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An Overview on Intuitionistic Fuzzy Multi-Hypergroup

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Abstract: The major goal of this work is to introduce the Intuitionistic Fuzzy Multi-Hypergroup and to introduce non membership on multihyper group to provide new path way in the field of hypergroup.
Keywords: Multiset, Fuzzy multiset, Intuitionistic fuzzy multiset, Intuitionistic fuzzy multi-Hypergroup

I. INTRODUCTION

In 1934, Frederic Marty defined a hypergroup as a natural extension of a group. Composing two elements in a group results in an element, but in a hypergroup, the result is a non-empty set. The law characterizing suchhyperstructure theory refers to algebraic structures that have at least one multi-valued operation.
Marty's motivation for introducing hypergroups is that the quotient of a group modulo any subgroup (not necessarily normal) is a hypergroup.
Since the 1970s, there has been significant progress in hyperstructure theory, including the introduction of new structures such as hyperrings, hypermodules, hyperlattices, and hyperfields. There are several forms of hyperstructures are employed in various contexts, including automata theory, topology, cryptography, geometry, graphs and hypergraphs, convex system analysis, finite group character theory, fuzzy and rough set theory, probability theory, ethnology, and the economy.
Zadeh proposed a fuzzy set, which is comparable to a set but with degrees of membership to the generation of fuzzy set. Intuitionistic Fuzzy Multi-Hypergroup is a generalization of Fuzzy Multi-Hypergroup.
Intuitionistic Fuzzy Multi-Hypergroup can be further extended in the field of Plus weighted grammar[3,4,5]. Intuitionistic Fuzzy Multi-Hypergroup can be linked with fuzzy hyper graph in the field of medical diagnosis[6].
Intuitionistic Fuzzy Multi-Hypergroup have many scope in the field of graph labeling and Coloring[2, 7,8,9,10].
Section 1, gives the general notion of Fuzzy multi hyper group. Some basic concepts which are needed for the succeeding sections are discussed in Section 2. Intuitionistic Fuzzy Multi-Hypergroup is introduced in Section 3 and related theorems are proved.

II. PRELIMINARIES

In this section some basic ideas which are needed for the succeeding sections are discussed.

1) *Definition2.1:* Let X be a nonempty set. A fuzzy set F drawn from X is defined as $F=\{\langle a,\mu_F(a)\rangle : a\in X\}$, where $\mu_F(a): X\to[0,1]$ is the membership function of the fuzzy set F.
2) *Definition2.2:* Let X be a nonempty set. An intuitionistic fuzzy set F in X is an object having the form $F=\{\langle a,\mu_F(a),\nu_F(a)\rangle : a\in X\}$, where the functions $\mu_F(a),\nu_F(a): X\to[0,1]$ define respectively, the degree of membership and degree of non-membership of the element $a\in X$ to the set F, which is a subset of X, and for every element $a\in X, 0\le \mu_F(a)+\nu_F(a)\le 1$.
3) *Definition2.4:* A multiset M drawn from the set X is represented by a count function $CTM : X\to N$, where N represents the set of non-negative integers. $CTM(a)$ is the number of occurrence of the element a in the multiset M. The multiset M drawn from $X=\{a_1,a_2,...,a_n\}$ will be represented by $M=\left\{{a_1}/{m_1},{a_2}/{m_2},...,{a_n}/{m_n}\right\}$ where m_i is the number of occurrence of the element $a_i,(i=1,2,...,n)$ in the multiset M.

4) *Definition2.5:* Let X be a nonempty set. A fuzzy multiset F drawn from X is characterized by a "count membership" function of F denoted by CTM_F such that $CTM_F : X \to Q$ where Q is the set of all crisp multisets drawn from the unit interval $[0,1]$.

5) *Definition2.6:* LetX be a nonempty set. An Intuitionistic Fuzzy Multiset F denoted by IFMS drawn from X is characterized by two functions: 'count membership' of $F(CTN_F)$ and 'count non membership' of $F(CTN_F)$ given respectively by $CTM_F : X \to Q$ and $CTN_F : X \to Q$ where Q is the set of all crisp multisets drawn from the unit interval $[0,1]$ such that for each $a \in X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CTM_F(a)$ which is denoted by $(\mu_F^1(a), \mu_F^2(a),.., \mu_F^p(a))$ where $(\mu_F^1(a) \geq \mu_F^2(a) \geq,...,\geq, \mu_F^p(a))$ and the corresponding non membership sequence will be denoted by $(v_F^1(a), v_F^2(a),..., v_F^p(a))$ such that $0 \leq \mu_F^i(a) + v_F^i(a) \leq 1$ for every $a \in X$ and $i = 1,2,..., p$.

An IFMS F is denoted by $F = \left\{ \begin{array}{ll} < a : & (\mu_F^1(a), \mu_F^2(a),..., \mu_F^p(a)), \\ & (v_F^1(a), v_F^2(a),..., v_F^p(a)) >: a \in X \end{array} \right\}$

III. INTUITIONISTIC FUZZY MULTI-HYPERGROUP

1) *Definition3.1:* Let $(H, \circ)$ be any hypergroup and $c \in H$ be a fixed element. We define an intuitionistic fuzzy multiset F of H with intuitionistic fuzzy count function CTM_F and CTN_F as $CTM_F(a) = CTM_F(c)$ and $CTN_F(a) = CTN_F(c)$ for all $a \in H$ .Then F is an intuitionistic fuzzy multi-hypergroup of H (the constant intuitionistic fuzzy multi-hypergroup).

2) *Theorem 3.2:* Let $(H, \circ)$ be a hypergroup and F be an intuitionistic fuzzy multi-hypergroup of H .Then the following assertions are true.

1. $CTM_F(r) \geq CTM_F(a_1) \wedge ... \wedge CTM_F(a_n)$, for all $r \in a_1 \circ ... \circ a_n$ and $n \geq 2$
2. $CTM_F(r) \geq CTM_F(a)$, for all $r \in a^n$
3. $CTN_F(r) \leq CTN_F(a_1) \vee ... \vee CTN_F(a_n)$, for all $r \in a_1 \circ ... \circ a_n$ and $n \geq 2$
4. $CTN_F(r) \leq CTN_F(a)$, for all $r \in a^n$

3) *Theorem 3.3:* Let $(H, \circ)$ be the bisethypergroup and F be an intuitionistic fuzzy multiset of H. Then F is an intuitionistic fuzzy multi-hypergroup of H.

Proof: Let F be an intuitionistic fuzzy multiset of H. Since $(H, \circ)$ is a commutative hypergroup.

We know that, $\begin{array}{l} CTM_F(a) \wedge CTM_F(b) \leq \inf_{r \in a \circ b} CTM_F(r) \\ (or\ equivalently, CTM_F(a) \wedge CTM_F(b) \leq CTM_F(r)\ for\ all\ r \in a \circ b) \end{array}$

Let $a, b \in H$ and $r \in a \circ b = \{a, b\}$.

Since, $CTM_F(r) \geq CTM_F(a) \wedge CTM_F(b)$

We know that, $\begin{array}{l} CTN_F(a) \vee CTN_F(b) \geq \sup_{r \in a \circ b} CTN_F(r) \\ (or\ equivalently, CTN_F(a) \vee CTN_F(b) \geq CTN_F(r)\ for\ all\ r \in a \circ b) \end{array}$

Since, $CTN_F(r) \leq CTN_F(a) \vee CTN_F(b)$

For every $c, a \in H$ there exists $b \in H$ such that $a \in c \circ b$ and

$CTM_F(a) \wedge CTM_F(c) \leq CTM_F(b)$ and $CTN_F(a) \vee CTN_F(c) \geq CTN_F(b)$

Let $c, a \in H$ .Then there exists $b = a \in H$ such that $a \in c \circ b$ and

$CTM_F(b) = CTM_F(a) \geq CTM_F(c) \wedge CTM_F(a)$ and $CTN_F(b) = CTN_F(a) \leq CTN_F(c) \vee CTN_F(a)$

Therefore, F is a intuitionistic fuzzy multi hypergroupof H .

4) *Theorem 3.4:* Let $(H, \circ)$ be the total hypergroup and F be an intuitionistic fuzzy multisetof H . Then F is an intuitionistic fuzzy multi hypergroup of H if and only if F is an intuitionistic fuzzy multiset described the above definition.

Proof: If F is an intuitionistic fuzzy multiset.A fuzzy multiset F of H with intuitionistic fuzzy count function CTM_F and CTN_F as $CTM_F(a) = CTM_F(c)$ and $CTN_F(a) = CTN_F(c)$ for all $a \in H$ .Thus, F is an intuitionistic fuzzy multi-hypergroupof H .

Let F be anintuitionistic fuzzy multi-hypergroup of H and $c \in H$. For all $a \in H$, we have $a \in c \circ c = H$ and $c \in a \circ a$ .The latter and having F anintuitionistic fuzzy multi-hypergroup of H implies that $CTM_F(a) \geq CTM_F(c)$ and $CTM_F(c) \geq CTM_F(a)$

$CTN_F(a) \leq CTN_F(c)$ and $CTN_F(c) \leq CTN_F(a)$.

Thus, $CTM_F(a) = CTM_F(c)$ and $CTN_F(a) = CTN_F(c)$ for all $a \in H$.

Therefore, F is an intuitionistic fuzzy multiset. Let $(H, \circ)$ be a hypergroup, F be a intuitionistic fuzzy multiset H and CTM is counting membershipdenoted by

1. $CTM_F(a) = (\mu_F^1(a), \mu_F^2(a), \ldots, \mu_F^p(a))$. We say that $CTM_F(a) > 0$,if $\mu_F^1(a) > 0$.

CTN is counting non-membershipdenoted by 2. $CTN_F(a) = (\nu_F^1(a), \nu_F^2(a), \ldots, \nu_F^p(a))$. We say that $CTN_F(a) < 0$,if $\nu_F^1(a) < 0$.

5) *Definition3.5:* Let $(H, \circ)$ be a hypergroup and F be an intuitionistic fuzzy multiset of H . Then $F_* = \{a \in X : CTM_F(a) > 0\}$ and $F_* = \{a \in X : CTN_F(a) < 0\}$.

6) *Theorem 3.6:* Let $(H, \circ)$ be a hypergroup and F be an intuitionistic fuzzy multi hypergroup of H . Then F_* is either the empty set or a subhypergroupof H .

Proof: Let $c \in F_* \neq \varnothing$. We need to show that the reproduction axiom is satisfied for F_* .

We show that $c \circ F_* = F_*$ and $F_* \circ c = F_*$ .For all $a \in F_*$ and $r \in c \circ a$, we have

$CTM_F(r) \geq CTM_F(c) \wedge CTM_F(a) > 0$ and $CTN_F(r) \leq CTN_F(c) \vee CTN_F(a) < 0$.

The latter implies that $r \in F_*$ and hence, $F_* \circ c \subseteq F_*$.

Moreover, for all $a \in F_*$, For every $a, c \in H$ there exists $b \in H$ such that $a \in c \circ b$ and $CTM_F(a) \wedge CTM_F(c) \leq CTM_F(b) \Rightarrow CTM_F(b) \geq CTM_F(a) \wedge CTM_F(c) > 0$.

and $CTN_F(a) \vee CTN_F(c) \geq CTN_F(b) \Rightarrow CTN_F(b) \leq CTN_F(a) \vee CTN_F(c) < 0$.

The latter implies that $b \in F_*$ and $a \in c \circ F_*$. The latter implies that $r \in F_*$ and hence, $F_* \circ c \subseteq F_*$.

Thus, $F_* \subseteq c \circ F_*$.

7) *Theorem 3.7:* Let $(H, \circ)$ be a hypergroup and F be a intuitionistic fuzzy multiset of H . If F is an intuitionistic fuzzy multi-hypergroup of H .Then $F \circ F = F$.

Proof: Let $r \in H$ .Then $CTM_F(r) \geq CTM_F(a) \wedge CTM_F(b)$ for all $r \in a \circ b$.

$CTN_F(r) \geq CTN_F(a) \wedge CTN_F(b)$, for all $r \in a \circ b$

The latter implies that $CTM_F(r) \geq \vee\{CTM_F(a) \wedge CTM_G(b) : r \in a \circ b\} \geq CTM_{F \circ F}(r)$ and

$CTN_F(r) \geq \wedge\{CTN_F(a) \vee CTN_G(b) : r \in a \circ b\} \geq CTN_{F \circ F}(r)$

Thus $F \circ F \subseteq F$.

Having $(H, \circ)$ a hypergroup and F a fuzzy multi-hypergroup of H implies that for every $a \in H$ there exist $b \in H$ such that $a \in a \circ b$ and

$CTM_F(b) \geq CTM_F(a)$ and $CTN_F(b) \leq CTN_F(a)$.

Moreover, we have

$CTM_{F \circ F}(a) \geq \vee\{CTM_F(b) \wedge CTM_G(r) : a \in b \circ r\} \geq CTM_F(a) \wedge CTM_F(b) = CTM_F(a)$

$and\; CTN_{F \circ F}(a) = \wedge\{CTN_F(b) \vee CTN_G(r) : a \in b \circ r\} \geq CTN_F(a) \vee CTN_F(b) = CTN_F(a)$

Thus, $F \subseteq F \circ F$.

IV. CONCLUSION

Intuitionistic fuzzy multi hypergroup has been introduced and it is proved that if $(H, \circ)$ is a hypergroup and F be a intuitionistic fuzzy multiset of H, if F is an intuitionistic fuzzy multi-hypergroup of H ,then $F \circ F = F$. This work can be further extended in the field of Plus weighted grammar and Plus weighted automata which will give fruitful result in the field of automata theory.

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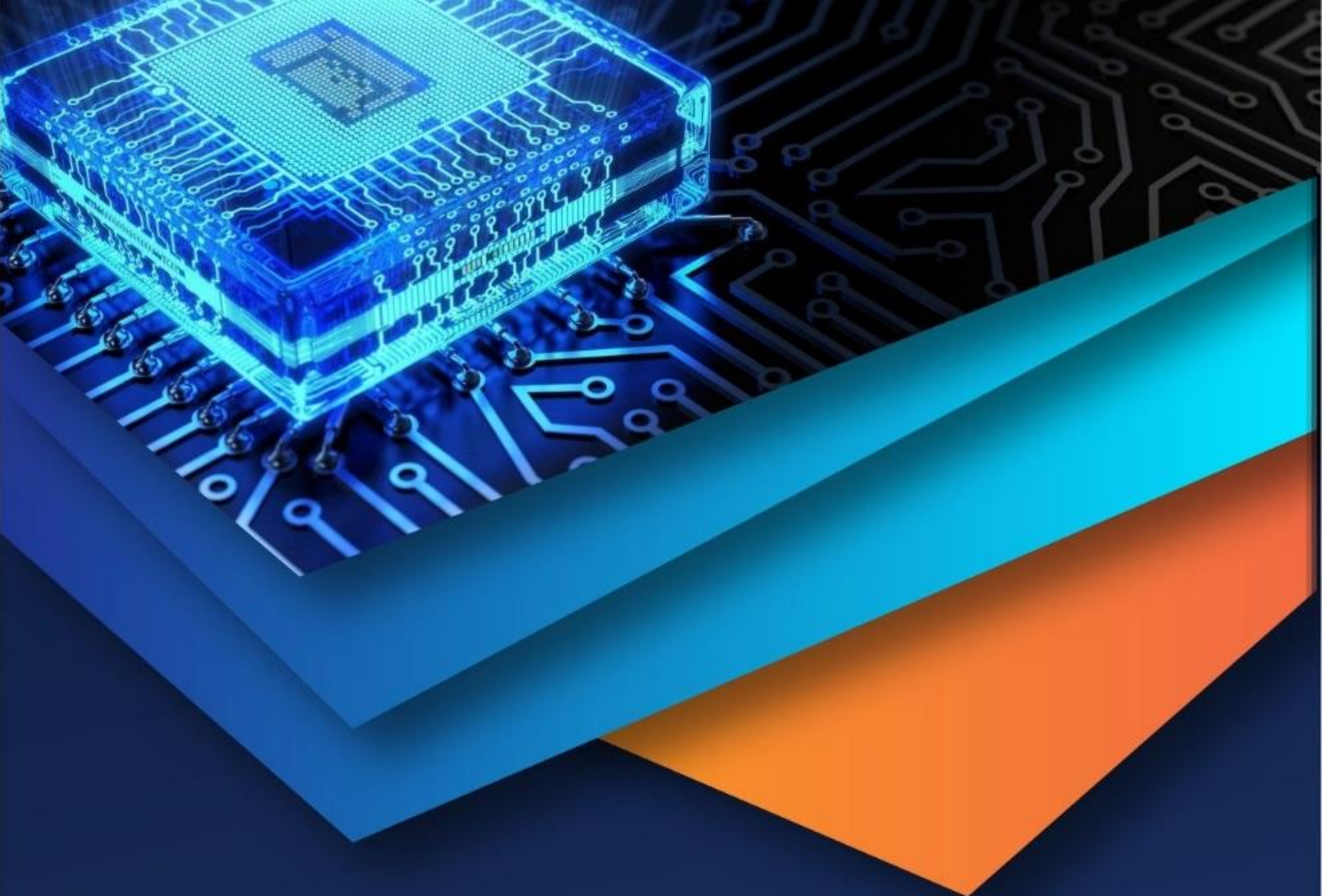

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