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An Economic Production Quantity Model Approach by Lowering the Impact of Carbon Emission under Uncertain Environment

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Abstract: The primary factor of rising global temperatures is the increase in carbon dioxide emissions, which have serious consequences on both ecological and social well-being worldwide. It contributes to environmental damage and poses various health hazards to society. This study aims to develop a fuzzy economic production quantity model with emission reduction strategies to promote economic profitability and ecological sustainability in inventory functions. Two models were developed: the fuzzy model and the crisp model. The parameters in this research work are taken as trapezoidal fuzzy numbers. The Yager ranking method and beta distribution methods are used for the solution procedure. A numerical example is used for a better understanding of the research work.
Keywords: Carbon emission, Sustainability, EPQ, Fuzzy, Green technology investment

I. INTRODUCTION

As global warming becomes a serious issue, several governments are increasingly pursuing carbon emission reduction throughout all phases of the inventory process. For many businesses, controlling inventory decisions while reducing waste is a crucial task. To address this, governments adopt various strategies, such as imposing carbon taxes at different production stages, setting emission limits, promoting recycling, and encouraging green investments. Recycling is widely regarded as an indispensable and globally acknowledged strategy for mitigating atmospheric pollution. It plays a vital role in protecting us from impending ecological catastrophes. Recycling consumes considerably less energy and also assists in protecting the planet's precious natural assets than producing stuff from scratch, which takes up plenty of energy. Some of the recent research investigations include Dwicahyani et.al (2017) proposed an inventory model with limitations in remanufacturing cycles and broader sustainability issues. In (2018), Daryanto et.al in their work presented a fundamental sustainable EPQ model that takes account of carbon emissions from waste disposal, warehousing, and production activities. Priyan et.al (2022) developed a manufacturing inventory model that incorporates emissions from transportation, production, and storage activities, which are meant to be decreased by green investment. In (2023), Shah et.al presented a production inventory model with manufacturing and inventory factors for deterioration with green technology investment. Singh et.al (2024) formulated an inventory model by incorporating machine learning for handling imperfect goods under green investment technology. Price shifts in many items on the marketplace result in an immense degree of ambiguity. The primary purpose of fuzzy set theory is the mathematical discussion of imprecision and unpredictability. Maity et.al (2008) offered a fuzzy environment optimum control recycling production inventory structure. Shekarian et.al (2016) established a reverse inventory model that includes imprecise demand and restoration rates for products that can be recovered, as well as the implications of the learning hypothesis on the recoverable manufacturing cycle.In this research study, Manufacturing inventory model with recycling and green investment is considered. Unpredictable variables in fuzzy approach are expressed by trapezoidal fuzzy numbers. Two methods were employed in fuzzy approach: Yager ranking and the beta distribution method.

II. DEFINITIONS

1) Definition 1 (Fuzzy set):

A fuzzy set $\tilde{B}$ in a universe of discourse X is defined as the following set of pairs $\tilde{B}=\{(x,\mu_{\tilde{B}}(x)):x\in X\}$. Here $\mu_{\tilde{B}}:X\rightarrow[0,1]$ is a mapping called the membership value of $x\in X$ in a fuzzy set $\tilde{B}$.

2) Definition 2 (Trapezoidal fuzzy number):
A trapezoidal fuzzy number $\tilde{B}_M = (m_1, m_2, m_3, m_4)$ is represented with membership function $\mu_{\tilde{B}_M}$ as

$$\mu_{\tilde{B}_M}(j) = \begin{cases} 0, j \le m_1; \\ \dfrac{j - m_1}{m_2 - m_1}, m_1 < x \le m_2; \\ 1, \ x = m_2 \\ \dfrac{m_3 - j}{m_4 - m_3}, m_3 < j \le m_4; \\ 0, otherwise \end{cases}$$

3) Definition 3 (Arithmetic operations): The arithmetic operations between Trapezoidal fuzzy numbers proposed are given below. Let us consider, $\tilde{U} = [u_1, u_2, u_3, u_4]$ and $\tilde{V} = [v_1, v_2, v_3, v_4]$ be two Trapezoidal fuzzy numbers.

$$\tilde{U} + \tilde{V} = (u_1 + v_1, u_2 + v_2, u_3 + v_3, u_4 + v_4)$$

$$\tilde{U} - \tilde{V} = (u_1 - v_4, u_2 - v_3, u_3 - v_2, u_4 - v_1)$$

$$\tilde{U} \times \tilde{V} = (u_1 v_1, u_2 v_2, u_3 v_3, u_4 v_4)$$

$$\frac{\tilde{U}}{\tilde{V}} = \left(\frac{u_1}{v_4}, \frac{u_2}{v_3}, \frac{u_3}{v_2}, \frac{u_4}{v_1} \right)$$

4) Definition 4 (Yager ranking): If the α cut of any fuzzy number $\tilde{A}$ is $[A_L(\alpha) + A_R(\alpha)]$. then its ranking index $I(\tilde{A})$ is said to be is $\frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)] d\alpha$.

5) Definition 5 (Alpha cut): The set of elements that belong to the fuzzy set $\tilde{A}$ atleast to the degree α is called the α level set or α cut. $A(\alpha) = x \in X : \mu_{\tilde{A}}(x) \ge \alpha$

6) Definition 6 (beta distribution): The trapezoidal fuzzy number $\tilde{B}_M = (m_1, m_2, m_3, m_4)$ are defuzzified by using the following beta distribution formula,

$$\eta_{\tilde{C}} = \frac{2m_1 + 7m_2 + 7m_3 + 2m_4}{18}$$

III. NOTATIONS

A. Crisp Parameters

S_{Ja} = Setup cost per cycle

S_{Jb} = Production cost per unit

S_{Jc} = Inventory cost per unit product in a time unit

S_{Jd} = Waste disposal fixed cost per cycle

S_{Jbk} = Average production emission cost per unit

S_{Jck} = Average inventory emission cost per unit

S_{Jdk} = Average waste disposal emission cost per unit

S_{Jek} = Average Recycling emission cost per unit

K_M = Production rate

T_M = Demand rate

R_q = Recycling rate

R_j = Recycling Cost

A_{epc} = Average production energy consumption per unit

A_{erc} = Average recycling energy consumption per unit

A_{cws} = Average energy consumption per warehouse space unit

u_s = Space occupied by a unit product

A_{sw} = Average weight of solid waste produced per unit product

E_{Gse} = Energy generated standard emission

E_{Dsw} = Disposal standard emission per ton of solid waste

X_{tax} = Carbon price or tax

S_c = Screening cost

G_{ct} = Green technology investment cost

Z_i = Proportion of the return of the demand

J_i = Social costs of vehicle emissions

W_p = Speed Average

x_i = Fixed cost for each trip

y_i = Variable cost

f = Mileage

J_{XT} = optimum order size

$TC(J_{XT})$= Total inventory cost

IV. FUZZY PARAMETERS

$\tilde{S}_{Ja}$ = Fuzzy Setup cost per cycle

$\tilde{S}_{Jb}$ = Fuzzy Production cost per unit

$\tilde{S}_{Jc}$ = Fuzzy Inventory cost per unit product in a time unit

$\tilde{S}_{Jd}$ = Fuzzy Waste disposal fixed cost per cycle

S_{Jbk} = Fuzzy average production emission cost per unit

$\tilde{S}_{Jck}$ = Fuzzy Average inventory emission cost per unit

$\tilde{S}_{Jdk}$ = Average waste disposal emission cost per unit

$\tilde{S}_{Jek}$ = Fuzzy Average Recycling emission cost per unit

$\tilde{R}_q$ = Fuzzy Recycling rate

$\tilde{R}_j$ = Fuzzy Recycling Cost

$\tilde{S}_c$ = Fuzzy screening cost

$\tilde{G}_{ct}$ = Fuzzy green technology investment cost

$\tilde{x}_i$ = Fuzzy cost for each trip

$\tilde{J}_i$ = Fuzzy social costs of vehicle emissions

Assumptions
1. Demand is taken as known and constant.
2. uncertain parameters are taken as trapezoidal fuzzy numbers.

V. MATHEMATICAL MODEL

A. Mathematical Formulation in Crisp Sense

In this model, carbon emissions occur in the stages of production, shipping, storage, disposal of waste, and recycling. Carbon taxation is imposed on each unit of carbon released. Items that can be recycled are taken to the recycling facility, while those that cannot be recycled are go to the disposal process. To minimize the damaging effect of carbon emissions, investment in green technology is considered.

B. Crisp Sense

Total inventory cost can be calculated by adding the setup cost, production cost, inventory holding cost, waste disposal cost, recycling cost, transportation cost, screening cost and green technology cost

Setup expenses are given as, $\left[\dfrac{S_{Ja}T_M}{J_{XT}}\right]$

Production expenses are given as, $\left[\left(S_{Jb}+S_{Jbk}\right)T_M\right]$, where $S_{Jbk}=\left(A_{epc}\times E_{Gse}\times X_{tax}\right)$

Inventory holding expenses are given as, $\left[\dfrac{\left(S_{Jc}+S_{Jck}\right)\left(K_M - T_M\right)J_{XT}}{2K_M}\right]$, where $S_{Jck}=\left(u_s\times A_{cws}\times E_{Gse}\times X_{tax}\right)$

Expenses for waste disposal are given by, $\left[\dfrac{S_{Jd}T_M}{J_{XT}}+S_{Jdk}\,T_M\right]$, where $S_{Jdk}=\left(A_{sw}\times E_{Dsw}\times X_{tax}\right)$

Expenses for recycling are given as, $\left[\dfrac{\left(R_q\times R_j\right)T_M}{J_{XT}}+\dfrac{S_{Jek}T_M}{J_{XT}}\right]$, where $S_{Jek}=\left(A_{erc}\times E_{Gse}\times X_{tax}\right)$

Transportation cost is given as, $\left[\dfrac{2x_iT_M}{J_{XT}}+\text{y}_i\,fT_M+1+\text{z}_i+\dfrac{2J_i\,fT_M}{J_{XT}}\right]$

Screening cost is expressed as, $\left[\dfrac{s_cT_M}{J_{XT}}\right]$

Green technology cost is given as, $\left[\dfrac{G_{ct}T_M}{J_{XT}}\right]$

Total inventory cost

$$TC(J_{XT}) = \left[\begin{array}{l} \frac{S_{Ja} T_M}{J_{XT}} + S_{Jb} T_M + S_{Jbk} T_M + \frac{S_{Jc}(K_M - T_M)}{2K_M} + \frac{S_{Jck}(K_M - T_M) J_{XT}}{2K_M} + \frac{S_{Jd} T_M}{J_{XT}} \\ + S_{Jdk} T_M + \frac{(R_q \times R_j) T_M}{J_{XT}} + \frac{S_{Jek} T_M}{J_{XT}} + \frac{2x_i T_M}{J_{XT}} + y_i f T_M + 1 + z_i + \frac{2J_i f T_M}{J_{XT}} + \frac{s_c T_M}{J_{XT}} + \frac{G_{ct} T_M}{J_{XT}} \end{array} \right] \quad \ldots\ldots (1)$$

By differentiating the equation (1) with respect to J_{XT} and equating to 0. We get optimal order quantity,

$$J_{XT}^{*} = \sqrt{\frac{2T_M \left[S_{Ja} + S_{Jd} + (R_q \times R_j) + S_{Jek} + 2x_i + S_c + G_{ct} + \left(\frac{2fJ_i}{w_p} \right) \right]}{[S_{Jc} + S_{Jck}] \left(1 - \frac{T_M}{K_M} \right)}} \quad \ldots\ldots (2)$$

C. Fuzzy sense

$$T\tilde{C}(J_{XT}) = \left[\begin{array}{l} \frac{\tilde{S}_{Ja} T_M}{J_{XT}} + \tilde{S}_{Jb} T_M + \tilde{S}_{Jbk} T_M + \frac{\tilde{S}_{Jc}(K_M - T_M)}{2K_M} + \frac{\tilde{S}_{Jck}(K_M - T_M) J_{XT}}{2K_M} + \frac{\tilde{S}_{Jd} T_M}{J_{XT}} \\ + \tilde{S}_{Jdk} T_M + \frac{(\tilde{R}_q \times \tilde{R}_j) T_M}{J_{XT}} + \frac{\tilde{S}_{Jek} T_M}{J_{XT}} + \frac{2\tilde{x}_i T_M}{J_{XT}} + y_i f T_M + 1 + z_i + \frac{2\tilde{J}_i f T_M}{J_{XT}} + \frac{\tilde{s}_c T_M}{J_{XT}} + \frac{\tilde{G}_{ct} T_M}{J_{XT}} \end{array} \right] \quad \ldots\ldots\ldots\ldots.. (3)$$

Parameters are taken as trapezoidal fuzzy numbers.

$\tilde{S}_{Ja} = (S_{Ja_1}, S_{Ja_2}, S_{Ja_3}, S_{Ja_4})$, $\tilde{S}_{Jb} = (S_{Jb_1}, S_{Jb_2}, S_{Jb_3}, S_{Jb_4})$, $\tilde{S}_{Jc} = (S_{Jc_1}, S_{Jc_2}, S_{Jc_3}, S_{Jc_4})$

$\tilde{S}_{Jd} = (S_{Jd_1}, S_{Jd_2}, S_{Jd_3}, S_{Jd_4})$, $\tilde{S}_{Jbk} = (S_{Jbk_1}, S_{Jbk_2}, S_{Jbk_3}, S_{Jbk_4})$, $\tilde{S}_{Jck} = (S_{Jck_1}, S_{Jck_2}, S_{Jck_3}, S_{Jck_4})$,

$\tilde{S}_{Jdk} = (S_{Jdk_1}, S_{Jdk_2}, S_{Jdk_3}, S_{Jdk_4})$, $\tilde{S}_{Jek} = (S_{Jek_1}, S_{Jek_2}, S_{Jek_3}, S_{Jek_4})$, $\tilde{R}_q = (R_{q_1}, R_{q_2}, R_{q_3}, R_{q_4})$,

$\tilde{R}_j = (R_{j_1}, R_{j_2}, R_{j_3}, R_{j_4})$

$\tilde{x}_i = (x_{i_1}, x_{i_2}, x_{i_3}, x_{i_4})$, $\tilde{J}_i = (J_{i_1}, J_{i_2}, J_{i_3}, J_{i_4})$, $\tilde{S}_c = (s_{c_1}, s_{c_2}, s_{c_3}, s_{c_4})$, $\tilde{G}_{ct} = (G_{ct_1}, G_{ct_2}, G_{ct_3}, G_{ct_4})$

D. Yager ranking Approach

Let $S_{Ja}, S_{Jb}, S_{Jc}, S_{Jd}, S_{Jbk}, S_{Jck}, S_{Jdk}, R_q, R_j, x_i, J_i, S_{Je_k}, S_c, G_{ct}$ be

$$S_{Ja}\left(\alpha_{S_{Ja}}\right)=\left[L^{-1}{}_{S_{Ja}}\left(\alpha_{S_{Ja}}\right), R^{-1}{}_{S_{Ja}}\left(\alpha_{S_{Ja}}\right)\right]$$

$$S_{Jb}\left(\alpha_{S_{Jb}}\right)=\left[L^{-1}{}_{S_{Jb}}\left(\alpha_{S_{Jb}}\right), R^{-1}{}_{S_{Jb}}\left(\alpha_{S_{Jb}}\right)\right]$$

$$S_{Jc}\left(\alpha_{S_{Jc}}\right)=\left[L^{-1}{}_{S_{Jc}}\left(\alpha_{S_{Jc}}\right), R^{-1}{}_{S_{Jc}}\left(\alpha_{S_{Jc}}\right)\right]$$

$$S_{Jd}\left(\alpha_{S_{Jd}}\right)=\left[L^{-1}{}_{S_{Jd}}\left(\alpha_{S_{Jd}}\right), R^{-1}{}_{S_{Jd}}\left(\alpha_{S_{Jd}}\right)\right]$$

$$S_{Jbk}\left(\alpha_{S_{Jbk}}\right)=\left[L^{-1}{}_{S_{Jbk}}\left(\alpha_{S_{Jbk}}\right), R^{-1}{}_{S_{Jbk}}\left(\alpha_{S_{Jbk}}\right)\right]$$

$$S_{Jck}\left(\alpha_{S_{Jck}}\right)=\left[L^{-1}{}_{S_{Jck}}\left(\alpha_{S_{Jck}}\right), R^{-1}{}_{S_{Jck}}\left(\alpha_{S_{Jck}}\right)\right]$$

$$S_{Jdk}\left(\alpha_{S_{Jdk}}\right)=\left[L^{-1}{}_{S_{Jdk}}\left(\alpha_{S_{Jdk}}\right), R^{-1}{}_{S_{Jdk}}\left(\alpha_{S_{Jdk}}\right)\right]$$

$$S_{Jek}\left(\alpha_{S_{Jek}}\right)=\left[L^{-1}{}_{S_{Jek}}\left(\alpha_{S_{Jek}}\right), R^{-1}{}_{S_{Jek}}\left(\alpha_{S_{Jek}}\right)\right]$$

$$R_q\left(\alpha_{R_q}\right)=\left[L^{-1}{}_{R_q}\left(\alpha_{R_q}\right), R^{-1}{}_{R_q}\left(\alpha_{R_q}\right)\right]$$

$$R_j\left(\alpha_{R_j}\right)=\left[L^{-1}{}_{R_j}\left(\alpha_{R_j}\right), R^{-1}{}_{R_j}\left(\alpha_{R_j}\right)\right]$$

$$x_i\left(\alpha_{x_i}\right)=\left[L^{-1}{}_{x_i}\left(\alpha_{x_i}\right), R^{-1}{}_{x_i}\left(\alpha_{x_i}\right)\right]$$

$$J_i\left(\alpha_{J_i}\right)=\left[L^{-1}{}_{J_i}\left(\alpha_{J_i}\right), R^{-1}{}_{J_i}\left(\alpha_{J_i}\right)\right]$$

$$S_c\left(\alpha_{S_c}\right)=\left[L^{-1}{}_{S_c}\left(\alpha_{S_c}\right), R^{-1}{}_{S_c}\left(\alpha_{S_c}\right)\right]$$

$$G_{ct}\left(\alpha_{G_{ct}}\right)=\left[L^{-1}{}_{G_{ct}}\left(\alpha_{G_{ct}}\right), R^{-1}{}_{G_{ct}}\left(\alpha_{G_{ct}}\right)\right]$$

$$K_1\left(\alpha_{S_{Ja}}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{S_{Ja}}\left(\alpha_{S_{Ja}}\right)d\alpha_{S_{Ja}}+\int_0^1 R^{-1}{}_{S_{Ja}}\left(\alpha_{S_{Ja}}\right)d\alpha_{S_{Ja}}\right]$$

$$K_2\left(\alpha_{S_{Jb}}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{S_{Jb}}\left(\alpha_{S_{Jb}}\right)d\alpha_{S_{Jb}}+\int_0^1 R^{-1}{}_{S_{Jb}}\left(\alpha_{S_{Jb}}\right)d\alpha_{S_{Jb}}\right]$$

$$K_3\left(\alpha_{S_{Jc}}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{S_{Jc}}\left(\alpha_{S_{Jc}}\right)d\alpha_{S_{Jc}}+\int_0^1 R^{-1}{}_{S_{Jc}}\left(\alpha_{S_{Jc}}\right)d\alpha_{S_{Jc}}\right]$$

$$K_4\left(\alpha_{S_{Jd}}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{S_{Jd}}\left(\alpha_{S_{Jd}}\right)d\alpha_{S_{Jd}}+\int_0^1 R^{-1}{}_{S_{Jd}}\left(\alpha_{S_{Jd}}\right)d\alpha_{S_{Jd}}\right]$$

$$K_5\left(\alpha_{R_q},\alpha_{R_j}\right)=\frac{1}{4}\begin{bmatrix}\int_0^1 L^{-1}{}_{R_q}\left(\alpha_{R_q}\right)\mathrm{d}\,\alpha_{R_q}.\int_0^1 L^{-1}{}_{\alpha_{R_j}}\left(\alpha_{\alpha_{R_j}}\right)\mathrm{d}\,\alpha_{R_j}+\\ \int_0^1 R^{-1}{}_{R_q}\left(\alpha_{R_q}\right)\mathrm{d}\,\alpha_{R_q}.\int_0^1 R^{-1}{}_{\alpha_{R_j}}\left(\alpha_{\alpha_{R_j}}\right)\mathrm{d}\,\alpha_{R_j}\end{bmatrix}$$

$$K_6\left(\alpha_{Sjbk}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{Sjbk}\left(\alpha_{Sjbk}\right)\mathrm{d}\,\alpha_{Sjbk}+\int_0^1 \mathrm{R}^{-1}{}_{Sjbk}\left(\alpha_{Sjbk}\right)\mathrm{d}\,\alpha_{Sjbk}\right]$$

$$K_7\left(\alpha_{Sjck}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{Sjck}\left(\alpha_{Sjck}\right)\mathrm{d}\,\alpha_{Sjck}+\int_0^1 \mathrm{R}^{-1}{}_{Sjck}\left(\alpha_{Sjck}\right)\mathrm{d}\,\alpha_{Sjck}\right]$$

$$K_8\left(\alpha_{Sjdk}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{Sjdk}\left(\alpha_{Sjdk}\right)\mathrm{d}\,\alpha_{Sjdk}+\int_0^1 \mathrm{R}^{-1}{}_{Sjdk}\left(\alpha_{Sjdk}\right)\mathrm{d}\,\alpha_{Sjdk}\right]$$

$$K_9\left(\alpha_{Sjek}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{Sjek}\left(\alpha_{Sjek}\right)\mathrm{d}\,\alpha_{Sjek}+\int_0^1 \mathrm{R}^{-1}{}_{Sjek}\left(\alpha_{Sjek}\right)\mathrm{d}\,\alpha_{Sjek}\right]$$

$$K_{10}\left(\alpha_{x_i}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{x_i}\left(\alpha_{x_i}\right)\mathrm{d}\,\alpha_{x_i}+\int_0^1 \mathrm{R}^{-1}{}_{x_i}\left(\alpha_{x_i}\right)\mathrm{d}\,\alpha_{x_i}\right]$$

$$K_{11}\left(\alpha_{J_i}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{J_i}\left(\alpha_{J_i}\right)\mathrm{d}\,\alpha_{J_i}+\int_0^1 \mathrm{R}^{-1}{}_{J_i}\left(\alpha_{J_i}\right)\mathrm{d}\,\alpha_{J_i}\right]$$

$$K_{12}\left(\alpha_{S_c}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{S_c}\left(\alpha_{S_c}\right)\mathrm{d}\,\alpha_{S_c}+\int_0^1 \mathrm{R}^{-1}{}_{S_c}\left(\alpha_{S_c}\right)\mathrm{d}\,\alpha_{S_c}\right]$$

$$K_{13}\left(\alpha_{G_{ct}}\right)=\frac{1}{2}\left[\int_0^1 L^{-1}{}_{G_{ct}}\left(\alpha_{G_{ct}}\right)\mathrm{d}\,\alpha_{G_{ct}}+\int_0^1 \mathrm{R}^{-1}{}_{G_{ct}}\left(\alpha_{G_{ct}}\right)\mathrm{d}\,\alpha_{G_{ct}}\right]$$

$$T\tilde{C}(J_{XT})=\begin{bmatrix}\frac{K_1\left(\alpha_{S_{Ja}}\right)T_M}{J_{XT}}+K_2\left(\alpha_{S_{Jb}}\right)T_M+K_6\left(\alpha_{S_{Jbk}}\right)T_M+\frac{K_3\left(\alpha_{S_{Jc}}\right)\left(K_M-T_M\right)}{2K_M}\\ +\frac{K_7\left(\alpha_{S_{jck}}\right)\left(K_M-T_M\right)J_{XT}}{2K_M}+\frac{K_4\left(\alpha_{S_{Jd}}\right)K_M}{J_{XT}}\\ +K_8\left(\alpha_{S_{Jdk}}\right)T_M+\frac{K_5\left(\alpha_{R_q},\alpha_{R_j}\right)T_M}{J_{XT}}+\frac{K_9\left(\alpha_{S_{Jek}}\right)T_M}{J_{XT}}+2\frac{K_{10}\left(\alpha_{x_i}\right)T_M}{J_{XT}}\\ +\mathrm{y}_i fT_M+1+\mathrm{z}_i+\frac{2K_{11}\left(\alpha_{J_i}\right)fT_M}{J_{XT}}+\frac{K_{12}\left(\alpha_{s_c}\right)T_M}{J_{XT}}+\frac{K_{13}\left(\alpha_{G_{ct}}\right)T_M}{J_{XT}}\end{bmatrix} \quad \ldots.(4)$$

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Differentiating equation (4) with respect to J_{XT} and equating to 0, we get fuzzy optimal order quantity in yager ranking method,

$$\tilde{J}_{XT}^{*} = \sqrt{\frac{2T_M\left[\begin{aligned}&K_1\left(\alpha_{S_{Ja}}\right)+K_4\left(\alpha_{S_{J_d}}\right)+K_5\left(\alpha_{R_q},\alpha_{R_j}\right)+K_9\left(\alpha_{S_{Jek}}\right)+\\&2K_{10}\left(\alpha_{x_i}\right)+K_{12}\left(\alpha_{s_c}\right)+K_{13}\left(\alpha_{G_{ct}}\right)+\frac{2K_{11}\left(\alpha_{J_i}\right)fT_M}{w_p}\end{aligned}\right]}{\left[K_3\left(\alpha_{S_{Jc}}\right)+K_7\left(\alpha_{S_{jck}}\right)\right]\left(1-\frac{T_M}{K_M}\right)}} \quad \ldots\ldots\ldots (5)$$

E. Beta Distribution Method

By using the beta distribution method formula, $\eta_{\tilde{C}} = \frac{2m_1 + 7m_2 + 7m_3 + 2m_4}{18}$, we get fuzzy total inventory cost

$$T\tilde{C}(J_{XT}) = \left[\begin{aligned}
&2\left(\begin{aligned}&\frac{S_{Ja_1}T_M}{J_{XT}}+\left(S_{Jb_1}+S_{Jbk_1}\right)T_M+\frac{\left(S_{Jc_1}+S_{Jck_1}\right)\left(K_M-T_M\right)}{2K_M}+\frac{S_{Jd_1}K_M}{J_{XT}}+S_{Jdk_1}T_M\\&+\frac{\left(R_{q_1}\times R_{j_1}\right)T_M}{J_{XT}}+\frac{S_{Jek_1}T_M}{J_{XT}}+\frac{2x_{i_1}T_M}{J_{XT}}+y_i fT_M+1+z_i+\frac{2J_{i_1}fT_M}{J_{XT}}+\frac{S_{c_1}T_M}{J_{XT}}+\frac{G_{ct_1}T_M}{J_{XT}}\end{aligned}\right)\\
&+7\left(\begin{aligned}&\frac{S_{Ja_2}T_M}{J_{XT}}+\left(S_{Jb_2}+S_{Jbk_2}\right)T_M+\frac{\left(S_{Jc_2}+S_{Jck_2}\right)\left(K_M-T_M\right)}{2K_M}+\frac{S_{Jd_2}K_M}{J_{XT}}+S_{Jdk_2}T_M\\&+\frac{\left(R_{q_2}\times R_{j_2}\right)T_M}{J_{XT}}+\frac{S_{Jek_2}T_M}{J_{XT}}+\frac{2x_{i_2}T_M}{J_{XT}}+y_i fT_M+1+z_i+\frac{2J_{i_2}fT_M}{J_{XT}}+\frac{S_{c_2}T_M}{J_{XT}}+\frac{G_{ct_2}T_M}{J_{XT}}\end{aligned}\right)\\
&+7\left(\begin{aligned}&\frac{S_{Ja_3}T_M}{J_{XT}}+\left(S_{Jb_3}+S_{Jbk_3}\right)T_M+\frac{\left(S_{Jc_3}+S_{Jck_3}\right)\left(K_M-T_M\right)}{2K_M}+\frac{S_{Jd_3}K_M}{J_{XT}}+S_{Jdk_3}T_M\\&+\frac{\left(R_{q_3}\times R_{j_3}\right)T_M}{J_{XT}}+\frac{S_{Jek_3}T_M}{J_{XT}}+\frac{2x_{i_3}T_M}{J_{XT}}+y_i fT_M+1+z_i+\frac{2J_{i_3}fT_M}{J_{XT}}+\frac{S_{c_3}T_M}{J_{XT}}+\frac{G_{ct_3}T_M}{J_{XT}}\end{aligned}\right)\\
&+2\left(\begin{aligned}&\frac{S_{Ja_4}T_M}{J_{XT}}+\left(S_{Jb_4}+S_{Jbk_4}\right)T_M+\frac{\left(S_{Jc_4}+S_{Jck_4}\right)\left(K_M-T_M\right)}{2K_M}+\frac{S_{Jd_4}K_M}{J_{XT}}+S_{Jdk_4}T_M\\&+\frac{\left(R_{q_4}\times R_{j_4}\right)T_M}{J_{XT}}+\frac{S_{Jek_4}T_M}{J_{XT}}+\frac{2x_{i_4}T_M}{J_{XT}}+y_i fT_M+1+z_i+\frac{2J_{i_4}fT_M}{J_{XT}}+\frac{S_{c_4}T_M}{J_{XT}}+\frac{G_{ct_4}T_M}{J_{XT}}\end{aligned}\right)
\end{aligned}\right] \quad \ldots. (6)$$

Differentiating equation (6) with respect to J_{XT} and equating to 0. We get fuzzy optimal order quantity using beta distribution formula.

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$$\tilde{J}_{XT}{}^{*}=\sqrt{\frac{2T_M\left[\begin{array}{l}\left(2S_{Ja_1}+7S_{Ja_2}+7S_{Ja_3}+2S_{Ja_4}\right)+\left(2S_{Jd_1}+7S_{Jd_2}+7S_{Jd_3}+7S_{Jd_4}\right)+\\ \left(2\left(R_{q_1}\times R_{j_1}\right)+7\left(R_{q_2}\times R_{j_2}\right)+7\left(R_{q_3}\times R_{j_3}\right)+2\left(R_{q_4}\times R_{j_4}\right)\right)+\\ \left(\left(2S_{Jek_1}+7S_{Jek_2}+7S_{Jek_3}+2S_{Jek_4}\right)\right)+2\left(2x_{i_1}+7x_{i_2}+7x_{i_3}+2x_{i_4}\right)\\ \left(2S_{c_1}+7S_{c_2}+7S_{c_3}+2S_{c_4}\right)+\left(2G_{ct_1}+7G_{ct_2}+7G_{ct_3}+2G_{ct_4}\right)+\\ \left(\frac{2f}{W_p}\left(2J_{i_1}+7J_{i_2}+7J_{i_3}+2J_{i_4}\right)\right)\end{array}\right]}{\left[\left(2S_{Jc_1}+7S_{Jc_2}+7S_{Jc_3}+2S_{Jc_4}\right)+\left(2S_{Jck_1}+7S_{Jck_2}+7S_{Jck_3}+2S_{Jck_4}\right)\right]\left(1-\frac{T_M}{K_M}\right)}}\quad \ldots\ldots (7)$$

VI. NUMERICAL EXAMPLE

A. Crisp Data Values

$T_M=140$, $K_M=200$, $S_{Ja}=25$, $S_{Jb}=9$, $S_{Jc}=4.5$, $S_{Jd}=8$, $X_{tax}=130$, $R_q=50$, $R_j=120$, $A_{epc}=50$, $A_{erc}=42.30$, $A_{cws}=5$, $u_s=1.7$, $E_{Dsw}=0.3$ $A_{sw}=10$, $E_{Gse}=0.5$, $S_c=0.25$, $G_{ct}=300$, $Z_i=0.75$, $J_i=120$, $W_p=80$, $x_i=60$, $y_i=35$, $f=170$

$S_{Jbk}=\left(50\times\left(\frac{0.5}{1000}\right)\times 130\right)$=3.25, $S_{Jck}=\left(1.7\times 5\times\left(\frac{0.5}{1000}\right)\times 130\right)$=0.5525,

$S_{Jdk}=\left(\left(\frac{10}{1000}\right)\times 0.3\times 130\right)=0.39$, $S_{Jek}=\left(42.30\times\left(\frac{0.5}{1000}\right)\times 130\right)=2.75$

B. Fuzzy data values

$\tilde{S}_{Ja}=(23,24,26,27)$, $\tilde{S}_{Jb}=(7,8,10,11)$, $\tilde{S}_{Jc}=(4.3,4.4,4.6,4.7)$, $\tilde{S}_{Jd}=(6,7,9,10)$, $\tilde{x}_i=(58,59,61,62)$
$\tilde{R}_q=(118,119,121,122)$, $\tilde{R}_j=(48,49,51,52)$, $\tilde{S}_c=(0.23,0.24,0.26,0.27)$, $\tilde{G}_{ct}=(298,299,301,302)$,
$\tilde{J}_i=(118,119,121,122)$, $\tilde{S}_{Jbk}=(3.248,3.249,3.251,3.252)$, $\tilde{S}_{Jdk}=(0.37,0.38,0.41,0.42)$, $\tilde{S}_{Jek}=(2.73,2.74,2.76,2.77)$,
$\tilde{S}_{Jck}=(0.5523,0.5524,0.5526,0.5527)$

Solution in crisp model
Using the above data in the equation (2) we obtain the optimal order quantity as $v=80.21$. Using the equation (1) we obtain the total inventory cost as $TC=Rs.\ 846990.69$

Solution in fuzzy model
Yager ranking solution
Using the above data in the equation (5) we obtain the optimal order quantity as $v^{*}=60.53$. Using the equation (4) we obtain the total inventory cost as $TC^{*}=Rs.\ 843992.778$

Beta distribution solution

Using the above data in the equation (7) we obtain the optimal order quantity as $v^* = 80.22$. Using the equation (6) we obtain the total inventory cost as $TC^* = Rs.\ 846992.768$

VII. CONCLUSION

Recycling is indispensable for the advancement of environmental preservation in a number of ways. On an economic scale, it lowers the systematic consumption of essential resources, ultimately resulting in cost savings. It has an advantageous ecological influence through reducing CO2 emissions and solid waste generation. This paper developed economic manufacturing model with ecological concerns of lowering emissions by adopting recycling and green technology investment in inventory functions. The findings of the research suggest that the Yager ranking method provides a lower solution results than beta distribution method.

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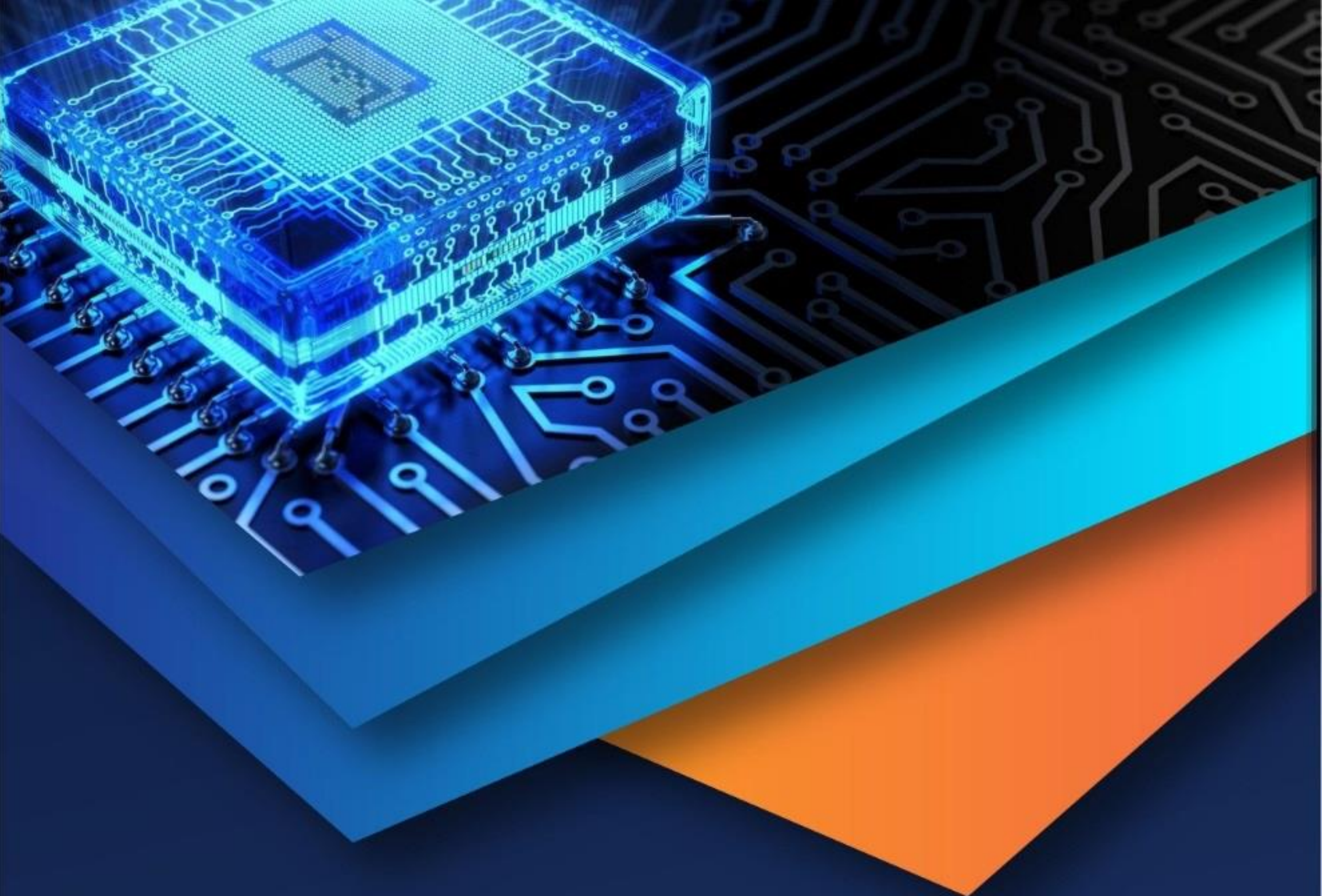

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