iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 **Issue:** VIII **Month of publication:** Aug 2023

DOI: https://doi.org/10.22214/ijraset.2023.55479

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue VIII Aug 2023- Available at www.ijraset.com

An Empirical Study of Stock Price Returns Using Black-Scholes Option Pricing Model

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Abstract: This study examined empirically the predictive ability of Black-Scholes Option pricing model in Nigerian Stock Market by testing whether there is any significant difference between the market (underlying) price of the stock and the theoretical prices. The data used for this study was Coca-Cola Stock prices from 2018 to 2022 obtained from http://www.investing.com and were analyzed using Statistical packages such as Microsoft Excel and Minitab to obtain the result. The results showed that there is a significant difference between the underlying and the theoretical price (Black-Scholes Option Pricing Model). Based on the findings of this study, we conclude that the Black- Scholes model is not accurate in its price predictive ability on the Coca-Cola Stock prices over the years under study.
***Keywords*: Black-Scholes, Option Pricing, Stock Price, Paired Sample t-Test**

I. INTRODUCTION

Stock is the proportion of a company's equity that is traded. Put differently, stock is one the instruments of a financial Market, otherwise, known as trading economy. A trading economy that concentrates on stock, only (a single portfolio) can be referred to as stock market. The price of stock is controlled by demand and supply vis-a-vis buyers and sellers, respectively. Particularly, the stock market prices fluctuate, due to some market forces. However, before investing into trading options, investors should have a good understanding of factors that determine the value of an option. These factors include the current stock price, the intrinsic value, time to expiration, volatility, cash dividends paid and macro-economic factors.

Speaking of the financial market, which is a trading environment that is composed of anything of value, that can be sold, bought or even exchanged, such as; stock bond, money, gold, structures, and many more. However, in this research our interest is in the first. In order to institutionalize and professionalize the trading in stock, the need to talk about institutes like the Nigerian stock exchange becomes necessary. The Nigerian stock exchange is a financial institution that trades exclusively on stock. Therein, we have the stock brokers, investors, a network of computer connected to a server, and many more. The Nigerian Stock Exchange started trading operation since 27th April, 1999, operates Automated Trading System (A. T. S). Unarguably, the stock exchange market is one of the important sectors of the Nigerian economy amongst which there is the Coca-Cola, a branch of the Nigerian Bottling Company which was introduced into the Nigerian market in 1951 and has since become a premium brand. With Nigeria as a developing country and the unstable nature of the economy, it is therefore important to test the market with a well-known model that is universally accepted for determining the price of derivative contracts.

The Black-Scholes Model for option pricing was developed in 1969 by Fisher Black and Myron Scholes, but was published in 1973 with the appearance in Chicago Board as the first regulated market of negotiable options. This model was subsequently developed by Merton (1976). It is noteworthy that for this scientific contribution, Myron Scholes and Robert C. Merton received the Nobel Prize for Economics in 1997 (the Swedish Academy of Sciences highlights the contribution of Fisher Black who was no longer alive at the time of the award).

The theory of Option pricing estimates a value of an option's contract by assigning a price, known as a premium, based on the calculated probability that the contract will finish in the money (ITM) at expiration. Essentially, option pricing theory provides an evaluation of an option's fair value, which traders incorporate into their strategies.

While this model is useful, it is based on the following market assumptions that may hinder its accuracy, such as:

1) The short-rate interest rate and volatility are known to be constant through time.
2) No transaction costs or services associated with buying or selling of the option.
3) The options are European-style options which can only be exercised on the expiration date.
4) The returns on the underlying stock prices are normally distributed.
5) The Black-Scholes model assumes that markets are perfectly liquid and it is possible to purchase or sell any amount of options or their fractions at any given time.

Black and Scholes (1973) introduced a theoretical method to determine the options values, and they stated that the model follows a fixed systematic pattern based on relevant market indicators such as volatility, spot prices, time to expiration and expected risk-free rate of return. The first article that empirically examined the Black-Scholes model was written by MacBeth and Merville (1979). More recently, other empirical studies on the applications of Black-Scholes formula for option pricing were found in many well-structured articles by Karoui, et al (1998); Kou (2002), Haug and Taleb (2011) and, hence, a review of recent developments in the Black-Scholes models were synthesized in Saedi and Tularam (2018) and several other researchers like Frino and Khan (1991), Bakshi, et al (1997), Kim, et al (1997), Genkay and Salih (2003) found out that the BSOPM model was not the appropriate pricing tool in high volatility than in a low volatility.

Angeli and Bonz (2010) tested the applicability and relevance of the Black–Scholes model for price stock index options and they determined the theoretical prices of options under the BSOPM model assumptions and then compared these prices with the real market values to find out the degree of variation in two different time zones and the result finally concluded that Black-Scholes model performed differently in the period before and after the financial crisis.

Sharma and Arora (2015) tested the relevance of Black Scholes Model in the Indian Stock market for the Option prices by using the model to calculate the theoretical option prices using the equation and then comparing it with the actual values. All the necessary assumptions were taken into consideration for option price calculation and the result concluded that the Black Scholes model values were not relevant to the market values of the stock options. Sethi and Nilakantan (2016) in their study explained that there was a critical contrast between the BSOPM call price and the market call price. As the quantity of perceptions expanded, the deviation of BSOPM price from the genuine market price expanded. Several approaches have also been developed over the years to evaluate the real options value of an investment, see for example; Mckenzie, et al (2007); Grundy (1991);Kumar and Agrawal (2017); Cetin, et al (2006); Del Giudice et al., (2013), Sarkar, S (1995); Shinde and Takale (2012), Ugomma, et al (2023) and Ugomma and Benjamin (2023) From the literature reviewed so far, we have seen the contributions of some authors and researchers that have made some remarkable contributions in the applications of Black-Scholes option pricing models in option or stock prices.

The pricing of Options in the market is dependent on certain factors such as spot price, volatility, and many more. Based on these factors, it is practically difficult to estimate the Option prices. Black-Scholes, in their contribution, developed the Black-Scholes Option pricing model, where they made some assumptions for the pricing of the option that makes it applicable to European Option pricing effectively. The question that begs for answer is whether Black-Scholes Option pricing model will still be effective for pricing real stock prices in the Nigeria Stock Market, since stock market experiences variations, with time in the spot price.

II. MATERIALS AND METHODS

A. *The Black-Scholes Formula*

The Black-Scholes formula can be derived for a call option on a non-dividend paying stock with strike price K and maturity T. We assumed that the stock price follows a Geometric Brownian Motion. By the using the Ornstein-Uhlenbeck Processs as a solution to

$$dX_t = \mu X_t dt + \sigma W_t \tag{1}$$

With an initial condition $X_0 = x_0$.

This SDE is solved by using the integrating factor $e^{-\mu t}$ given by

$$e^{-\mu t} dX_t = \mu e^{-ut} X_t d_t + \sigma e^{-\mu t} dW_t \tag{2}$$

Letting $f\left(x,t\right) = xe^{-\mu t}$, then

$$\frac{\partial f}{\partial t} = -\mu e^{-\mu t} Xdt, \frac{\partial f}{\partial x} = e^{-\mu t} \text{ and } \frac{\partial f}{\partial x^2} = 0$$

By the application of Ito's lemma, we have,

$$df\left(X_t,t\right) = \frac{\partial f}{\partial t}\left(X_t,t\right)dt + \frac{\partial f}{\partial x}\left(X_t,t\right)dX_t + \frac{\partial^2 f}{\partial x^2}\left(X_t,t\right)dX_t^2 \tag{3}$$

$$d\left(e^{-\mu t} X_t\right) = -\mu e^{-\mu t} X_t dt + e^{-\mu t} dX_t$$

$$e^{-\mu t} dX_t = d\left(e^{-\mu t} X_t\right) + e^{-\mu t} X_t dt \tag{4}$$

Therefore, the SDE with integrating factor rewritten as

$$d\left(e^{-\mu t}X_t\right)+\mu e^{-\mu t}X_t dt=\mu e^{-\mu t}X_t dt+\sigma e^{-\mu t}dW_t \tag{5}$$

$$\Rightarrow d\left(e^{-\mu t}X_t\right)=\sigma e^{-\mu t}dW_t$$

Now, we integrate both sides from 0 to t

$$\int_0^t d\left(e^{-\mu_s}X_s\right)=\int_0^t \sigma e^{-\mu_s}dW_s$$

$$e^{-\mu t}X_t-X_0=\int_0^t \sigma e^{-\mu_s}dW_s \tag{6}$$

$$X_t=e^{\mu t}X_0+e^{\mu t}\int_0^t\left(\sigma e^{\mu_s}\right)dW_s \tag{7}$$

Let's consider the stock model given by

$$dS_t=\mu S_t dt+\sigma S_t dW_t \tag{8}$$

With solution

$$S_t=S_0 e^{\left(\mu-\frac{1}{2}\sigma^2\right)t}+\sigma W_t \tag{9}$$

Hence, we calculate the probability density function of S_t as follows

$$P\left(S_t\le S\right)=P\left(S_0 e^{\left(\mu-\frac{1}{2}\sigma^2\right)t}+\sigma W_t\le S\right)$$

$$P\left(e^{\left(\mu-\frac{1}{2}\sigma^2\right)t}+\sigma W_t\le\frac{S}{S_0}\right)$$

$$P\left(\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma W_t\le\ln\left(\frac{S}{S_0}\right)\right)$$

$$\frac{1}{\sqrt{2\pi\sigma^2 t}}\int_{-\infty}^{\ln\left(\frac{S}{S_0}\right)} e^{-\frac{\left(x-\left(\mu-\frac{1}{2}\sigma^2\right)t\right)^2}{2\sigma^2 t}}dx \tag{10}$$

The solution to Equation (10) in the case of our call option is given by;

$$C\left(S_t,T\right)=S_t N\left(d_1\right)-e^{rT}KN\left(d_2\right); \tag{11}$$

$$d_2=d_1-\sigma\sqrt{T}; \tag{12}$$

where,

C is the Call Premium,

S_t is the current stock price,

T is the time to maturity,

K is the strike (exercise) price,

r is the risk-free interest rate,

$N(\bullet)$ is the cumulative distribution function of a standardized normal distribution

e is the exponential function.

B. Method of Data Analysis

In this study, we used pair-wise t test to compare two population means where two samples can be paired as one observation. That is, "before – and – after" observation on the same subject. A paired t-test is used to compare two population means having two samples in which observations in one sample can be paired with observations in the other sample.

Here, we assumed the null hypothesis of no significant difference between the means of the market prices of the stock and Black-Scholes prices of the same stock at 5% level of significance.

The pair wise t-test used in this study is given as

$$t = \frac{\bar{d}}{SE\left(\bar{d}\right)} \qquad (13)$$

Where,

$$\bar{d} = \frac{\sum d_i}{n} \qquad (14)$$

$$SE\left(\bar{d}\right) = \frac{S_d}{\sqrt{n}} \qquad (15)$$

and

$$S_d = \sqrt{\frac{n\sum d_i^2 - \left(\sum d_i\right)^2}{n\left(n-1\right)}} \qquad (16)$$

III. EMPIRICAL EVIDENCE

A. Descriptive Statistics of Absolute Returns of Coca-Cola for 2018

The result in Table 1 showed that the stock price is positively Skewed, meaning that the stock price of Coca-Cola for 2018 is normally distributed. The result also showed low volatility of approximately 0%, signifying no risk of buying shares or investing in the company during 2018.

Table 1 Descriptive statistics for Absolute Returns of Coca-Cola for 2018

Trading days	Mean	Variance	Standard Dev	Skewness	Kurtosis	Volatility
250	0.001497	0.00031	0.0175	4.9446	36.6929	1.9606×10^{-5}

Fig 1 Plot of the underlying price of Coca-Cola for 2018

Figure 1 is a plot of the underlying price of Coca-Cola Company for year 2018.From the plot, we observed that there are fluctuations during the trading days.

1) Descriptive Statistics of Absolute Returns of Coca-Cola for 2019

Table 2 Descriptive statistics for Absolute Returns of Coca-Cola for 2019

Trading days	Mean	Variance	Standard Dev	Skewness	Kurtosis	Volatility
251	0.0161	0.00028	0.0167	2.3761	7.7701	1.7634×10^{-5}

From the result in Table 2, we observed that the Skewness is positive (2.4), ie, the data moves towards the right and the volatility is about 0%, meaning low risk in the investment of Coca-Cola Company for 2019.

Fig 2 Plot of the underlying price of Coca-Cola for 2019

Figure 2 is a plot of the underlying price of Coca-Cola Company for year 2019. From the plot, we observed that there are fluctuations during the trading days.

2) Descriptive Statistics of Absolute Returns of Coca-Cola for 2020

Table 3 Descriptive statistics for Absolute Returns of Coca-Cola for 2020

Trading days	Mean	Variance	Standard Dev	Skewness	Kurtosis	Volatility
252	0.0193	0.0004	0.0204	2.8991	11.8313	2.5148×10^{-5}

From the result in Table 3, we observed that the Skewness is positive (2.8991) and high kurtosis of about 12 with low volatility of about 0%, meaning low risk in the investment of Coca-Cola Company for 2020.

Fig 3 Plot of the underlying price of Coca-Cola for 2020

Figure 3 is a graph of the underlying price of Coca-Cola Company for year 2020. From the plot, we observed fluctuations during the trading days.

3) Descriptive Statistics of Absolute Returns of Coca-Cola for 2021

Table 4 Descriptive statistics for Absolute Returns of Coca-Cola for 2021

Trading days	Mean	Variance	Standard Dev	Skewness	Kurtosis	Volatility
251	0.0145	0.0002	0.0151	2.8113	12.6233	1.82×10^{-2}

The result in Table 4 showed that the Skewness is positive (2.8113) and high kurtosis of about 13 with low volatility of about 0%, meaning low risk in the investment of Coca-ColaCompany for 2021.

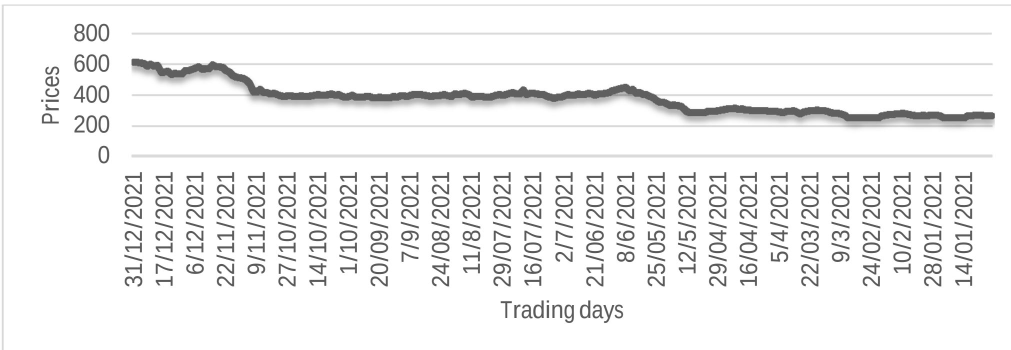

Fig 4 Plot of the underlying price of Coca-Cola for 2021

Figure 4 is a plot of the underlying price of Coca-Cola Company for year 2020. From the graph, we observed the prices maintained a steady decline from 7th of January; 2021and appreciates on the 8th of June 2021 and swings between 30th June to 31st December 2021 during the 251 trading days.

4) Descriptive Statistics of Absolute Returns of Coca-Cola for 2022

Table 5 Descriptive statistics for Absolute Returns of Coca-Cola for 2022

Trading days	Mean	Variance	Standard Dev	Skewness	Kurtosis	Volatility
224	0.02184	0.00051	0.02250	4.9370	42.771	1.20×10^{-3}

From the result in Table 5, we observed that the Skewness is positive (4.94) and highest kurtosis of about 42.8 with low volatility of about 0%, meaning low risk in the investment of Coca-ColaCompany for 2022.

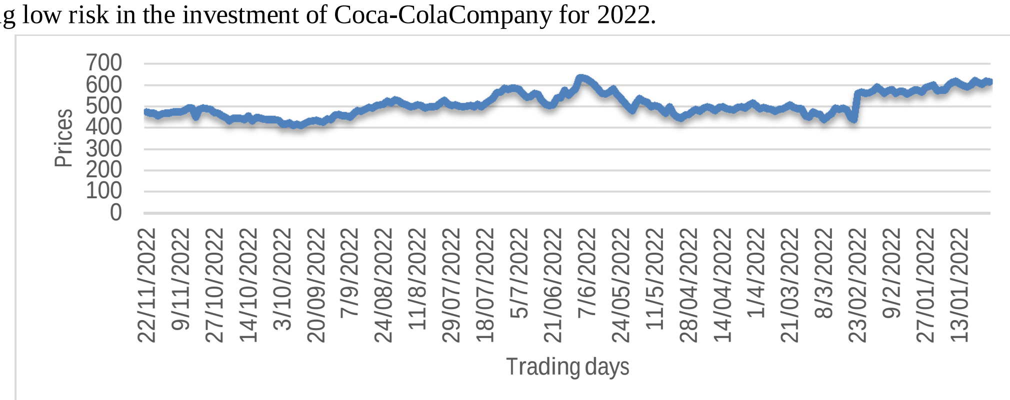

Fig 5 Plot of the underlying price of Coca-Cola for 2022

Figure 5 is a graph of the underlying price of Coca-Cola Company for year 2022. From the graph, we observed high degree of fluctuations during the 224 trading days in 2022. This means that the stocks prices of Coca-Cola Company never remain steady in the stock market for the 2022 trading days.

B. Paired Sample t-Test

1) Paired T-Test for 2018 trading days

$$H_0 : \mu = 0$$
$$H_1 : \mu \neq 0$$

Table 6 The output of the paired t-test for 2018

	Trading days	Mean	Standard Deviation	Standard Error	T-value	P-value	Decision
Underlying price	250	170.67	25.07	1.59	107.63	0.00	Reject
Black-Scholes call	250	0.00	0.00	0.00			
Difference	250	170.67	25.07	1.59			

Table 6 showed that the critical value is 107.63 and the P-value is 0.00. Since the p-value (0.00) is less than the 0.05, we reject the null hypothesis of no significant difference; hence, we conclude that there is significant difference between the underlying price and Black-Scholes call price of the stocks of Coca-Cola during the trading days in 2018.

2) Paired T-Test for 2019 trading days

$$H_0 : \mu = 0$$
$$H_1 : \mu \neq 0$$

Table 7 The output of the paired t-test for 2019

	Trading days	Mean	Standard Deviation	Standard Error	T-value	P-value	Decision
Underlying price	251	248.36	40.81	2.58	110.38	0.00	Reject
Black-Scholes call	251	0.00	0.00	0.00			
Difference	251	248.36	40.81	2.58			

From the result in Table 7, we observed that the critical value is 110.38 and the P-value is 0.00. Since the p-value (0.00) is less than the 0.05, we reject the null hypothesis of no significant difference; hence, we conclude that there is significant difference between the underlying price and the Black-Scholes call price of the stocks of Coca-Cola during the trading days in 2019.

3) Paired T-Test for 2020 trading days

$$H_0 : \mu = 0$$
$$H_1 : \mu \neq 0$$

Table 8 The output of the paired t-test for 2020

	Trading days	Mean	Standard Deviation	Standard Error	T-value	P-value	Decision
Underlying price	252	247.45	40.81	22.96	171.10	0.00	Reject
Black-Scholes call	252	0.00	0.00	0.00			
Difference	252	247.45	40.81	22.96			

From the result in Table 8, we observed that the critical value is 171.10 and the P-value is 0.00.We reject the null hypothesis of no significant difference since the (P=0.00), hence, we conclude that there is significant difference between the underlying price and the Black-Scholes call price of the stocks of Coca-Cola during the trading days in 2020.

4) Paired T-Test for 2021 trading days

$$H_0 : \mu = 0$$

$$H_1 : \mu \neq 0$$

Table 9 The output of the paired t-test for 2021

	Trading days	Mean	Standard Deviation	Standard Error	T-value	P-value	Decision
Underlying price	251	380.77	93.48	5.90	64.54	0.00	Reject
Black-Scholes call	251	0.00	0.00	0.00			
Difference	251	380.77	93.48	5.90			

From the result in Table 9, we observed that the critical value is 64.54 and the P-value is 0.00. We reject the null hypothesis of no significant difference since the (P=0.00), hence, we conclude that there is significant difference between the underlying price and the Black-Scholes call price of the stocks of Coca-Cola during the trading days in 2021.

5) Paired T-Test for 2022 trading days

$$H_0 : \mu = 0$$

$$H_1 : \mu \neq 0$$

Table 10 The output of the paired t-test for 2022

	Trading days	Mean	Standard Deviation	Standard Error	T-value	P-value	Decision
Underlying price	224	508.18	53.55	3.58	142.04	0.00	Reject
Black-Scholes call	224	0.00	0.00	0.00			
Difference	224	508.18	53.55	3.58			

Table 10, showed that the critical value of142.04 and the P-value is 0.00. We, therefore, reject the null hypothesis of no significant difference since the (P=0.00), and conclude that there is significant difference between the underlying price and the Black-Scholes call price of the stocks of Coca-Cola during the 2022 trading days.

IV. CONCLUSION

The findings of this study reviewed that there is significant difference between the underlying price and the Black-Scholes Call price of Coca Cola's stock for each of the years, 2018 through 2022. Based on the findings, we conclude that the Black- Scholes model is not accurate in its price predictive ability on the Coca-Cola Stock prices over the years under study.

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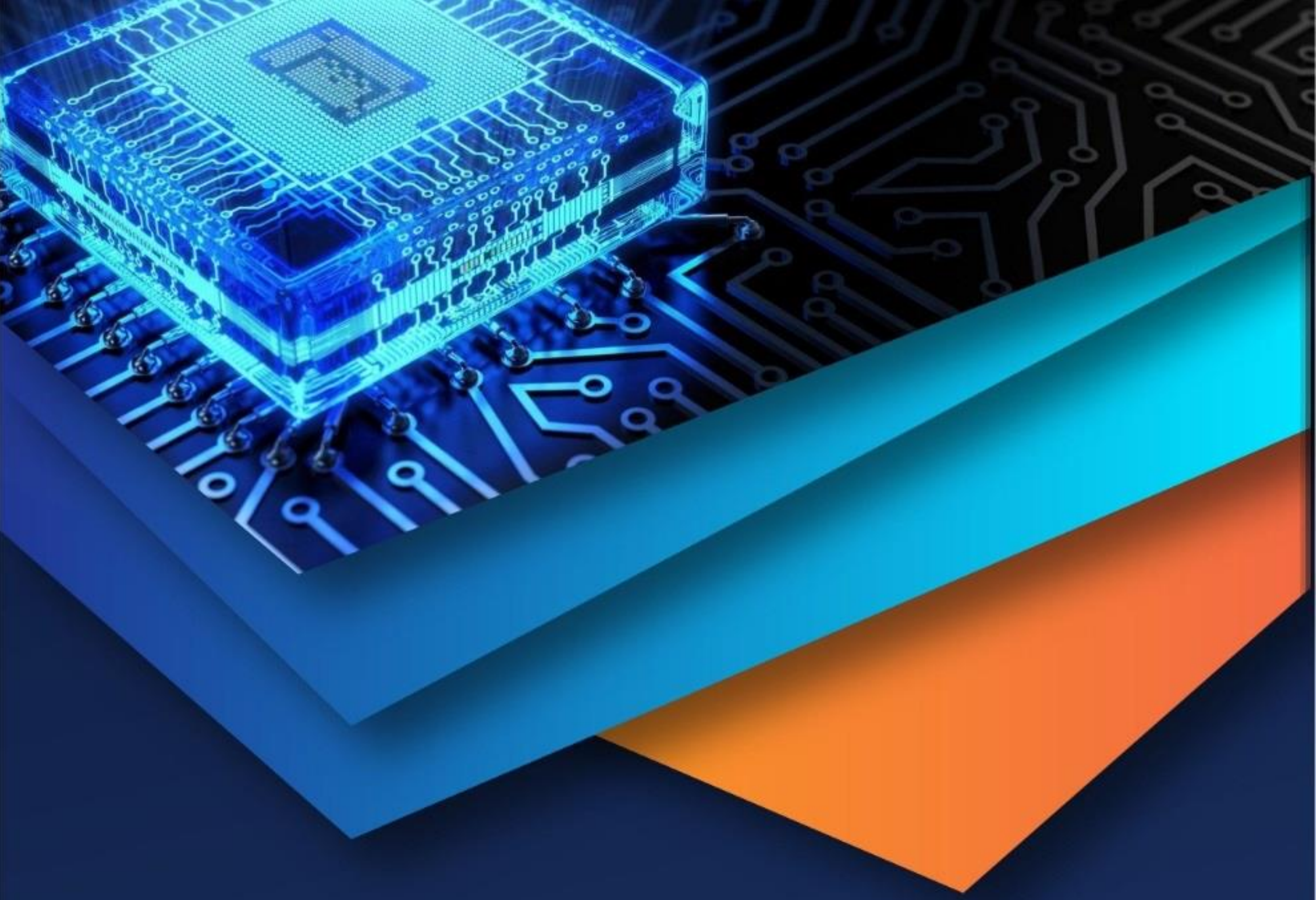

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