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# An Encourage Arrival of Single-Server Queuing System with Defeat Balking and Feedback Customers

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**Abstract:** In this paper, we develop a finite capacity Single-server Markovian queuing system with encourage arrival, defeat balking and feedback customers. The steady state solution and the system characteristic are derived for this model and various probabilistic and performance measures of the system are derived.

**Keywords:** Single-server, defeat balking, feedback, encourage arrival.

## I. INTRODUCTION

Consumer behavior is one of the most uncertain characteristics of business environment. Customers often look for profit-making deals offered by various firms. To attract a greater number of customers, organizations forces by giving a variety of offers to the customers to improve the business. By giving attractive offers, the organizations will encourage the arrivals. This encourage customers are attack by the following offers offered by company and entered into the scheme is termed as defeat balking. Due to that, probability of balking customer is reduce. Attracting customers are termed as encouraged arrivals in this paper. The phenomenon of encouraged arrivals can also be understood as contrary to discouraged arrivals, as discussed by Kumar and Sharma [1]. Eventually, encouraged arrivals put the service facility under pressure, which can make customers dissatisfied with the service. Such customers those who re-join the queue to complete the service are termed as feedback customers in queuing literature. Feedback in queuing system is studied by Takacs [2]. Sasikala and Thiagarajan [3], [4], [5], [6] studied reverse balking with controllable arrival with single server finite capacity. Sasikala and Abinaya [7] studied reverse balking with impatient customers. M.Thiagarajan, S.Premalatha [8], [9], [10], studied the Markovian feedback queuing system with discouraged arrivals and retention of renege customers with controllable arrival rates.

The concept of defeat balking is the most recent one and is suited to many practical situations as described in the introduction section. We incorporate the concept of defeat balking into a finite capacity Markovian single server queuing model with encourage and feedback. We derive the steady-state solution and perform the sensitivity analysis of the model.

## II. MODEL ASSUMPTIONS

- 1) The arrival process is Poisson with parameter  $\lambda$ .
- 2) The service times follow exponential distribution with parameter  $\mu$ .
- 3) The system capacity is taken as finite, say K.
- 4) The queue discipline is First-Come, First-Served.
- 5) (a) When the system is empty, the customers balk with probability  $q'$  and may not balk with probability  $p' = (1 - q')$ . (b)

When the system is not empty, customers balk with a probability  $1 - \frac{n}{N-1}$  and do not balk with probability  $\frac{n}{N-1}$ .

- 6) When a customer may rejoin the system as a feedback customer for receiving another regular service with probability  $\alpha$  and may not join with complementary probability  $\beta$ .

## III. STOCHASTIC MODEL FORMULATION

Let  $P_n(t)$  be the probability that there are n customers in the system at time t.

The Chapman -Kolmogorov equations of the model are:

$$\frac{dP_0(t)}{dt} = -\lambda (1 + \gamma) p' P_0(t) + \mu \beta P_1(t); \quad n=0 \quad (1)$$

$$\frac{dP_1(t)}{dt} = \lambda (1 + \gamma) p' P_0(t) - \left\{ \lambda (1 + \gamma) \left( \frac{1}{K-1} \right) + \mu \beta \right\} P_1(t) + \mu \beta P_2(t); \quad n=1 \quad (2)$$

$$\frac{dP_n(t)}{dt} = \lambda (1 + \gamma) \left( \frac{n-1}{K-1} \right) P_{n-1}(t) - \left\{ \lambda (1 + \gamma) \left( \frac{n}{K-1} \right) + \mu \beta \right\} P_n(t) + \mu \beta P_{n+1}(t);$$

$$2 \leq n \leq K \quad (3)$$

$$\frac{dP_K(t)}{dt} = \lambda (1 + \gamma) P_{n-1}(t) - \mu \beta P_K(t); \quad n=K \quad (4)$$

#### IV. STEADY- STATE SOLUTION

In steady state  $\lim_{t \rightarrow \infty} P_n(t) = 0$ . Therefore the equations (1) to (5) becomes:

$$0 = -\lambda (1 + \gamma) p' P_0 + \mu \beta P_1; \quad n=0 \quad (5)$$

$$0 = \lambda (1 + \gamma) p' P_0 - \left\{ \lambda (1 + \gamma) \left( \frac{1}{K-1} \right) + \mu \beta \right\} P_1 + \mu \beta P_2; \quad n=1 \quad (6)$$

$$0 = \lambda (1 + \gamma) \left( \frac{n-1}{K-1} \right) P_{n-1} - \left\{ \lambda (1 + \gamma) \left( \frac{n}{K-1} \right) + \mu \beta \right\} P_n + \mu \beta P_{n+1};$$

$$2 \leq n \leq K \quad (7)$$

$$0 = \lambda (1 + \gamma) P_{n-1} - \mu \beta P_K; \quad n=K \quad (8)$$

Solving (5) – (8) we obtain:

$$P_n = \left[ \frac{\lambda(1+\gamma)}{\mu\beta} \right]^n \prod_{n=1}^K \left[ \frac{(n-1)!}{(K-1)^{n-1}} \right] P_0 P' \quad (9)$$

Using condition of normality  $\sum_{n=0}^K P_n = 1$ , we get

$$P_0 + \sum_{n=1}^K \left\{ \left[ \frac{\lambda(1+\gamma)}{\mu\beta} \right]^n \prod_{n=1}^K \left[ \frac{(n-1)!}{(K-1)^{n-1}} \right] P_0 P' \right\} = 1$$

Therefore

$$P_0 = \left\{ 1 + \sum_{n=1}^K \left[ \frac{\lambda(1+\gamma)}{\mu\beta} \right]^n \prod_{n=1}^K \left[ \frac{(n-1)!}{(K-1)^{n-1}} \right] P' \right\}^{-1} \quad (10)$$

### V. MEASURES OF PERFORMANCE

Expected system size ( $L_s$ )

$$L_s = \sum_{n=1}^K n P_n$$

$$L_s = \sum_{n=1}^K n \left[ \frac{\lambda(1+\gamma)}{\mu\beta} \right]^n \prod_{n=1}^K \left[ \frac{(n-1)!}{(K-1)^{n-1}} \right] P_0 P'$$
(11)

Average time a customer spends in the system ( $W_s$ )

$$W_s = \frac{L_s}{\lambda(1+\gamma)}$$
(12)

Average rate of defeat balking

$$D_b' = q' \lambda P_0 + \sum_{n=1}^{K-1} \left( 1 - \frac{n}{K-1} \right) \lambda P_n$$

$$D_b' = q' \lambda P_0 + \sum_{n=1}^{K-1} \left( 1 - \frac{n}{K-1} \right) \lambda \left[ \frac{\lambda(1+\gamma)}{\mu\beta} \right]^n \prod_{n=1}^K \left[ \frac{(n-1)!}{(K-1)^{n-1}} \right] P_0 P'$$

### VI. NUMERICAL ILLUSTRATIONS

In this section, the expected queue length is computed and tabulated for specific values of the system parameters. The following values of the parameters are considered in the tables are

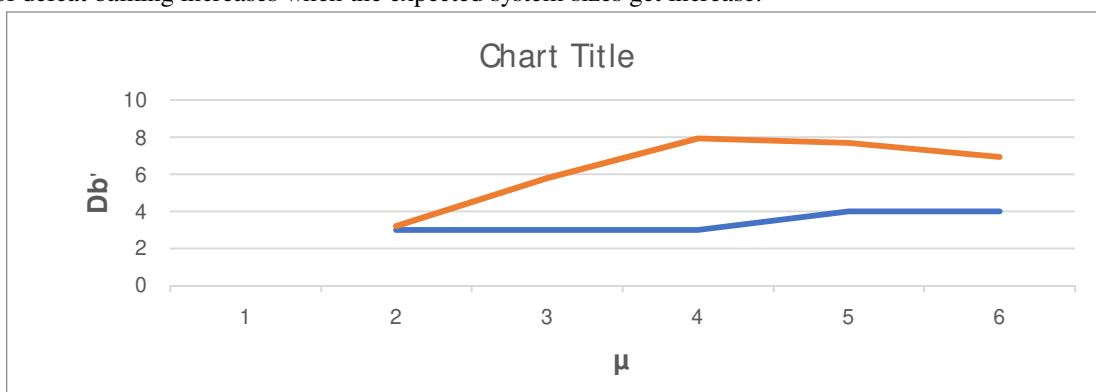
**K=6**  
Table 1

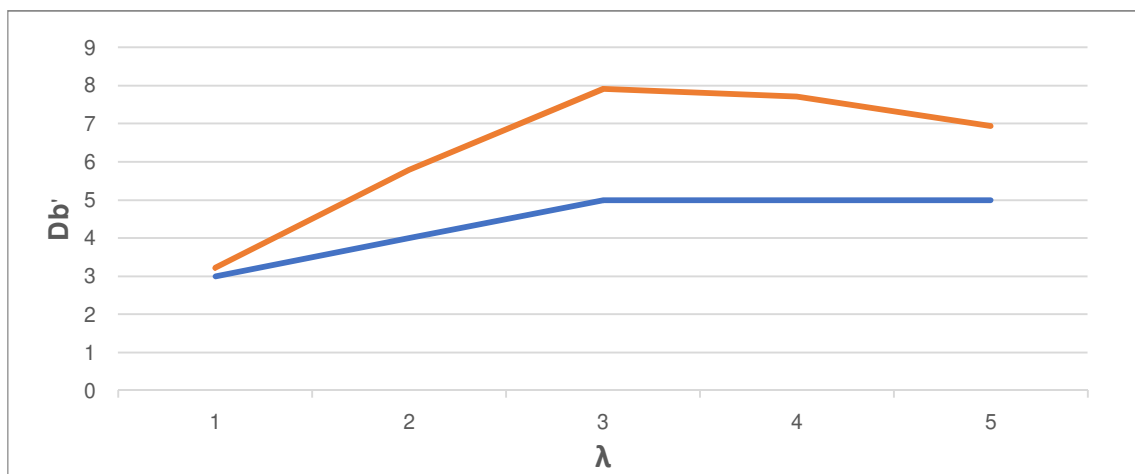
$\lambda$	$\mu$	$\gamma$	$p'$	$q'$	$\beta$	$L_s$	$W_s$	$D_b'$
2	3	0.1	0.1	0.9	0.3	13.4779	6.1263	3.2120
3	3	0.1	0.1	0.9	0.3	19.5825	5.9341	5.7778
4	3	0.1	0.1	0.9	0.3	20.6472	4.6925	7.9178
4	4	0.1	0.1	0.9	0.3	19.5825	4.4506	7.7037
4	4	0.1	0.1	0.9	0.4	15.9159	3.6173	6.93445

### VII. CONCLUSION

The observations made from the Table 1 are

- 1) The expected system size increases with increase in average rate of arrival.
- 2) The rate of defeat balking increases when the expected system sizes get increase.





### REFERENCES

- [1] L.Takacs, A Single Server Queue with Feedback, The Bell System Tech. Journal, 42 (1963) 134-149
- [2] S.Sasikala, and M.Thiagarajan, The M/M/1/N Interdependent Queuing Model with Controllable Arrival Rates and Reverse Balking, International Journal of Current Research,8, (01), 25311-25316
- [3] S.Sasikala, and Thiagarajan, The M/M/c/N/K Interdependent Queuing Model with Controllable Arrival Rates and Reverse Balking, International Journal of Current Research, 2016
- [4] S.Sasikala, and Thiagarajan, M/M/c/N/K loss and delay interdependent queuing model with Controllable arrival rates and reverse balking, International Journal for research in applied Science and engineering technology, 6, (3), 717-724, 2018
- [5] Thiagarajan and S.Sasikala, The M/M/1/N Interdependent Queuing Model with Controllable Arrival Rates, Reverse Balking and Reverse Reneging, International Journal of Innovative research and development 5(4), 306-310, 2016
- [6] S.Sasikala, and V.Abinaya, The M/M/1/N Interdependent Inter Arrival Queuing Model with Controllable Arrival Rates and Reverse Balking and Impatient Customers, Arya Bhatta Journal of Mathematics and Informatics, 15(1), 1-10, 2023
- [7] M.Thiagarajan, S.Premalatha, A Single Server Markovian Queuing System with Discouraged Arrivals Retention of Reneged Customers and Controllable Arrival Rates, International Journal of Mathematical Archive – 7(2), 2016
- [8] M.Thiagarajan, S.Premalatha, A Multi-Server Markovian Queuing System with Discouraged Arrivals and Retention of Reneged Customers with Controllable Arrival Rates, International Journal of Current Research, Vol.8, Issue, 02, pp.26763-26770, 2016
- [9] M.Thiagarajan, S.Premalatha, A Single Server Markovian Feedback Queuing System with Discouraged Arrivals and Retention of Reneged Customers with Controllable Arrival Rates, International Journal of Advanced Scientific and Technical Research. Issue 6, Vol.2, 2016.



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