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An Integral Solution of Negative Pell Equation $x^2 = 5y^2 - 9^t$

S. Vidhya[1], B. Amala[2]
[1]Assistant Professor, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 18.
[2]PG Student, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 18.

***Abstract:** We look for non–trivial integer solution to the equation $x^2 = 5y^2 - 9^t, t \in N$ for the singular choices of particular by (i) t = 2k (ii) t = 2k+1, $\forall k \in N$. Additionally, recurrence relations on the solutions are obtained.*
***Keywords:** Negative Pell equations, Pell equations, Diophantine equations, Integer solutions.*

I. INTRODUCTION

It is well known the Pell equation $x^2 - Dy^2 = 1$ (D > 0 and square free) has at all times positive integer solutions. When N$\neq 1$, the Pell equation $x^2 - Dy^2 = -N$ possibly will not boast at all positive integer solutions. In favour of instance, the equations $x^2 = 3y^2 - 1$ and $x^2 = 7y^2 - 4$ comprise refusal integer solutions.

This manuscript concerns the negative Pell equation $x^2 = 5y^2 - 9^t$, where t > 0 and infinitely numerous positive integer solutions are obtained for the choices of t known by (i) *t = 2k* (ii) *t = 2k+1*. Supplementary recurrence relationships on the solutions are consequent.

II. PRELIMINARY

The Pell equation is a Diophantine equation of the form $x^2 - dy^2 = 1$. Given *d*, we would like to find all integer pairs (*x, y*) that satisfy the equation. Since any solution (*x, y*) yields multiple solutions ($\pm x, \pm y$), we may restrict our attention to solutions where *x* and *y* nonnegative integer. We usually take *d* in the equation $x^2 - dy^2 = 1$ to be a positive non square integer. Otherwise, there are only uninteresting solutions: if *d < 0*, then (*x, y*) = ($\pm$ 1, 0) in the case *d<-1*, and (*x, y*)=(0, ± 1) or ($\pm$ 1, 0) in the case *d = -1*; if *d = 0*, then *x* = $\pm$ *1* (*y* arbitrary); and if a nonzero square, then dy^2 and x^2 are consecutive squares, implying that (*x, y*)=($\pm$ 1, 0). Notice that the Pell equation always has trivial solution (*x, y*) = (1, 0). We now investigate an illustrate case of Pell's equation and its solution involving recurrence relations.

Let p be a prime. The negative Pell's equation $x^2 - py^2 = -1$ is solvable if and only if *p = 2* (or) p$\equiv 1$ (mod 4).

III. METHOD OF ANALYSIS

Consider the negative Pell equation $x^2 = 5y^2 - 9^t, t \in N$

1) Choice 1: t = 2k, k > 0

The Pell equation is

$$x^2 = 5y^2 - 9^{2k}, k > 0 \quad (1)$$

Let (x_0, y_0) be the initial solution of (1) specified by

$$x_0 = 18.9^k ; y_0 = 9.9^k$$

Applying Brahma Gupta Lemma Connecting (x_0, y_0) and ($\tilde{x}_n, \tilde{y}_n$), the progression of non-zero separate solution to (1) are obtained as

$$x_{n+1} = \frac{1}{2}[18.9^k fn + 9.9^k \sqrt{5} gn] \quad (2)$$

$$y_{n+1} = \frac{1}{2\sqrt{5}}[9.9^k gn + 18.9^k \sqrt{5} fn] \quad (3)$$

The recurrence relationship fulfilled by the solutions of (1) are specified by

$$x_{n+3} - 18x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 18y_{n+2} + y_{n+1} = 0.$$

2) *Choice 2: t = 2k+1*, k > 0

The Pell equation is

$$x^2 = 5y^2 - 9^{2k+1}, k > 0 \quad (4)$$

Let (x_0, y_0) be the initial solution of (4) given by

$$x_0 = 6.9^k ; y_0 = 3.9^k$$

Applying Brahma Gupta Lemma Connecting (x_0, y_0) and ($\tilde{x}_n, \tilde{y}_n$), the progression of non-zero dissimilar integer way out to (4) is obtained as

$$x_{n+1} = \frac{1}{2}[6.9^k fn + 3.9^k \sqrt{5} gn] \quad (5)$$

$$y_{n+1} = \frac{1}{2\sqrt{5}}[3.9^k gn + 6.9^k \sqrt{5} fn] \quad (6)$$

The recurrence relationships fulfilled employing the solutions of (4) are convinced utilizing

$$x_{n+3} - 18x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 18y_{n+2} + y_{n+1} = 0.$$

IV. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the represented by the negative Pell equation $x^2 = 5y^2 - 9^t$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of negative Pell equations and determine their integer solutions along with suitable properties.

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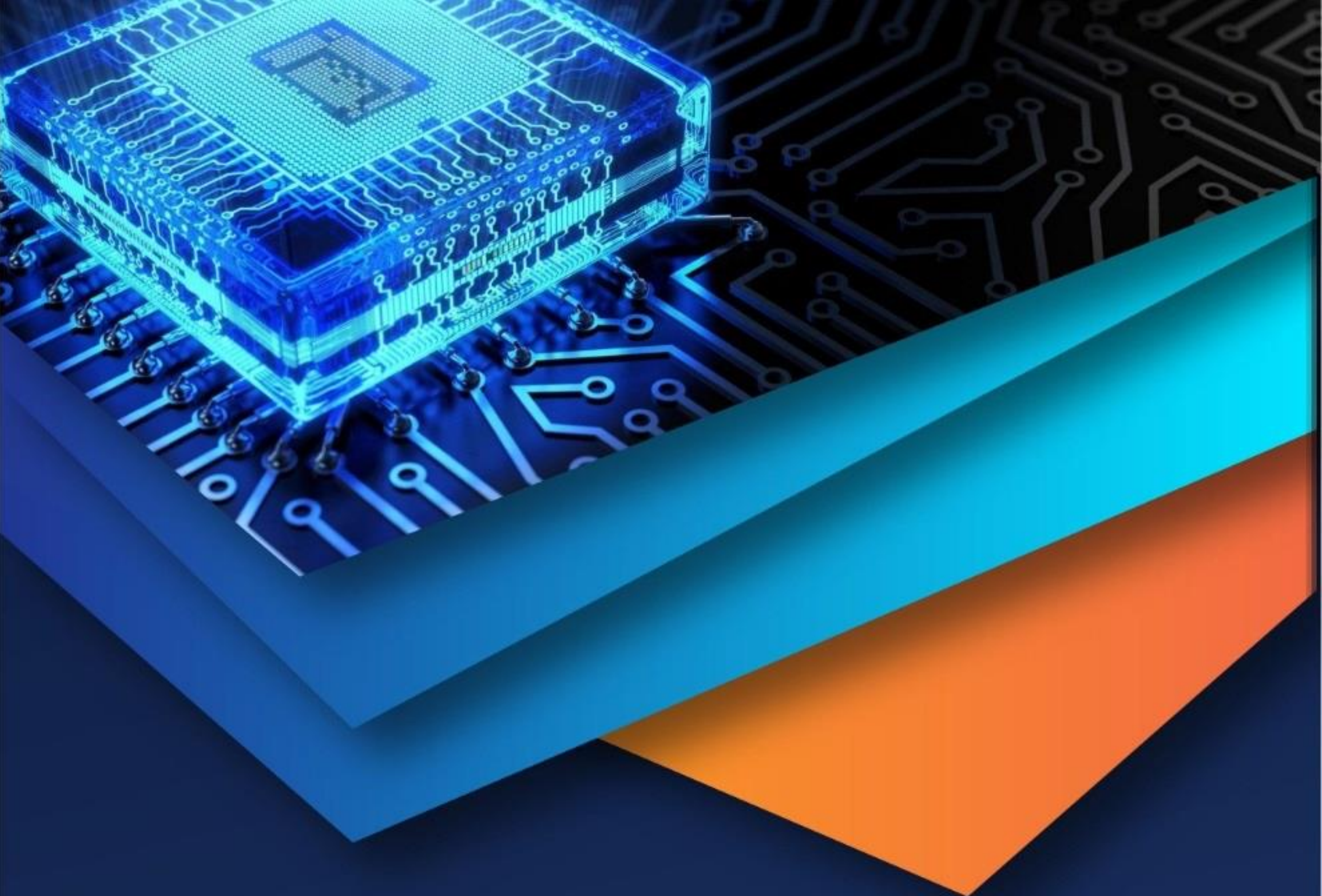

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