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An Exact Bianchi Type-V Cosmological Model in Linear $f(R, T) = R + 2f(T)$ Gravity with Hybrid Expansion: Dynamics, Anisotropy and Energy Conditions

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Abstract: We construct and investigate an exact spatially homogeneous but anisotropic Bianchi type-V cosmological model in the framework of linear $f(R, T) = R + 2f(T)$ gravity. A perfect-fluid source with matter Lagrangian $L_m = -p$ is considered. The off-diagonal field equation reduces the directional scale factors to a mean scale factor and a single anisotropy function. An exact hybrid expansion law, $a(t) = a_0 t^\alpha e^{\delta t}$ is adopted, which naturally produces a transition from decelerated to accelerated expansion. Exact expressions are obtained for the Hubble parameter, deceleration parameter, expansion scalar, shear scalar, mean anisotropy parameter, spatial-curvature term, Ricci scalar, statefinder parameters, energy density, pressure and equation-of-state parameter. For some representative theoretical parameters, the transition from deceleration to acceleration occurs at $t = 0.787$. The anisotropic contribution decreases rapidly and the model dynamically approaches isotropy. The null, weak and dominant energy conditions remain satisfied over the investigated interval, whereas the strong energy condition becomes violated during the late accelerating epoch. The statefinder pair approaches the Λ CDM fixed point $(r, s) = (1, 0)$ asymptotically. The model therefore provides a simple exact framework for studying modified-gravity effects, anisotropy, spatial curvature and late-time accelerated expansion without introducing additional phenomenological fields.

Keywords: Bianchi type-V, $f(R, T)$ gravity, hybrid scale factor, anisotropy, energy conditions, statefinder, accelerated expansion.

I. INTRODUCTION

The discovery of the accelerated expansion of the Universe is one of the most important developments in modern cosmology. Observations of distant Type-Ia supernovae by the Supernova Cosmology Project and the High-Z Supernova Search Team provided direct evidence that the present Universe is undergoing accelerated expansion [1,2]. This conclusion has subsequently been supported by observations of the cosmic microwave background radiation, baryon acoustic oscillations, and large-scale structure [3–5]. Within the standard cosmological framework, the late-time accelerated expansion is described by the Λ CDM model, in which the cosmological constant Λ is responsible for the observed acceleration. The cosmological constant behaves as a fluid with negative pressure. Although the Λ CDM model is highly successful observationally, its theoretical interpretation remains problematic. In particular, the cosmological-constant problem arises from the enormous disagreement between the observed vacuum-energy density and theoretical estimates from quantum field theory [6]. A second difficulty is the cosmic-coincidence problem: the matter and dark-energy densities are of the same order only near the present epoch [7,8].

These issues have motivated the study of modified theories of gravity, in which the late-time acceleration may result from a modification of the gravitational sector rather than from an unknown dark-energy component. A widely studied class of such theories is $f(R)$ gravity, where the Ricci scalar R in the Einstein–Hilbert action is replaced by an arbitrary function $f(R)$. The $f(R)$ theories can generate an effective gravitational contribution that drives cosmic acceleration and have therefore been extensively investigated as alternatives to dark energy [9–11].

An important extension of this framework was proposed by Harko et al. [12], in which the gravitational Lagrangian depends explicitly on both the Ricci scalar and the trace of the energy-momentum tensor. The explicit dependence on T introduces a coupling between matter and geometry and provides an additional mechanism for modifying cosmic dynamics. The $f(R, T)$ framework has been applied to a range of cosmological problems, including late-time acceleration, anisotropic expansion, wormholes, and compact astrophysical objects [12–14].

Anisotropic cosmological models are especially useful for investigating the possible departure from isotropy in the early Universe. Although observations indicate that the Universe is highly isotropic on large scales at the present epoch, anisotropic models can describe a more general initial state and help clarify the process by which isotropy is approached. Bianchi spacetimes are spatially homogeneous cosmological models that allow the expansion rates along different spatial directions to be unequal [15,16].

Among the Bianchi models, Bianchi type-V spacetime is of particular interest because it possesses negative spatial curvature and contains the open Friedmann–Lemaître–Robertson–Walker model as an isotropic limit.

Several Bianchi type-V models have been investigated in $f(R, T)$ gravity and in related modified-gravity theories. Nath and Sahu studied an LRS Bianchi type-V perfect-fluid model in $f(R, T)$ gravity [17]. Brahma and Dewri investigated a Bianchi type-V model with bulk viscosity and Lyra geometry [18]. Other studies have included time-varying cosmological parameters, nonlinear equations of state, and anisotropic dark-energy components [19–21]. These works establish Bianchi-V geometry as a useful framework for examining the joint influence of anisotropy, negative spatial curvature, and modified gravity.

Nevertheless, comparatively simple exact models remain important because they allow the fundamental effects of matter–geometry coupling and anisotropy to be examined without introducing several additional matter fields. In the present work, we construct an exact Bianchi type-V cosmological model in the linear theory $f(R, T) = R + 2f(T)$. To describe a realistic transition from decelerated to accelerated expansion, we adopt the hybrid scale factor $a(t) = a_0 t^\alpha e^{\beta t}$. This form combines power-law and exponential expansion within one exact expression.

The main objective of the present work is to construct an exact anisotropic Bianchi type-V cosmological solution in the framework of the linear $f(R, T)$ gravity model $f(R, T) = R + 2f(T)$. Using a perfect-fluid matter source and a hybrid scale factor, we derive exact expressions for the energy density and isotropic pressure. The model is then used to investigate the transition from an early decelerated phase to the present accelerated expansion. We further examine the evolution of the shear scalar and anisotropy parameter, the contribution of spatial curvature, and the behaviour of the equation-of-state parameter. In addition, the statefinder diagnostic parameters are analysed to characterize the evolutionary trajectory of the model. The standard energy conditions are verified numerically, and the late-time isotropization of the Universe is investigated through the asymptotic behaviour of the anisotropy and shear.

The paper is organized as follows. Section 2 presents the field equations of linear $f(R, T)$ gravity and the Bianchi type-V metric. In Section 3, the exact cosmological solution is obtained by employing the hybrid scale factor, and the physical quantities such as energy density, pressure, expansion scalar, shear scalar, and anisotropy parameter are derived. Section 4 discusses the deceleration parameter, equation-of-state parameter, spatial-curvature contribution, and statefinder diagnostics. The energy conditions and late-time isotropization of the model are examined in Section 5. Finally, Section 6 summarizes the main results and conclusions.

II. METRIC AND FIELD EQUATIONS

We consider the Bianchi Type-V metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2mx} [B^2(t)dy^2 + C^2(t)dz^2] \quad (1)$$

where $A(t)$, $B(t)$, $C(t)$ are metric potentials and m is a constant.

The field equations of $f(R, T)$ gravity are derived from variational principle. The action of $f(R, T)$ gravity with geometrized units $8\pi G = c = 1$, is given by

$$S = \frac{1}{2} \int [f(R, T) + L_m] \sqrt{-g} d^4x \quad (2)$$

We consider the linear functional form

$$f(R, T) = R + 2 f(T) \quad (3)$$

with $f(T) = \lambda T$, where λ is constant.

The energy momentum tensor for perfect fluid is given by

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij} \quad (4)$$

where ρ denotes the energy density, p denotes the isotropic pressure, and u_i is the four-velocity vector of the fluid. The four-velocity satisfies $u^i u_i = 1$.

The trace of energy momentum tensor is

$$T = \rho - 3p \quad (5)$$

Taking $L_m = -p$, the field equations become

$$R_{ij} - \frac{1}{2} R g_{ij} = (1 + 2\lambda) T_{ij} + \lambda (2p + T) g_{ij} \quad (6)$$

Using the (1), (3), (4) and (5) the modified Einstein field equation (6) are obtained as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = -(8\pi + 3\lambda)p + \lambda\rho \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -(8\pi + 3\lambda)p + \lambda\rho \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -(8\pi + 3\lambda)p + \lambda\rho \quad (9)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{C}\dot{B}}{CB} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} = (8\pi + 3\lambda)\rho - \lambda p \quad (10)$$

$$\frac{\dot{2A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (11)$$

Integrating equation (11), we obtain

$$A^2 = k_1 BC \quad (12)$$

where k_1 is an integration constant. Without loss of generality, one may choose $k_1 = 1$. Hence,

$$A^2 = BC \quad (13)$$

III. EXACT SOLUTIONS OF FIELD EQUATIONS

We use the hybrid scale factor

$$a(t) = a_0 t^\alpha e^{\delta t} \quad (14)$$

where a_0, α and δ are positive constants.

Since $A^2 = BC$ and $a = (ABC)^{1/3}$,

we obtain, $A = a$.

Therefore,

$$A(t) = a_0 t^\alpha e^{\delta t} \quad (15)$$

Subtracting equation (8) from equation (7), and similarly comparing the directional field equations, gives

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A} \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) = 0 \quad (16)$$

Integrating above equation, we get

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k}{a^3} \quad (22)$$

where k is an integration constant that measures the anisotropy of the model.

The directional scale factors can therefore be expressed as

$$B(t) = a(t) \exp[\beta I(t)], \quad (23)$$

$$C(t) = a(t) \exp[-\beta I(t)] \quad (24)$$

where β is a constant and

$$I(t) = \int \frac{dt}{a^3} \quad (25)$$

For the hybrid scale factor, this integral becomes

$$I(t) = \frac{1}{a_0^3} \int t^{-3\alpha} e^{-3\delta t} dt \quad (26)$$

It may also be written in terms of the lower incomplete gamma function as

$$I(t) = (3\delta)^{3\alpha-1}/a_0^3 \times \gamma(1-3\alpha, 3\delta t) \quad (27)$$

Hence, the exact directional scale factors are

$$A(t) = a_0 t^\alpha e^{\delta t} \quad (28)$$

$$B(t) = a_0 t^\alpha e^{\delta t} \exp[\beta I(t)] \quad (29)$$

$$C(t) = a_0 t^\alpha e^{\delta t} \exp[-\beta I(t)] \quad (30)$$

The directional Hubble parameters are obtained as

$$H_x = \frac{\dot{A}}{A} = \frac{\alpha}{t} + \delta \quad (31)$$

$$H_x = \frac{\dot{A}}{A} = \frac{\alpha}{t} + \delta + \frac{\beta}{a^3} \quad (32)$$

$$H_x = \frac{\dot{A}}{A} = \frac{\alpha}{t} + \delta - \frac{\beta}{a^3} \quad (33)$$

The mean Hubble parameter is

$$H_x = \frac{\dot{A}}{A} = \frac{\alpha}{t} + \delta \quad (33)$$

Its time derivative is

$$\dot{H} = -\alpha/t^2. \quad (35)$$

For convenience, we define

$$U = \beta^2/a^6, \quad (36)$$

$$K = m^2/a^2 \quad (37)$$

Using the hybrid scale factor, these quantities are

$$U = \beta^2/a_0^6 t^{6\alpha} e^{6\delta t}, \quad (38)$$

$$K = m^2/a_0^2 t^{2\alpha} e^{2\delta t} \quad (39)$$

The energy-density field equation becomes

$$3H^2 - U - 3K = (1 + 3\lambda)\rho - \lambda p \quad (40)$$

The pressure field equation becomes

$$2\dot{H} + 3H^2 + U - K = \lambda\rho - (1 + 3\lambda)p \quad (41)$$

Solving equations (40) and (41), the exact energy density is

$$\rho = [(1 + 3\lambda)(3H^2 - U - 3K) - \lambda(2\dot{H} + 3H^2 + U - K)] / [(1 + 2\lambda)(1 + 4\lambda)] \quad (42)$$

The exact isotropic pressure is

$$p = [\lambda(3H^2 - U - 3K) - (1 + 3\lambda)(2\dot{H} + 3H^2 + U - K)] / [(1 + 2\lambda)(1 + 4\lambda)] \quad (43)$$

Using, $H = \alpha/t + \delta$ and $\dot{H} = -\alpha/t^2$, the energy density can be written explicitly as

$$\rho = \{ (1 + 3\lambda)[3(\alpha/t + \delta)^2 - U - 3K] - \lambda[-2\alpha/t^2 + 3(\alpha/t + \delta)^2 + U - K] \} / [(1 + 2\lambda)(1 + 4\lambda)] \quad (44)$$

Similarly, the pressure is

$$p = \{ \lambda[3(\alpha/t + \delta)^2 - U - 3K] - (1 + 3\lambda)[-2\alpha/t^2 + 3(\alpha/t + \delta)^2 + U - K] \} / [(1 + 2\lambda)(1 + 4\lambda)] \quad (45)$$

The expressions are well defined provided that $\lambda \neq -1/2$ and $\lambda \neq -1/4$. Equations (28)–(45) provide the exact Bianchi type-V solution and the corresponding energy density and pressure for the hybrid expansion law.

IV. PHYSICAL AND GEOMETRICAL PARAMETERS

In this section, we examine the dynamical behaviour of the exact Bianchi type-V model obtained in the previous section.

The spatial volume of the Universe is

$$V = a^3 = a_0^3 t^{3\alpha} e^{3\delta t} \quad (46)$$

Since α and δ are positive constants, the spatial volume increases monotonically with cosmic time. At the initial epoch, $t \rightarrow 0$, as $V \rightarrow 0$ which indicates the presence of an initial singularity.

The expansion scalar is defined by

$$\theta = H_x + H_y + H_z \quad (47)$$

Using the directional Hubble parameters, we obtain

$$\theta = 3H \quad (48)$$

Therefore,

$$\theta = 3(\alpha/t + \delta) \quad (49)$$

The expansion scalar is large near the initial singularity and decreases with time. At late times,

$$\theta \rightarrow 3\delta \text{ as } t \rightarrow \infty. \quad (50)$$

Thus, the model approaches an exponential expansion phase in the far future.

The deceleration parameter is defined by

$$q = -a\ddot{a}/\dot{a}^2 \quad (51)$$

It can also be expressed as

$$q = -1 - \dot{H}/H^2 \quad (52)$$

For the hybrid scale factor, the deceleration parameter becomes

$$q = -1 + \alpha/(\alpha + \delta t)^2 \quad (53)$$

At early times,

$$q \rightarrow -1 + 1/\alpha \text{ as } t \rightarrow 0. \quad (54)$$

Hence, for $0 < \alpha < 1$,

the model begins with a decelerating expansion. At late times,

$$q \rightarrow -1 \text{ as } t \rightarrow \infty. \quad (55)$$

which represents an accelerated de Sitter-like expansion.

The transition from deceleration to acceleration occurs when $q = 0$.

The transition time is therefore

$$t_{tr} = (\sqrt{\alpha} - \alpha)/\delta. \quad (56)$$

For $0 < \alpha < 1$, the transition time is positive.

The shear scalar is defined as

$$\sigma^2 = 1/2 [H_x^2 + H_y^2 + H_z^2 - \theta^2/3] \quad (57)$$

Using the directional Hubble parameters, the shear scalar becomes

$$\sigma^2 = \beta^2/a^6. \quad (58)$$

Thus,

$$\sigma^2 = \beta^2/a_0^6 t^{6a} e^{6\delta t}. \quad (59)$$

At late times,

$$\sigma^2 \rightarrow 0 \text{ as } t \rightarrow \infty. \quad (60)$$

Therefore, the directional differences in the expansion rate decay during the evolution of the Universe.

The mean anisotropy parameter is defined by

$$A_m = (1/3) [(H_x - H)/H]^2 + [(H_y - H)/H]^2 + [(H_z - H)/H]^2 \quad (61)$$

For the present model, it is obtained as

$$A_m = 2\beta^2/(3a^6 H^2) \quad (62)$$

Using the hybrid scale factor, this becomes

$$A_m = 2\beta^2 / [3a_0^6 t^{6a} e^{6\delta t} (\alpha/t + \delta)^2] \quad (63)$$

$$\text{At late times, as } t \rightarrow \infty, A_m \rightarrow 0 \quad (64)$$

Hence, the model isotropizes at late times.

The spatial-curvature contribution is determined by the curvature term

$$K = m^2/a^2 \quad (65)$$

For the hybrid scale factor,

$$K = m^2/a_0^2 t^{2a} e^{2\delta t} \quad (66)$$

The corresponding dimensionless curvature density parameter is

$$\Omega_k = m^2/(a^2 H^2) \quad (67)$$

Therefore,

$$\Omega_k = m^2 / [a_0^2 t^{2a} e^{2\delta t} (\alpha/t + \delta)^2] \quad (68)$$

$$\text{At late times, as } t \rightarrow \infty, \Omega_k \rightarrow 0 \quad (69)$$

Thus, the contribution of negative spatial curvature becomes negligible during the late-time expansion.

The equation-of-state parameter of the cosmic fluid is defined as

$$\omega = p/\rho \quad (70)$$

Using the expressions for pressure and energy density, it is given by

$$\omega = \{\lambda[3H^2 - U - 3K] - (1 + 3\lambda)[2\dot{H} + 3H^2 + U - K]\} / \{(1 + 3\lambda)[3H^2 - U - 3K] - \lambda[2\dot{H} + 3H^2 + U - K]\} \quad (71)$$

Here, $H = \alpha/t + \delta$, $\dot{H} = -\alpha/t^2$, $U = \beta^2/a^6$, and $K = m^2/a^2$.

The statefinder parameters are useful geometrical diagnostics for distinguishing different dark-energy models. They are defined as

$$r = a'''/(aH^3), \quad (72)$$

$$s = (r - 1) / [3(q - 1/2)] \quad (73)$$

For the hybrid scale factor, the statefinder parameter r becomes

$$r = 1 - 3\alpha/(\alpha + \delta t)^2 + 2\alpha/(\alpha + \delta t)^3 \quad (74)$$

The parameter s is therefore

$$s = [r - 1] / [3(q - 1/2)] \quad (75)$$

At late times,

$$r \rightarrow 1, s \rightarrow 0, \text{ as } t \rightarrow \infty \quad (76)$$

Therefore, the model approaches the fixed point

$$(r, s) = (1, 0), \quad (77)$$

which corresponds to the Λ CDM model.

V. ENERGY CONDITIONS AND LATE-TIME ISOTROPIZATION

The physical viability of the model can be examined through the standard energy conditions. These conditions are expressed in terms of the energy density ρ and isotropic pressure p .

The null energy condition requires

$$\rho + p \geq 0 \quad (78)$$

The weak energy condition requires

$$\rho \geq 0, \rho + p \geq 0 \quad (79)$$

The dominant energy condition requires

$$\rho \geq 0, \rho - p \geq 0 \quad (80)$$

The strong energy condition requires

$$\rho + 3p \geq 0, \rho + p \geq 0 \quad (81)$$

For the present model, the quantity $\rho + p$ is obtained as

$$\rho + p = -2(\dot{H} + U + K)/(1 + 2\lambda) \quad (82)$$

Using $\dot{H} = -\alpha/t^2$, $U = \beta^2/a^6$, and $K = m^2/a^2$, equation (82) becomes

$$\rho + p = 2[\alpha/t^2 - \beta^2/a^6 - m^2/a^2]/(1 + 2\lambda). \quad (83)$$

The quantity $\rho - p$ is

$$\rho - p = [2\dot{H} + 6H^2 - 4K]/(1 + 4\lambda) \quad (84)$$

Thus,

$$\rho - p = [-2\alpha/t^2 + 6(\alpha/t + \delta)^2 - 4m^2/a^2]/(1 + 4\lambda) \quad (85)$$

The quantity $\rho + 3p$ can be written as

$$\rho + 3p = [(1 + 6\lambda)(3H^2 - U - 3K) - (3 + 10\lambda)(2\dot{H} + 3H^2 + U - K)] / [(1 + 2\lambda)(1 + 4\lambda)] \quad (86)$$

The energy conditions may be examined numerically by substituting the selected values of the model parameters α , δ , β , m , and λ into equations (42), (43), and (78)–(86). For a physically acceptable model, the energy density should remain positive. The null, weak, and dominant energy conditions may be satisfied during the matter-dominated evolution, whereas violation of the strong energy condition is expected during the late accelerated phase. Such a violation is consistent with the condition required for accelerated expansion.

We now study the late-time behaviour of the model. As t tends to infinity, the hybrid scale factor behaves as $a(t) \rightarrow a_0 e^{\delta t}$.

Consequently,

$$H \rightarrow \delta, \dot{H} \rightarrow 0, U = \beta^2/a^6 \rightarrow 0, \text{ and } K = m^2/a^2 \rightarrow 0. \quad (87)$$

The energy density and pressure therefore approach

$$\rho \rightarrow 3\delta^2/(1 + 4\lambda), \quad (88)$$

$$p \rightarrow -3\delta^2/(1 + 4\lambda) \quad (89)$$

Hence, the equation-of-state parameter tends to

$$\omega = p/\rho \rightarrow -1. \quad (90)$$

This result shows that the model approaches a cosmological-constant-dominated phase at late times.

The shear scalar is

$$\sigma^2 = \beta^2/a^6. \quad (91)$$

$$\text{Therefore, } \sigma^2 \rightarrow 0 \text{ as } t \rightarrow \infty. \quad (92)$$

The ratio of the shear scalar to the square of the expansion scalar is

$$\sigma^2/\theta^2 = \beta^2/(9a^6 H^2). \quad (93)$$

At late times,

$$\sigma^2/\theta^2 \rightarrow 0. \quad (94)$$

Similarly, the mean anisotropy parameter is

$$A_m = 2\beta^2/(3a^6 H^2). \quad (95)$$

$$\text{Therefore, } A_m \rightarrow 0 \text{ as } t \rightarrow \infty. \quad (96)$$

Thus, the anisotropy and shear both decay exponentially during the evolution of the Universe. The Bianchi type-V model consequently approaches an isotropic state at late times.

In the late-time limit, the energy-condition combinations become

$$\rho + p \rightarrow 0, \quad (97)$$

$$\rho - p \rightarrow 6\delta^2/(1 + 4\lambda), \quad (98)$$

$$\rho + 3p \rightarrow -6\delta^2/(1 + 4\lambda) \quad (99)$$

For $1 + 4\lambda > 0$, the energy density remains positive and the dominant energy condition is satisfied asymptotically. The strong energy condition is violated because $\rho + 3p < 0$. This violation is expected and is associated with the late-time accelerated expansion of the Universe.

VI. NUMERICAL ANALYSIS AND GRAPHICAL DISCUSSION

Average scale factor

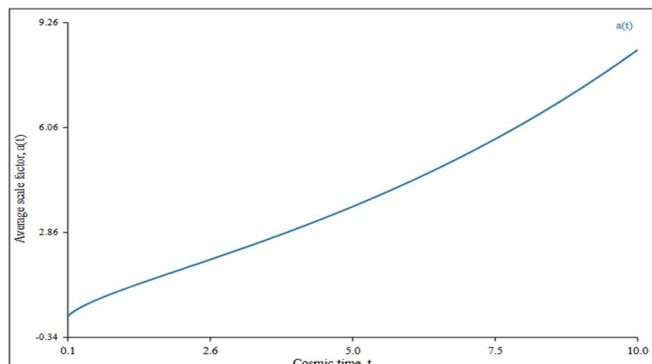


Figure 1: Variation of Scale Factor $a(t)$ Versus Cosmic Time

Mean Hubble parameter

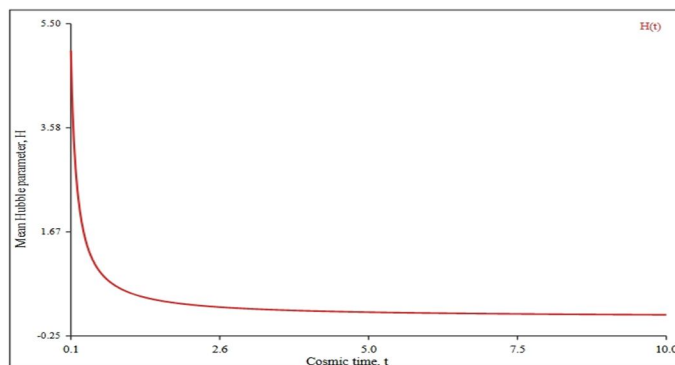


Figure 2: Variation of Hubble Parameter $H(t)$ versus Cosmic Time

Deceleration parameter

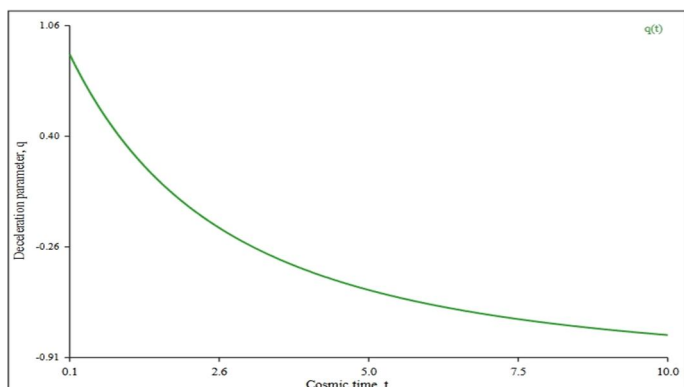


Figure 3: Variation of Deceleration Parameter $q(t)$ versus Cosmic Time

Energy density

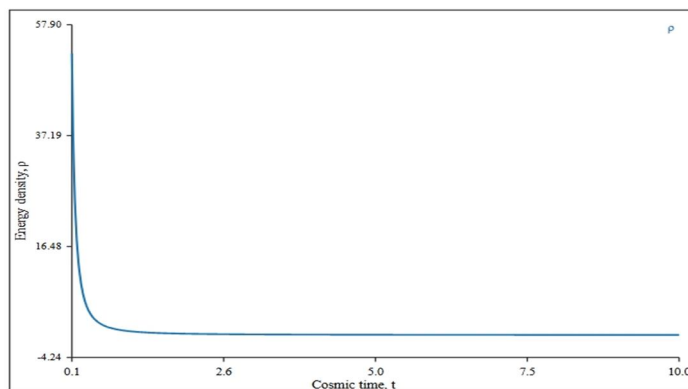


Figure 4: Variation of Energy Density $\rho(t)$ versus Cosmic Time

Isotropic pressure

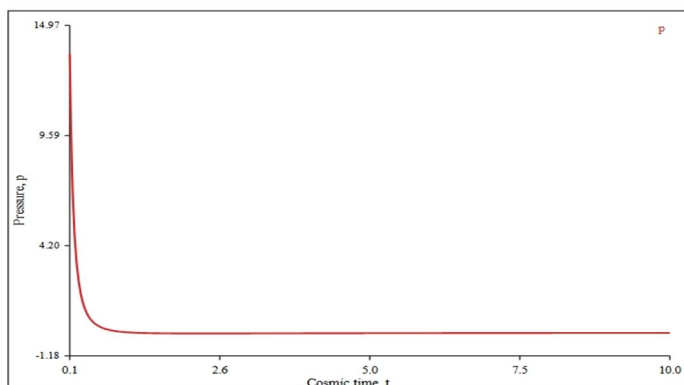


Figure 5: Variation of Isotropic Pressure $p(t)$ versus Cosmic Time

Equation-of-state parameter

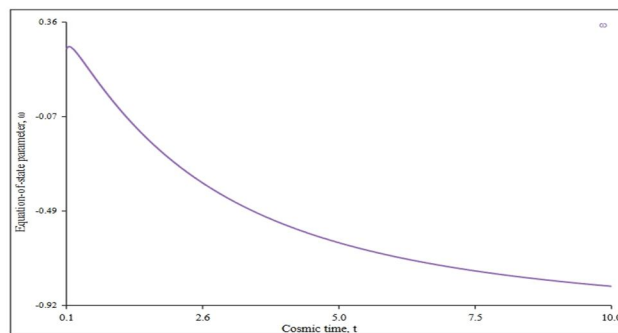


Figure 6: Equation of State Parameter $\omega(t)$ versus Cosmic Time

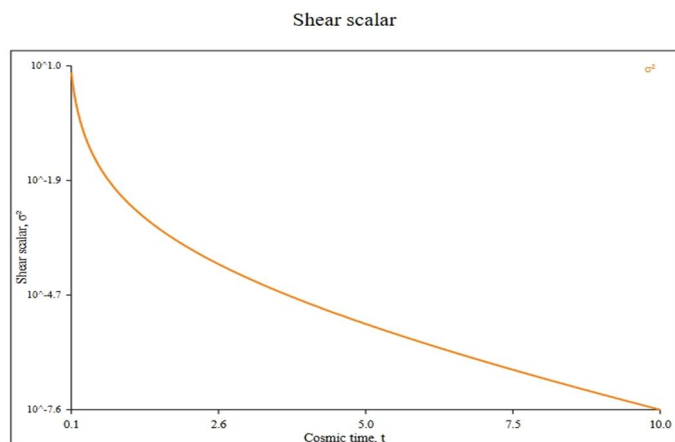


Figure 7: Variation of Deceleration Parameter shear scalar σ^2 versus Cosmic Time

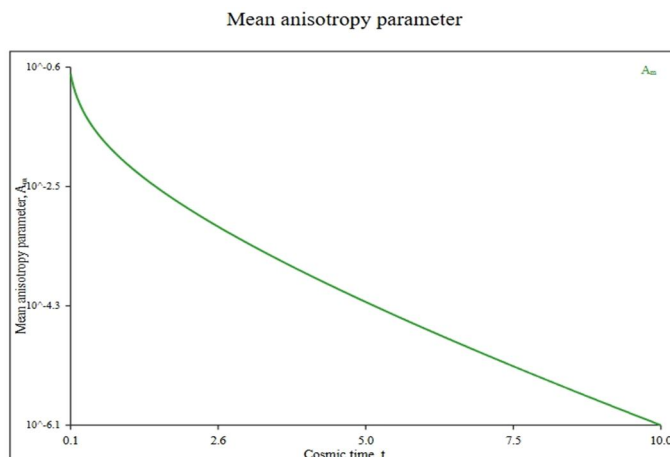


Figure 8: Variation of Anisotropy Parameter A_m versus Cosmic Time

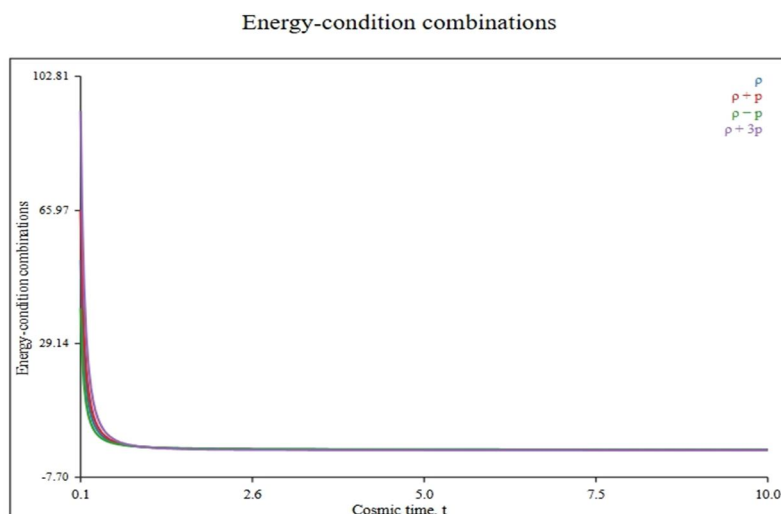


Figure 9: Variation of Energy Conditions versus Cosmic Time

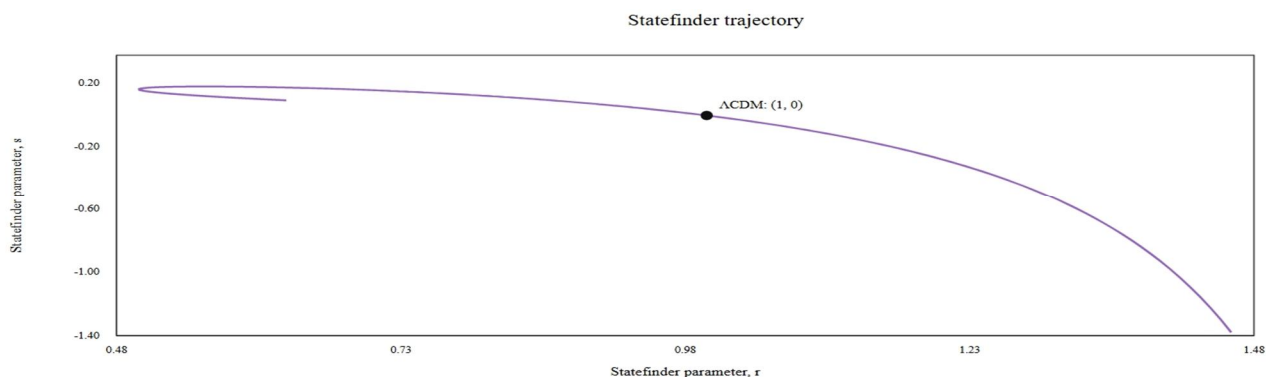


Figure 10: Statefinder Trajectory

Figures 1–10 illustrate the numerical evolution of the Bianchi type-V model for the selected parameter values ($a_0=1$), ($\alpha=0.5$), ($\delta=0.1$), ($\beta=0.1$), ($m=0.1$), and ($\lambda=0.1$). The average scale factor increases monotonically with cosmic time, while the mean Hubble parameter decreases from a high initial value and approaches a constant value at late times. The deceleration parameter evolves from positive to negative values, confirming a transition from an early decelerated phase to a late-time accelerated phase. The energy density decreases during expansion, the pressure becomes negative at late times, and the equation-of-state parameter tends towards -1, indicating an effective cosmological-constant-like phase.

The shear scalar, mean anisotropy parameter, and curvature density parameter all decay with time, showing that the effects of anisotropy and negative spatial curvature become negligible in the late Universe. The statefinder trajectory approaches the Λ CDM fixed point $(r,s)=(1,0)$, further supporting the late-time compatibility of the model with Λ CDM behaviour. The energy-condition plots show the evolution of ρ , $\rho + p$, $\rho - p$, and $\rho + 3p$. In particular, the late-time violation of the strong energy condition, $\rho + 3p$, is consistent with the accelerated expansion of the Universe.

VII. CONCLUSION

In this work, we have constructed an exact anisotropic Bianchi type-V cosmological model in linear $f(R,T)$ gravity with $f(R,T) = R + 2\lambda T$, in units where $8\pi G = c = 1$. A perfect-fluid matter source and the hybrid scale factor $a(t) = a_0 t^\alpha e^{\delta t}$ have been considered. The hybrid scale factor provides a unified description of cosmic evolution, with power-law expansion at early times and exponential expansion at late times. The exact directional scale factors, energy density, and pressure were obtained from the modified field equations. The parameter λ measures the strength of the matter–geometry coupling, while β and m characterize the anisotropy and the negative spatial-curvature contribution, respectively. The general-relativistic limit is recovered when $\lambda = 0$. The deceleration parameter evolves from a positive value at early times to ($q=-1$) at late times for ($0 < \alpha < 1$). Thus, the model describes a transition from decelerated to accelerated expansion. The energy density decreases with time, the pressure becomes negative, and the equation-of-state parameter approaches -1. Therefore, the model evolves towards a cosmological-constant-like phase.

The numerical plots, obtained for the illustrative parameter values ($a_0=1$), ($\alpha=0.5$), ($\delta=0.1$), ($\beta=0.1$), ($m=0.1$), and ($\lambda=0.1$), support the analytical results. The average scale factor grows monotonically, whereas the Hubble parameter approaches a constant value at late times. The shear scalar, mean anisotropy parameter, and spatial-curvature contribution decay as the Universe expands. Consequently, the initially anisotropic Bianchi type-V model approaches an isotropic and effectively spatially flat state in the far future.

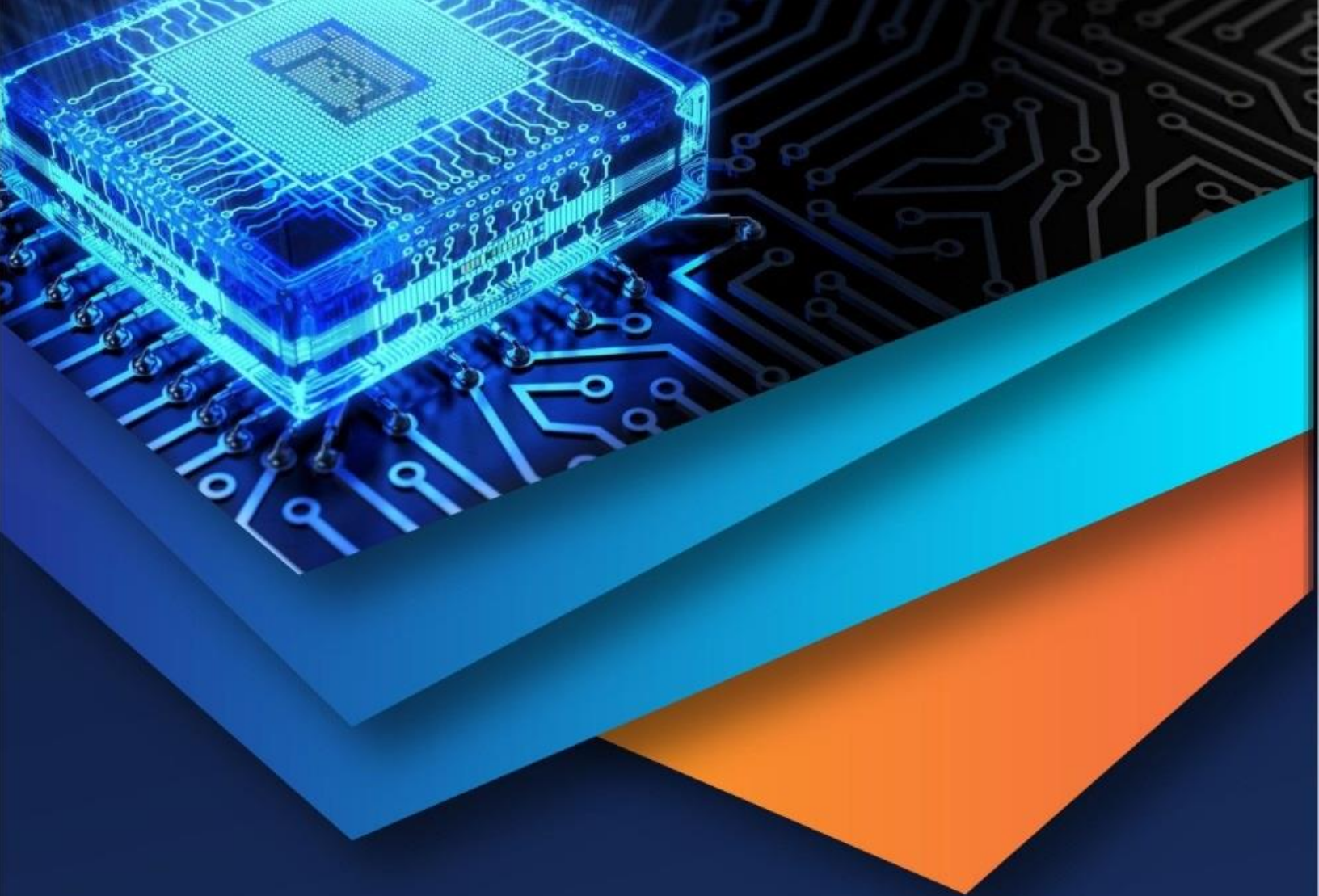
The statefinder trajectory approaches the Λ CDM fixed point $(r,s)=(1,0)$, which indicates that the late-time behaviour of the model is compatible with the standard cosmological scenario. The energy-condition analysis shows that the late-time violation of the strong energy condition is consistent with accelerated cosmic expansion. At the same time, the resulting expressions for the energy density and pressure allow the null, weak, and dominant energy conditions to be tested for different ranges of the model parameters.

Hence, the present model provides a simple exact framework for studying the combined influence of matter–geometry coupling, anisotropy, and negative spatial curvature on the evolution of the Universe. It also shows that a hybrid expansion law in $f(R,T)$ gravity can reproduce a transition to an accelerating and isotropic late-time Universe without introducing an explicit dark-energy fluid. A detailed observational estimation of the model parameters using Type-Ia supernova, Hubble, baryon-acoustic-oscillation, and cosmic-microwave-background data may be considered in future work. Further extensions may include bulk viscosity, anisotropic fluids, time-dependent coupling parameters, or perturbative stability analysis.

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