525, 14.3053, 5.3466, 9.2336, 9.8095, 14.048, 12.9022, 5.7375, 9.2235, 14.7255, 12.032, 11.0999, 11.1613, 11.0015, 13.4497, 1.5654, 9.229, 7.532, 9.2601,5.1058, 4.16, 10.6834, 7.1786,15.6324, 15.5459, 10.2004, 4.8993, 11.7692, 4.8925, 1.9229, 6.3267, 15.0327, 10.4623 5.1379, 8.8921, 9.7573, 6.8669, 5.6638, 3.673, 4.6123, 14.8917, 3.2619, 7.5334, 13.0579, 4.5075, 4.47, 4.0035, 7.5741, 11.5513, 4.4443, 7.1219, 15.0204, 11.0285, 8.9159, 4.9497, 2.9556, 12.5241, 4.8403, 11.988, 9.2739, 10.677, 5.8483 2.2035, 14.9888, 1.1909, 6.0836, 11.896, 12.5218, 10.3565, 13.5211, 4.1092, 9.327, 3.5398,

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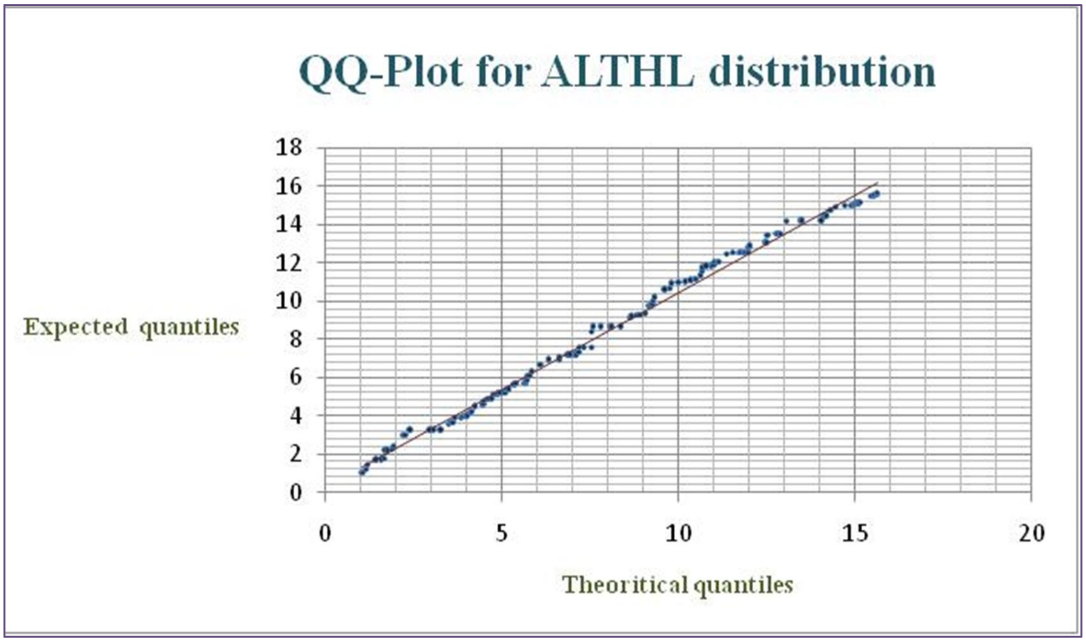
11.3636, 8.822, 14.2073, 10.6229, 3.6075, 10.6248, 1.1524, 3.8488, 8.373, 8.1056, 3.0582, 1.054, 5.0889, 1.6647, 15.1563, 4.745 1, 14.1332, 3.9765, 10.5053, 9.9976, 15.0135, 9.8141, 2.3921, 6.6384, 1.6908, 4.4594.

Graph 4.3



The quantile function in (2.1.1) and the r^th raw moments in (3.1.3) of the ALTHL distribution are used to obtain the values of mean, median, variance, skewness and kurtosis for some selected values of σ^2 and α for sample size n=10. These values are displayed in Table 1.

Table 1
Mean, Median, Variance, Skewness and Kurtosis for ALTHL distribution

σ	θ	α	Mean	Median	Variance	Skewness	Kurtosis
0.5	0.5	0.5	4.103	1.269	0.801	1.074	1.072
	1.5	1.5	4.104	1.306	0.799	1.072	1.066
	2.5	2.5	4.109	1.332	0.788	1.066	1.042
	3.5	3.5	4.115	1.395	0.746	1.042	1.033
	4.5	4.5	4.123	1.426	0.734	1.036	0.978
	5.5	5.5	4.128	1.432	0.734	1.035	0.961
	6.5	6.5	4.141	1.448	0.73	1.033	0.931
	7.5	7.5	4.147	1.463	0.653	0.99	0.897
1.5	0.5	0.5	4.108	1.245	0.826	1.088	1.087
	1.5	1.5	4.118	1.256	0.824	1.087	1.053
	2.5	2.5	4.124	1.294	0.784	1.064	1.009
	3.5	3.5	4.126	1.305	0.773	1.058	0.991
	4.5	4.5	4.127	1.306	0.764	1.053	0.962
	5.5	5.5	4.127	1.416	0.719	1.027	0.947
	6.5	6.5	4.127	1.461	0.717	1.026	0.883
	7.5	7.5	4.131	1.518	0.714	1.024	0.778
2.5	0.5	0.5	4.11	1.28	0.825	1.087	1.084
	1.5	1.5	4.111	1.327	0.82	1.084	1.033
	2.5	2.5	4.112	1.351	0.82	1.084	0.968

	3.5	3.5	4.113	1.474	0.781	1.062	0.91
	4.5	4.5	4.113	1.607	0.753	1.047	0.863
	5.5	5.5	4.114	1.622	0.747	1.043	0.861
	6.5	6.5	4.124	1.65	0.746	1.043	0.849
	7.5	7.5	4.129	1.691	0.745	1.042	0.834
3.5	0.5	0.5	4.103	1.252	0.801	1.074	1.074
	1.5	1.5	4.109	1.369	0.8	1.073	1.035
	2.5	2.5	4.113	1.378	0.795	1.07	1.008
	3.5	3.5	4.113	1.383	0.753	1.046	0.937
	4.5	4.5	4.113	1.4	0.733	1.035	0.935
	5.5	5.5	4.115	1.421	0.732	1.035	0.931
	6.5	6.5	4.12	1.487	0.724	1.03	0.909
	7.5	7.5	4.121	1.498	0.718	1.026	0.829

The quantile function in (2.1.1) and the r^{th} raw moments in (3.1.3) of the ALTHL distribution are used to obtain the values of mean, median, variance, skewness and kurtosis for some selected values of σ^2 and α for sample size n=20. These values are displayed in Table 2.

Table 2

Mean, Median, Variance, Skewness and Kurtosis for ALTHL distribution

σ	θ	α	Mean	Median	Variance	Skewness	Kurtosis
0.5	0.5	0.5	4.103	1.237	0.79	1.067	1.055
	1.5	1.5	4.115	1.257	0.769	1.055	1.021
	2.5	2.5	4.115	1.268	0.761	1.051	0.975
	3.5	3.5	4.115	1.289	0.745	1.042	0.934
	4.5	4.5	4.116	1.296	0.723	1.029	0.875
	5.5	5.5	4.117	1.33	0.708	1.021	0.855
	6.5	6.5	4.118	1.334	0.692	1.012	0.778
	7.5	7.5	4.12	1.345	0.69	1.011	0.759
1.5	0.5	0.5	4.109	1.208	0.831	1.091	1.091
	1.5	1.5	4.11	1.269	0.817	1.082	1.066
	2.5	2.5	4.111	1.279	0.797	1.071	1.017
	3.5	3.5	4.114	1.282	0.788	1.066	0.945
	4.5	4.5	4.116	1.298	0.782	1.063	0.82
	5.5	5.5	4.125	1.315	0.742	1.04	0.782
	6.5	6.5	4.126	1.351	0.702	1.017	0.774
	7.5	7.5	4.129	1.377	0.701	1.017	0.759
2.5	0.5	0.5	4.105	1.236	0.827	1.088	1.087
	1.5	1.5	4.106	1.242	0.826	1.087	0.978
	2.5	2.5	4.109	1.316	0.759	1.05	0.975
	3.5	3.5	4.111	1.337	0.738	1.038	0.966
	4.5	4.5	4.113	1.361	0.664	0.996	0.957
	5.5	5.5	4.115	1.377	0.661	0.994	0.906
	6.5	6.5	4.116	1.379	0.632	0.978	0.866
	7.5	7.5	4.116	1.391	0.627	0.975	0.863
3.5	0.5	0.5	4.148	1.218	0.8	1.073	0.812
	1.5	1.5	4.152	1.224	0.794	1.069	0.759
	2.5	2.5	4.153	1.235	0.785	1.064	0.758

	3.5	3.5	4.153	1.272	0.771	1.056	0.747
	4.5	4.5	4.154	1.288	0.764	1.053	0.732
	5.5	5.5	4.156	1.373	0.763	1.052	0.728
	6.5	6.5	4.157	1.378	0.756	1.048	0.677
	7.5	7.5	4.158	1.398	0.739	1.038	0.656

The quantile function in (2.1.1) and the r^{th} raw moments in (3.1.3) of the ALTHL distribution are used to obtain the values of mean, median, variance, skewness and kurtosis for some selected values of σ^2 and α for sample size n=40. These values are displayed in Table 3.

Table 3
Mean, Median, Variance, Skewness and Kurtosis For ALTHL distribution

σ	θ	α	Mean	Median	Variance	Skewness	Kurtosis
0.5	0.5	0.5	4.132	1.475	0.641	0.983	0.763
	1.5	1.5	4.136	1.478	0.636	0.98	0.754
	2.5	2.5	4.139	1.494	0.633	0.978	0.735
	3.5	3.5	4.14	1.554	0.616	0.969	0.724
	4.5	4.5	4.143	1.593	0.614	0.968	0.701
	5.5	5.5	4.145	1.596	0.602	0.961	0.691
	6.5	6.5	4.145	1.608	0.588	0.953	0.681
	7.5	7.5	4.154	1.716	0.574	0.945	0.645
1.5	0.5	0.5	4.13	1.538	0.694	1.013	0.831
	1.5	1.5	4.135	1.54	0.687	1.009	0.819
	2.5	2.5	4.136	1.549	0.68	1.005	0.818
	3.5	3.5	4.137	1.574	0.655	0.991	0.751
	4.5	4.5	4.138	1.574	0.608	0.964	0.706
	5.5	5.5	4.142	1.588	0.604	0.962	0.705
	6.5	6.5	4.146	1.614	0.588	0.953	0.701
	7.5	7.5	4.152	1.614	0.577	0.947	0.683
2.5	0.5	0.5	4.121	1.708	0.73	1.033	0.774
	1.5	1.5	4.128	1.749	0.72	1.028	0.747
	2.5	2.5	4.129	1.778	0.69	1.011	0.742
	3.5	3.5	4.13	1.781	0.654	0.99	0.73
	4.5	4.5	4.131	1.785	0.637	0.981	0.725
	5.5	5.5	4.132	1.922	0.62	0.971	0.721
	6.5	6.5	4.132	1.93	0.619	0.971	0.678
	7.5	7.5	4.133	2.08	0.614	0.968	0.653
3.5	0.5	0.5	4.121	1.505	0.695	1.016	0.754
	1.5	1.5	4.125	1.521	0.685	1.008	0.748
	2.5	2.5	4.13	1.548	0.657	0.992	0.747
	3.5	3.5	4.132	1.568	0.649	0.988	0.744
	4.5	4.5	4.137	1.632	0.583	0.95	0.739
	5.5	5.5	4.138	1.64	0.56	0.937	0.688
	6.5	6.5	4.141	1.68	0.556	0.935	0.655
	7.5	7.5	4.143	1.697	0.553	0.933	0.652

The quantile function in (2.1.1) and the r^{th} raw moments in (3.1.3) of the ALTHL distribution are used to obtain the values of mean, median, variance, skewness and kurtosis for some selected values of σ^2 and α for sample size n=60. These values are displayed in Table 4.

Table 4

Mean, Median, Variance, Skewness and Kurtosis For ALTHL distribution

σ	θ	α	Mean	Median	Variance	Skewness	Kurtosis
0.5	0.5	0.5	4.13	1.351	0.657	0.992	0.748
	1.5	1.5	4.134	1.353	0.651	0.989	0.735
	2.5	2.5	4.137	1.358	0.644	0.985	0.705
	3.5	3.5	4.138	1.362	0.644	0.985	0.69
	4.5	4.5	4.14	1.387	0.627	0.975	0.686
	5.5	5.5	4.143	1.404	0.594	0.956	0.677
	6.5	6.5	4.146	1.47	0.586	0.952	0.667
	7.5	7.5	4.146	1.5	0.554	0.934	0.642
1.5	0.5	0.5	4.119	1.38	0.654	0.991	0.814
	1.5	1.5	4.122	1.398	0.614	0.968	0.793
	2.5	2.5	4.123	1.461	0.583	0.95	0.79
	3.5	3.5	4.124	1.469	0.574	0.945	0.711
	4.5	4.5	4.128	1.481	0.53	0.92	0.708
	5.5	5.5	4.134	1.557	0.511	0.909	0.656
	6.5	6.5	4.136	1.632	0.478	0.891	0.649
	7.5	7.5	4.137	1.649	0.474	0.889	0.637
2.5	0.5	0.5	4.161	1.404	0.611	0.966	0.802
	1.5	1.5	4.167	1.422	0.595	0.957	0.787
	2.5	2.5	4.168	1.422	0.571	0.943	0.777
	3.5	3.5	4.173	1.458	0.527	0.918	0.75
	4.5	4.5	4.177	1.515	0.512	0.91	0.749
	5.5	5.5	4.178	1.619	0.505	0.906	0.747
	6.5	6.5	4.181	1.667	0.487	0.896	0.739
	7.5	7.5	4.181	1.672	0.458	0.88	0.708
3.5	0.5	0.5	4.157	1.415	0.716	1.026	0.806
	1.5	1.5	4.162	1.427	0.707	1.02	0.787
	2.5	2.5	4.165	1.477	0.704	1.019	0.762
	3.5	3.5	4.166	1.529	0.683	1.007	0.739
	4.5	4.5	4.17	1.584	0.679	1.004	0.721
	5.5	5.5	4.172	1.688	0.67	0.999	0.72
	6.5	6.5	4.176	1.745	0.663	0.996	0.716
	7.5	7.5	4.179	1.791	0.652	0.989	0.71

The quantile function in (2.1.1) and the r^{th} raw moments in (3.1.3) of the ALTHL distribution are used to obtain the values of mean, median, variance, skewness and kurtosis for some selected values of σ^2 and α for sample size n=80. These values are displayed in Table 5.

Table 5

Mean, Median, Variance, Skewness and Kurtosis For ALTHL distribution

σ	θ	α	Mean	Median	Variance	Skewness	Kurtosis
0.5	0.5	0.5	4.159	1.811	0.568	0.942	0.795

	1.5	1.5	4.162	1.816	0.563	0.939	0.782
	2.5	2.5	4.163	1.843	0.549	0.931	0.764
	3.5	3.5	4.166	1.85	0.53	0.92	0.749
	4.5	4.5	4.168	1.885	0.526	0.918	0.726
	5.5	5.5	4.17	1.896	0.51	0.909	0.718
	6.5	6.5	4.172	1.933	0.503	0.905	0.688
	7.5	7.5	4.176	1.964	0.488	0.897	0.681
1.5	0.5	0.5	4.134	1.67	0.567	0.941	0.809
	1.5	1.5	4.135	1.684	0.56	0.937	0.799
	2.5	2.5	4.139	1.751	0.463	0.883	0.781
	3.5	3.5	4.141	1.768	0.387	0.839	0.753
	4.5	4.5	4.143	1.797	0.361	0.825	0.705
	5.5	5.5	4.143	1.81	0.336	0.81	0.69
	6.5	6.5	4.149	1.847	0.335	0.81	0.687
	7.5	7.5	4.15	1.849	0.294	0.787	0.681
2.5	0.5	0.5	4.154	2.101	0.585	0.951	0.808
	1.5	1.5	4.155	2.114	0.557	0.935	0.765
	2.5	2.5	4.156	2.13	0.535	0.923	0.764
	3.5	3.5	4.156	2.141	0.531	0.921	0.761
	4.5	4.5	4.158	2.149	0.512	0.91	0.757
	5.5	5.5	4.16	2.155	0.5	0.903	0.738
	6.5	6.5	4.173	2.165	0.492	0.899	0.722
	7.5	7.5	4.179	2.186	0.429	0.863	0.721
3.5	0.5	0.5	4.146	1.759	0.549	0.931	0.801
	1.5	1.5	4.154	1.761	0.509	0.909	0.799
	2.5	2.5	4.161	1.795	0.503	0.905	0.796
	3.5	3.5	4.165	1.807	0.501	0.904	0.793
	4.5	4.5	4.169	1.808	0.449	0.874	0.777
	5.5	5.5	4.17	1.887	0.434	0.866	0.772
	6.5	6.5	4.171	1.907	0.394	0.843	0.745
	7.5	7.5	4.172	1.934	0.391	0.842	0.73

The quantile function in (2.1.1) and the r^{th} raw moments in (3.1.3) of the ALTHL distribution are used to obtain the values of mean, median, variance, skewness and kurtosis for some selected values of σ^2 and α for sample size n=100. These values are displayed in Table 6.

Table 6

Mean, Median, Variance, Skewness and Kurtosis For ALTHL distribution

σ	θ	α	Mean	Median	Variance	Skewness	Kurtosis
0.5	0.5	0.5	4.141	1.524	0.504	0.906	0.687
	1.5	1.5	4.145	1.627	0.493	0.899	0.676
	2.5	2.5	4.147	1.645	0.458	0.879	0.674
	3.5	3.5	4.153	1.72	0.45	0.875	0.67
	4.5	4.5	4.157	1.724	0.431	0.864	0.668
	5.5	5.5	4.168	1.74	0.414	0.855	0.666
	6.5	6.5	4.177	1.765	0.412	0.854	0.65
	7.5	7.5	4.179	1.785	0.402	0.848	0.64
1.5	0.5	0.5	4.181	1.661	0.462	0.882	0.709

	1.5	1.5	4.184	1.663	0.457	0.879	0.704
	2.5	2.5	4.194	1.767	0.439	0.869	0.704
	3.5	3.5	4.199	1.782	0.395	0.844	0.678
	4.5	4.5	4.203	1.884	0.393	0.843	0.66
	5.5	5.5	4.205	2.003	0.359	0.824	0.66
	6.5	6.5	4.219	2.02	0.353	0.82	0.659
	7.5	7.5	4.222	2.117	0.349	0.818	0.652
2.5	0.5	0.5	4.183	1.699	0.457	0.879	0.679
	1.5	1.5	4.186	1.794	0.451	0.876	0.677
	2.5	2.5	4.187	1.802	0.444	0.872	0.677
	3.5	3.5	4.19	1.81	0.434	0.866	0.675
	4.5	4.5	4.192	1.826	0.429	0.863	0.673
	5.5	5.5	4.196	1.826	0.412	0.853	0.654
	6.5	6.5	4.2	1.911	0.408	0.851	0.641
	7.5	7.5	4.204	1.919	0.404	0.849	0.64
3.5	0.5	0.5	4.182	1.906	0.603	0.962	0.677
	1.5	1.5	4.182	1.935	0.551	0.932	0.672
	2.5	2.5	4.188	1.958	0.537	0.924	0.66
	3.5	3.5	4.19	1.971	0.536	0.924	0.654
	4.5	4.5	4.195	2.086	0.529	0.92	0.653
	5.5	5.5	4.198	2.087	0.503	0.905	0.649
	6.5	6.5	4.202	2.132	0.5	0.903	0.646
	7.5	7.5	4.203	2.149	0.484	0.894	0.636

The quantile function in (2.1.1) and the r^{th} raw moments in (3.1.3) of the ALTHL distribution are used to obtain the values of mean, median, variance, skewness and kurtosis for some selected values of σ^2 and α for sample size n=250. These values are displayed in Table 7.

Table 7
Mean, Median, Variance, Skewness and Kurtosis For ALTHL distribution

σ	θ	α	Mean	Median	Variance	Skewness	Kurtosis
0.5	0.5	0.5	4.1509	2.0028	0.4862	0.9418	0.7024
	1.5	1.5	4.156	1.9519	0.4762	0.9386	0.6943
	2.5	2.5	4.1619	1.837	0.4762	0.9312	0.6928
	3.5	3.5	4.1668	1.8214	0.4547	0.9205	0.6624
	4.5	4.5	4.1847	1.7936	0.4088	0.9182	0.6613
	5.5	5.5	4.1875	1.7857	0.3393	0.909	0.6412
	6.5	6.5	4.1894	1.6588	0.3111	0.9049	0.6386
	7.5	7.5	4.1912	1.6368	0.2828	0.8965	0.634
1.5	0.5	0.5	4.1802	1.8306	0.4245	0.9414	0.7238
	1.5	1.5	4.182	1.7207	0.4069	0.9369	0.723
	2.5	2.5	4.187	1.7013	0.4035	0.8826	0.7187
	3.5	3.5	4.1882	1.4004	0.3831	0.8394	0.6943
	4.5	4.5	4.1886	1.3417	0.3768	0.8247	0.6774
	5.5	5.5	4.1919	1.2072	0.3725	0.8103	0.6336
	6.5	6.5	4.1943	1.2071	0.3514	0.8098	0.6331
	7.5	7.5	4.1947	1.1814	0.349	0.7866	0.6328
2.5	0.5	0.5	4.1729	1.3732	0.3871	0.9513	0.7863

	1.5	1.5	4.1751	1.3659	0.368	0.9354	0.7721
	2.5	2.5	4.1859	1.2003	0.3563	0.9232	0.7696
	3.5	3.5	4.1955	1.1467	0.3406	0.921	0.7309
	4.5	4.5	4.1965	0.8468	0.3157	0.9099	0.7233
	5.5	5.5	4.1965	0.8129	0.2719	0.9031	0.706
	6.5	6.5	4.1997	0.6499	0.267	0.8989	0.706
	7.5	7.5	4.2014	0.6343	0.2295	0.863	0.7027
3.5	0.5	0.5	4.1855	1.1244	0.3996	0.9312	0.6954
	1.5	1.5	4.186	0.9143	0.3767	0.9085	0.6683
	2.5	2.5	4.1941	0.8157	0.3498	0.9051	0.6556
	3.5	3.5	4.1997	0.7694	0.3344	0.9038	0.6497
	4.5	4.5	4.2067	0.7585	0.2789	0.8742	0.6483
	5.5	5.5	4.2096	0.712	0.2452	0.8659	0.6423
	6.5	6.5	4.2111	0.7118	0.236	0.8432	0.6423
	7.5	7.5	4.2114	0.635	0.225	0.8418	0.6324

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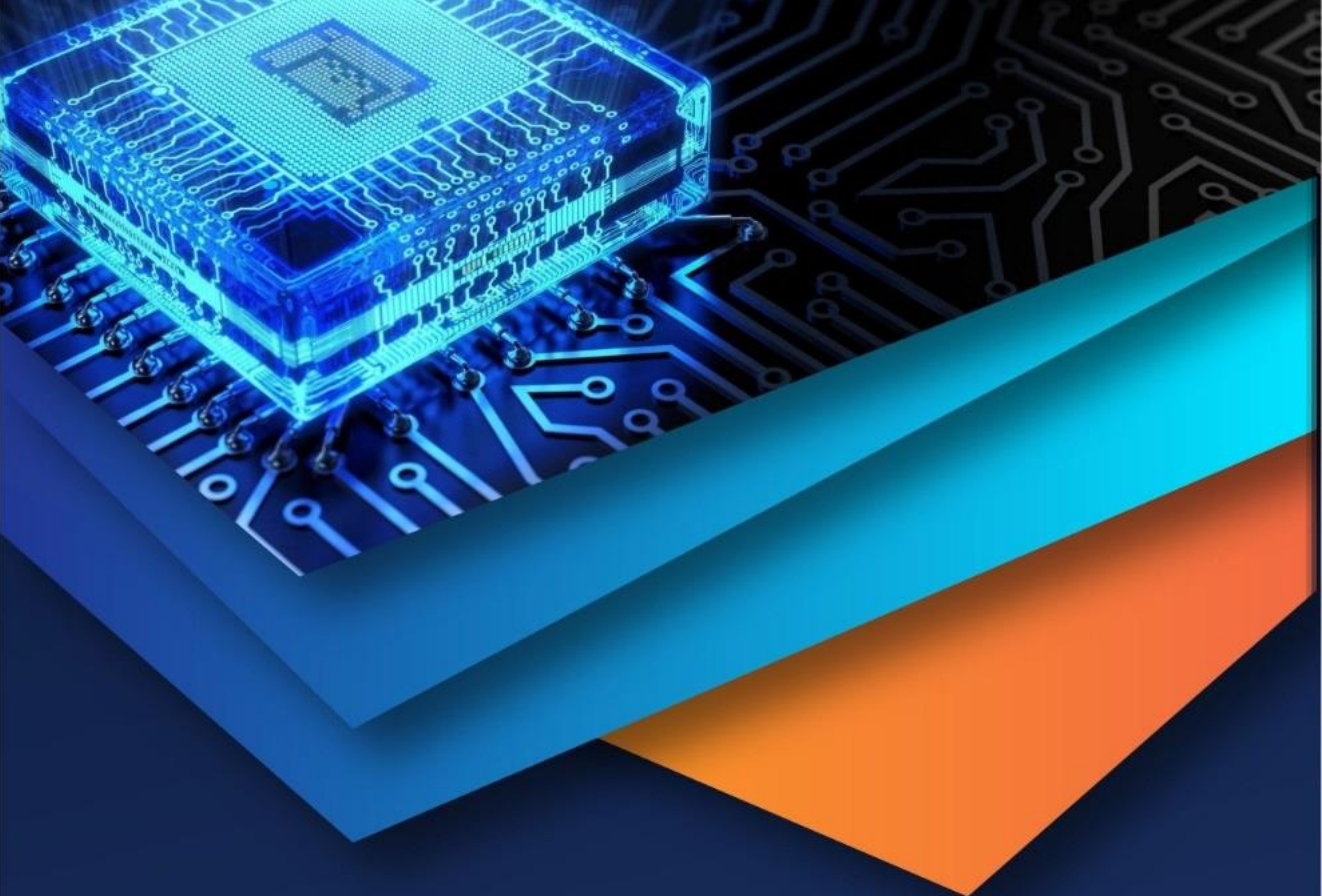

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