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## An Integral Solution of Negative Pell Equation $x^2 = 5y^2 - 9^t$

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Abstract: We look for non-trivial integer solution to the equation  $x^2 = 5y^2 - 9^t$ ,  $t \in N$  for the singular choices of particular by (i) t = 2k (ii) t = 2k+1,  $\forall k \in N$ . Additionally, recurrence relations on the solutions are obtained. Keywords: Negative Pell equations, Pell equations, Diophantine equations, Integer solutions.

#### I. INTRODUCTION

It is well known the Pell equation  $x^2 - Dy^2 = 1$  (D > 0 and square free) has at all times positive integer solutions. When N  $\neq 1$ , the Pell equation  $x^2 - Dy^2 = -N$  possibly will not boast at all positive integer solutions. In favour of instance, the equations  $x^2 = 3y^2 - 1$  and  $x^2 = 7y^2 - 4$  comprise refusal integer solutions.

This manuscript concerns the negative Pell equation  $x^2 = 5y^2 - 9^t$ , where t > 0 and infinitely numerous positive integer solutions are obtained for the choices of t known by (i) t = 2k (ii) t = 2k+1. Supplementary recurrence relationships on the solutions are consequent.

#### **II. PRELIMINARY**

The Pell equation is a Diophantine equation of the form  $x^2 - dy^2 = 1$ . Given *d*, we would like to find all integer pairs (*x*, *y*) that satisfy the equation. Since any solution (*x*, *y*) yields multiple solutions ( $\pm x, \pm y$ ), we may restrict our attention to solutions where *x* and *y* nonnegative integer. We usually take *d* in the equation  $x^2 - dy^2 = 1$  to be a positive non square integer. Otherwise, there are only uninteresting solutions: if d < 0, then (*x*, *y*) = ( $\pm 1$ , 0) in the case d < -1, and (*x*, *y*)=(0,  $\pm 1$ ) or ( $\pm 1$ , 0) in the case d = -1; if d = 0, then  $x = \pm 1$  (*y* arbitrary); and if a nonzero square, then  $dy^2$  and  $x^2$  are consecutive squares, implying that (*x*, *y*)=( $\pm 1$ , 0). Notice that the Pell equation always has trivial solution (*x*, *y*) = (1, 0). We now investigate an illustrate case of Pell's equation and its solution involving recurrence relations.

Let p be a prime. The negative Pell's equation  $x^2 - py^2 = -1$  is solvable if and only if p = 2 (or)  $p \equiv 1 \pmod{4}$ .

#### **III. METHOD OF ANALYSIS**

Consider the negative Pell equation  $x^2 = 5y^2 - 9^t, t \in N$ 

1) Choice 1: t = 2k, k > 0The Pell equation is  $x^2 = 5y^2 - 9^{2k}$ , k > 0

Let  $(x_0, y_0)$  be the initial solution of (1) specified by

$$x_0 = 18.9^k$$
;  $y_0 = 9.9^k$ 

(1)



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Applying Brahma Gupta Lemma Connecting  $(x_0, y_0)$  and  $(\tilde{x}_n, \tilde{y}_n)$ , the progression of non-zero separate solution to (1) are obtained as

$$x_{n+1} = \frac{1}{2} [18.9^k fn + 9.9^k \sqrt{5}gn]$$
<sup>(2)</sup>

$$y_{n+1} = \frac{1}{2\sqrt{5}} \left[9.9^k \, gn + 18.9^k \, \sqrt{5} \, fn\right] \tag{3}$$

The recurrence relationship fulfilled by the solutions of (1) are specified by

$$x_{n+3} - 18x_{n+2} + x_{n+1} = 0$$
  

$$y_{n+3} - 18y_{n+2} + y_{n+1} = 0.$$
  
Choice 2:  $t = 2k+1, k > 0$   
Pell equation is  
 $= 5y^2 - 9^{2k+1}, k > 0$ 
(4)

Let  $(x_0, y_0)$  be the initial solution of (4) given by

$$x_0 = 6.9^k$$
;  $y_0 = 3.9^k$ 

 $\frac{2}{\text{The}}$  $x^2$ 

Applying Brahma Gupta Lemma Connecting  $(x_0, y_0)$  and  $(\tilde{x}_n, \tilde{y}_n)$ , the progression of non-zero dissimilar integer way out to (4) is obtained as

$$x_{n+1} = \frac{1}{2} [6.9^k fn + 3.9^k \sqrt{5} gn]$$
(5)

$$y_{n+1} = \frac{1}{2\sqrt{5}} [3.9^k gn + 6.9^k \sqrt{5} fn]$$
(6)

The recurrence relationships fulfilled employing the solutions of (4) are convinced utilizing

$$x_{n+3} - 18x_{n+2} + x_{n+1} = 0$$
  
$$y_{n+3} - 18y_{n+2} + y_{n+1} = 0.$$

#### **IV. CONCLUSION**

In this paper, we have presented infinitely many integer solutions for the represented by the negative Pell equation  $x^2 = 5y^2 - 9^t$ . As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of negative Pell equations and determine their integer solutions along with suitable properties.

#### REFERENCES

- [1] Carmichael R.D, History of Theory of numbers and Diophantine Analysis, Dover Publication, New York, 1959.
- [2] Nagell T, Introduction to Number Theory, Chelsea publishing company, New York, 1982.
- [3] Mordell L.J, Diophantine equations, Academic press, London, 1969.
- [4] Janaki G, Vidhya S (2016), On the integer solutions of the Pell equation  $x^2 79y^2 = 9^k$ , International Journal Of Scientific Research in Science, Engineering and Technology, 2(2), 1195-1197.
- [5] Janaki G, Vidhya S (2016), On the integer solutions of the Pell equation  $x^2 = 20y^2 4^t$ , International Journal Multidisciplinary Research and Development, 3(5), 39-42.
- [6] Janaki G, Vidhya S (2016), On the negative Pell equation  $y^2 = 21x^2 3$ , International Journal Of Applied Research, 2(11), 462-466.



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- [7] Vidhya S, Janaki G (2017), Observation on  $y^2 = 6x^2 + 1$ , International Journal Of Statistics and Applied Mathematics, 2(3), 04-05.
- [8] Vidhya S, Janaki G (2018) An integral solution of negative Pell's equation involving two-digit sphenic numbers, International Journal Of Computer Sciences and Engineering, 6,444-445.
- [9] Vidhya S, Janaki G (2019), Observation on Remarkable Diophantine Equation, Compliance Engineering Journal, 10(12), 667-670.
- [10] Vidhya S, Janaki G (2019), Observation on  $y^2 = 11x^2 + 1$ , International Journal for science and Advance Research in Technology, 5(12), 232-233.
- [11] Janaki, G., & Vidhya, S. (2016). Rectangle with area as a special polygonal number. International Journal of Engineering Research, 4, 88-91.
- [12] Janaki, G., & Vidhya, S. Special pairs of rectangles and Sphenic number. International Journal for Research in Applied Science & Engineering Technology (IJRASET), 4, 376-378.











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