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Contact Stresses and Dynamic Analysis of involute Spur Gear with FEM Approach

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Abstract: This paper investigates the characteristics of a gear system including contact stresses, bending stresses, and the transmission errors of gears in mesh. The objective of this paper is to compare values of contact stress and dynamic analysis obtained by theoretical hertz equation with the ANSYS result. A two stage spur gear box has been designed for material handling application by manual calculation and then performs contact stress and dynamic Simulation to ensure its reliable working. The results of the two-dimensional Finite Element Method (FEM) analysis from ANSYS and theoretical results are well comparable.
Keywords: involute, spur gear, contact stress, dynamic analysis, finite element analysis

NOMENCLATURE

F = load per unit width.

R_i = radius of cylinder,

R_i di sin $\varphi/2$ for the gear teeth,

Φ= pressure angle

v_i=Poisson's ratio

E_i=Young's modulus of elasticity

FEA= Finite element analysis

I. INTRODUCTION

In this technological world there is a challenge for better design and optimization of work in every field of technology. As gears have a wide application many scientists and researchers had done significant work on various aspects of gearing. In this section we are going to discuss about various literatures which deals with numerous model, improving performance approach, optimal design, durability, major concerns in transmission error, stress analysis, gear noise and vibration, dynamic loading etc.

Harris, S. L. (1958), discussed about the transmission errors occurred during the meshing of gear tooth. The transmission error and photo-elastic gear models were used. The errors during manufacturing, tooth stiffness inequality (dissimilarity) and non-linearity in tooth stiffness were determined as all these three are internal sources of vibration due to contact loss between gear teeth. It revealed that the behavior of spur gears at low speeds can be summarized in a set of static transmission error curves. With the help of pitch errors and tooth profile errors, the vibration characteristics including stiffness variation of teeth mesh can be obtained.

Sushil, K. T., Upendra K. J. (2012), determines the contact stress and bending stress of loading and rotating involute spur gear teeth while meshing. For bending stress a Lewis formula is applied and for contact stress a Hertz equation is used. This paper provides the detailed study of gear stress analysis. The design of an effective and reliable gearing system is include its ability to withstand RBS (Root Bending Stress) and SCS (Surface Contact Stress). The RBS and SCS calculation is calculated for mating involute spur gear teeth for this purpose he uses FEA method and 3D gear design. The transmission error is defined in CAD system as well as involute spur gear without tooth modification in CAD (CATIA V5 and AUTODESK INVENTOR etc.) and FEA is done by using finite element software ANSYS. Obtained FEA results is comparable with theoretical and AGMA standard. In this work, for the calculation of gear stress he uses hertz equation as well as Lewis method and for detailed analysis of involute spur gear stress he uses FEM along with AGMA standards.

Jwan Khalil Mohammed, Younis Khalid Khdir, Safeen Yaseen Kasab, (2019) "Contact

Stress Analysis of Spur Gear Under the Different Rotational Speed by Theoretical and

Finite Element Method".In this thesis the contact stress whichproduced in spur gear is introducedunder the effect of rotational speed. With the help of ANSYS software the dynamic analysis of 3-dimensional modeled and designed gear has been done.Firstly the theoretical values of contact stress of spur gear are calculated with the help of hertz equation and after that use FEM analysis. The study of contact stress effect is done on the basis of different values of rotational speed. The comparison of the methods was done by determining error percent of contact stress and modelling of spur gear which is carried out using ANSYS and solid works respectively.

A. *Problem Formulation*
1) Excessive Noise of gear pair while shifting of gear.
2) Excessive vibration.
3) Stress generated due to incorrect meshing.

B. *Objective*
1) To solve the contact stresses and dynamic analysis by FEM approach.
2) To reduce the excessive noise and vibration of gear pair.
3) To satisfy the conditions of correct meshing.

C. *Research Gap*
1) As per study I have seen many researchers has been done the substantial amount of work before, in the improvement of gear design, to reduce contact stresses ,to find out correct meshing of gears ,improvement in gear speed. Much of the work was carried out in past and in earlier years for the design improvement, material selection, compatibility.
2) In this research I have done the comparison analysis on contact stresses by FEA method and HERTZ method along with the dynamic analysis of gear.

D. *Material Properties*
1) *Grey Cast Iron*

Proerties	Value
Density	7100 kg/m3
Modulus of elasticity	165 Gpa
Tensile strength	350 Mpa
Yield strength	250 Mpa
Hardness	97 RB

2) *Steel*

Density	7.872* 10^3 kg/m³
Modulus of elasticity	201 GPa
Specific heat capacity	486 J/(kg*K)
Tensile strength	565 MPa
Yield strength	310 MPa
Hardness	84 RB

II. EXPERIMENTAL SETUP AND METHODOLOGY

A. Hertz Contact Stress Equations

Generally the engineering problems being calculated using hertz equation which is derived for the contact between two cylindrical bodies. With the help of this equation we are able to find out the maximum contact stress, shear stress, von mises stress between two mating gears.
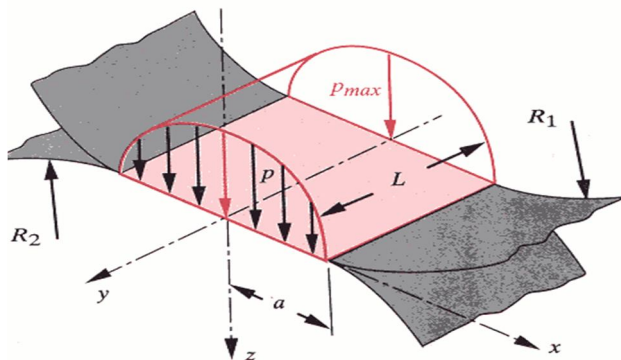
$$p_{max}= \sigma_H = 0.564 \frac{\sqrt{F\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}}{\sqrt{\left(\frac{1-v_1^2}{E_1}+\frac{1-v_2^2}{E_2}\right)}}$$

The maximum valuers are calculated with the help of maximum surface stress(σ_H) are

$$\sigma_{Von\ Mises} = 0.57\ \sigma_H$$

$$\sigma_{Max\ Shear} = 0.30\ \sigma_H$$

$$\sigma_{Ortho\ Shear} = 0.25\ \sigma_H$$

B. Theoretical Calculatios

Spur gear box is used in material handling equipment for transfer the material from ground level to top level.

Input Power (P) = 3 Kw

Motor rpm (N1) = 1500

Required Output rpm = 60

Velocity Ratio = Input rpm / output rpm

= 1500 / 60 = 25

This velocity ratio is too high for the single stage speed reduction because of on this bases the gear sizes become too large which increase the weight and overall size of the gear box and also increase the manufacturing cost of the gear box. So we are reduces the speed of motor in two stages for satisfy the coal conveyor requirement.

Single stage VR = $\sqrt{25}$
= 5.0

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C. *First stage Calculation of the Gear Box*

Standard Ratio from Design Data Book = 5.01

Deviation in Ratio = (Actual Ratio – Standard Ratio) / Standard ratio *100

$$= (5.01 - 5.00) / 5.01 * 100$$
$$= -0.1996 \%$$

By Design Data Book,

Total Number of Teeth (Z) = 85

Driver Gear Teeth (Z_1) = 14

Driven Gear Teeth (Z_2) = 71

Data assume from the Design Data book.

Normal Module (m) = 6

Pressure Angle(φ) = 20

Input Torque (T_1) = 60 * 10^6 * P / (2* π * (N1))

$$= 60 * 10^6 * 3 / (2 * 3.14 * (1500))$$
$$= 19108.2802 \text{ Nmm}$$

Pitch Circle Diameter of gear (d_1) = Z_1 * m

$$= 14 * 6$$
$$= 84 \text{ mm}$$

Pitch Circle Diameter of pinion (d_2) = Z_2 * m

$$= 71 * 6$$
$$= 426 \text{ mm}$$

Addendum Circle Diameter of Driver gear (d_{1a}) = d_1 + (2* m)

$$= 84 + (2*6)$$
$$= 96 \text{ mm}$$

Addendum Circle Diameter of Driven gear (d_{2a}) = d_2 + (2* m)

$$= 426 + (2*6)$$
$$= 438 \text{ mm}$$

Dedendum Circle Diameter of Driver gear (d_{1d}) = d_{1a} - (1.25* m)

$$= 84 - (1.25*6)$$
$$= 76.5 \text{ mm}$$

Dedendum Circle Diameter of Driven gear (d_{2d}) = d_{2a} - (1.25* m)

$$= 426 - (1.25*6)$$
$$= 418.5 \text{ mm}$$

Minimum Face Width Required = 8*m

$$= 8*6$$
$$= 48 \text{ mm}$$

Take Face Width (b) = 50 mm

Normal Circular Pitch (p) = π * m

$$= 3.14*6$$
$$p = 18.84 \text{ mm}$$

Pitch Line Velocity (v_1) = π * d_1*N_1/60

$$= 3.14 * 84 * 1500 / 60 * 1000$$
$$= 6.594 \text{ met/sec.}$$

D. *Force Calculation*

Force generated due to the application of the Spur Gear Box in coal handling plant for transfer the coal from ground level to furnace level, following loads are generated on the Spur Gear Box at Stage-1.

Tangential Force (F_{t1}) = 2*T_1/d_1

$$= 2*19108.28 / 84$$
$$= 454.96 \text{ N}$$

Radial Force (F_{r1}) = F_{t1} (tan φ)

$\qquad$ = 454.96 * (tan20)

$\qquad$ = 165.59 N

Velocity Factor (C_v) $\qquad$ = 5.6 / (5.6+sqrt(v))

$\qquad$ = 5.6 / (5.6+sqrt(6.594))

$\qquad$ = 0.68

Spur gear box is continuously worked in the coal handling plant with the medium shock load condition. On this bases the service condition for the spur gear box is 1.75 from the design data book.

$\qquad$ Service Factor (C_s) $\qquad$ = 1.75

$\qquad$ Effective Load (F_{eff}) $\qquad$ = C_s * F_{t1} / C_v

$\qquad$ = 1.75 * 454.96 / 0.68

$\qquad$ = 1161.2657 N

E. Calculation for Dynamic Load

Dynamic Load (F_{d1}) $\qquad$ $= \dfrac{21v(Ceb+Ft)}{21v+(Ceb+Ft)}$

Deformation factor (C) = 11400

Form Factor (Y) = 0.44

Take Grade-4 Type gear,

e= Sum of errors between two meshing gear

$\qquad$ For Driver Gear,

$\qquad$ e $\qquad$ = 3.20 + 0.25φ

$\qquad$ $φ_p$ $\qquad$ = m + 0.25*$\sqrt{d}$

$\qquad$ = 6 + 0.25*$\sqrt{84}$

$\qquad$ = 8.291

$\qquad$ e_p $\qquad$ = 3.20 + 0.25 $φ_p$

$\qquad$ = 3.20 +0.25*8.291

$\qquad$ = 5.272 micron

$\qquad$ For Driven Gear,

$\qquad$ $φ_g$ $\qquad$ = m + 0.25*$\sqrt{d}$

$\qquad$ = 6 + 0.25*$\sqrt{426}$

$\qquad$ = 11.16

$\qquad$ e_g $\qquad$ = 3.20 + 0.25 $φ_p$

$\qquad$ = 3.20 +0.25*11.16

$\qquad$ = 5.9899 micron

$\qquad$ e $\qquad$ = e_p + e_g

$\qquad$ = 0.011 mm

Dynamic Load (F_{d1}) $\qquad$ $= \dfrac{21v(Ceb+Ft)}{21v+(Ceb+Ft)}$

$$= \frac{21*6.594(11400*0.011*50+454.96)}{21*6.594+(11400*0.011*50++454.96)}$$

$\qquad$ = 4300.028 N

Effective Load Due to Dynamic Load (F_{eff})

$\qquad$ = $(C_sF_t + F_{d1})$

$\qquad$ = (1.75 * 454.96 + 4300.028)

$\qquad$ = 5096.20 N

Take Material for Cast Iron- Grade 35

Following are the properties,

Design Bending Stress $(\sigma_b) = 600$ Kg/cm^2

$= 60$ N/mm^2

BHN $= 230$

Beam Strength (S_b) $= m * b * \sigma_b * Y$

$= 6 * 50 * 60 * 0.44$

$= 7920$ N

This is less than Effective load.   So, the design of gear is Safe.

F. Calculation of Wear Strength

Ration Factor (Q) $= 2*Z_2 / (Z_1 + Z_2)$

$= 2*71 / (14+71)$

$= 1.67$

K $= 0.16 * (BHN/100)^2$

$= 0.16 * (230/100)^2$

$= 0.8464$

Wear Strength (S_w) $= bQd_1k$

$= 50*1.67*84*0.8464$

$= 5938.74$ N

This is less than Effective load. So, the design of gear is Safe.

G. Maximum Contact Stress

$$p_{max}= \sigma_H=0.564 \frac{\sqrt{F(\frac{1}{R_1}+\frac{1}{R_2})}}{\sqrt{(\frac{1-v_1^2}{E_1}+\frac{1-v_2^2}{E_2})}}$$

$=0.564*$sq root$(702.5(1/42+1/213))$

sq root$[2(1-0.26^2)]$

$1.65*10^5$

Pmax $= 1221.3$ Mpa

H. Modelling of Spur Gear

After designing gear by doing simple calculation, the same modeling has been performed on the Solid works 2015 version and then after, the investigation work has been performed on the ANSYS 14.0 version.
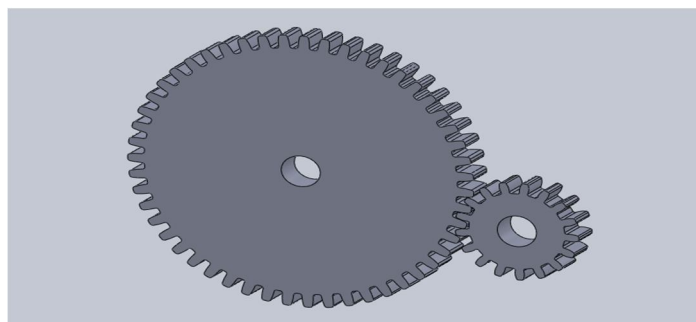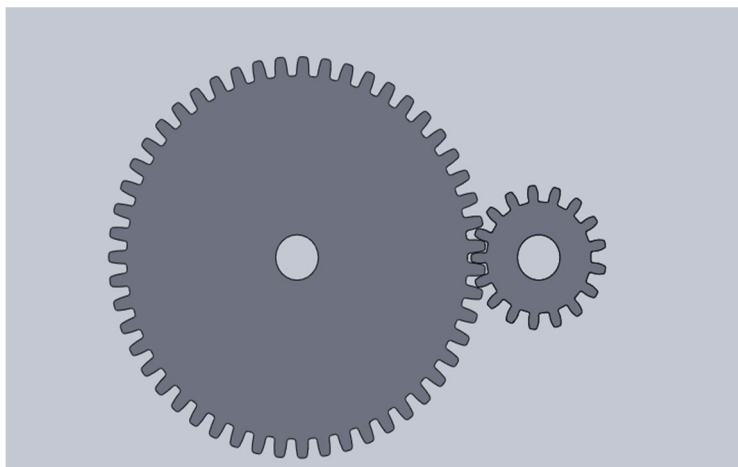
**Comparison of Finite Element With The Exact Solutions**
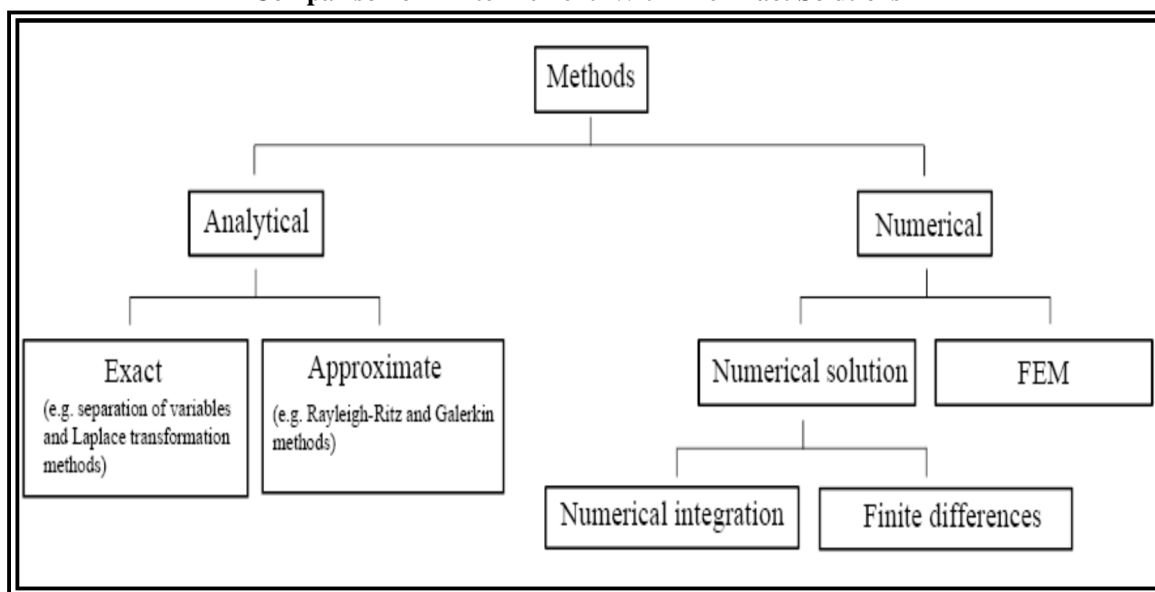
I. Basic Concept

The finite element method is the method of representation of structure or body by dividing the particular structure into subdivisions called finite elements which are shown in figure below. The corner and edge joining points called as nodes. After this choose the correct differential function as per requirement. The functions are expressed by polynomials. The equilibrium equation for each element are obtained by principle of minimum potential energy. Combine the equations of all the elements by which it will be formulated for entire body. Solve the differential equations by applying boundary conditions.

Following is the procedure of the finite element method:
1) Discretization: divide the body into many equivalent elements.
2) Formation of element stiffness matrix by choosing suitable displacement function.
3) Formation of global stiffness matrix.
4) Formation of global load vector.
5) Formation of global nodal displacement vector.
6) Assemblage of global nodal displacement load equation.
7) Incorporation of specified boundary condition.
8) Solution of simultaneous algebraic equation.
9) Computation of Elemental Stress, Strain.

III. PROCEDURE OF ANALYSIS

1) Firstly assembly is prepared in Solid works and Save as this assembly as .IGES for Exporting into ANSYS Workbench Environment. Import the .IGES file in ANSYS Workbench Simulation Module
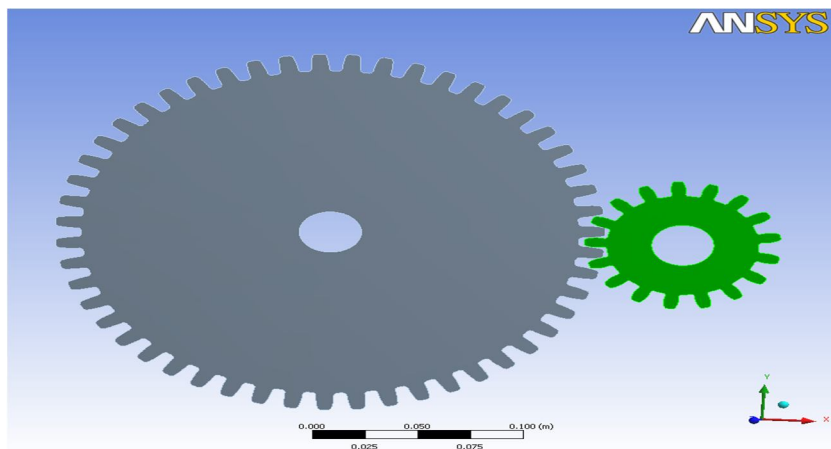
2) Check the Geometry for Meshing.
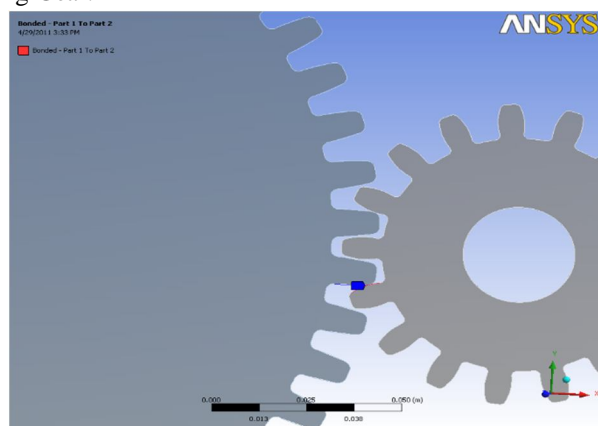
Apply Contact between Two Contacting Gear.

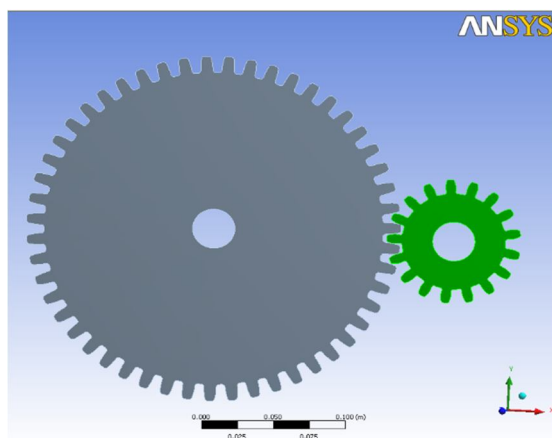Figure :- Defining Contact Between two Spur Gear Teeth

3) Apply Each Gear Material.

Figure 5.10 Apply Material for Pinion

4) Mesh the Contacting Gear.

Mesh Stastics:

Type of Element: Tetrahedrons

Number of Nodes   :   8632

Number of Elements: 7604
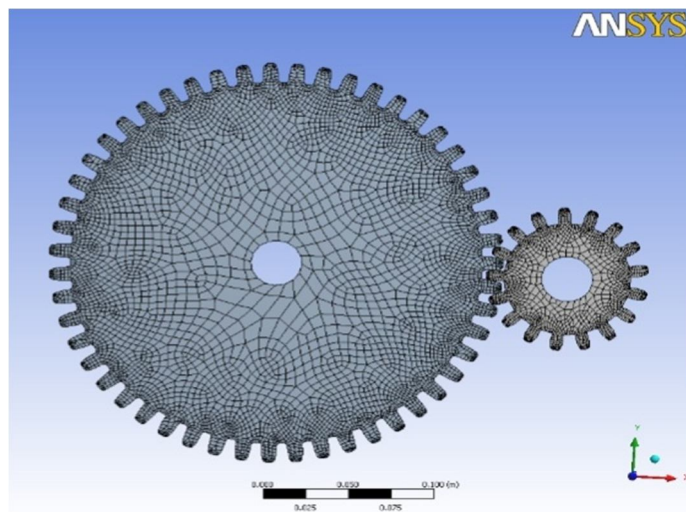
Figure : Meshed model Contacting Gear

5) Define type of Analysis

Type ofAnalysis :-Static Structural

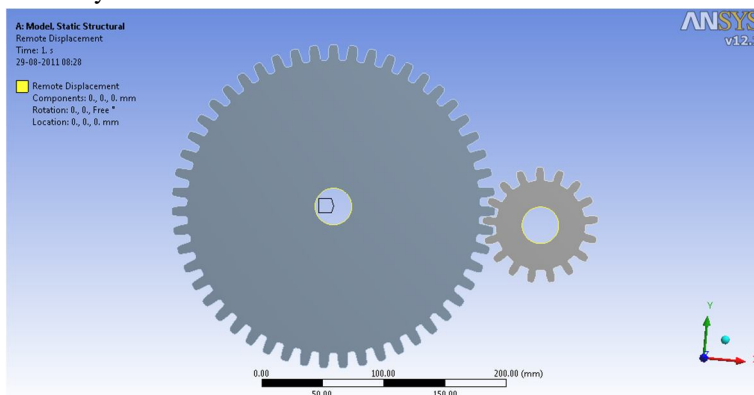Define Boundary Condition for Analysis

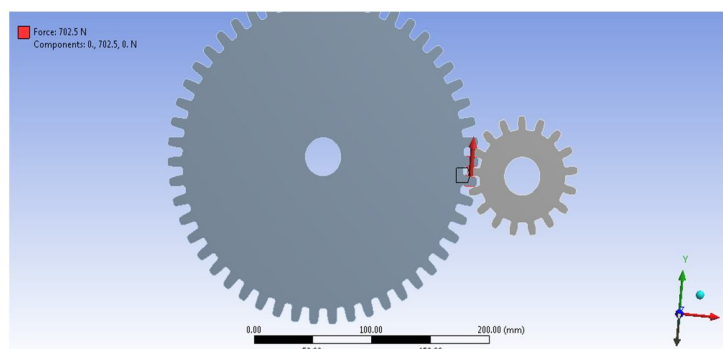Figure : Apply Remote Displacement for Bearing Support (having only one degree of freedom(Rotational))

Figure: Apply Force
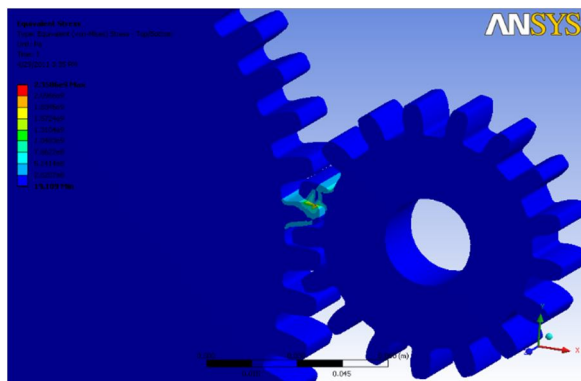
6) Run the Analysis
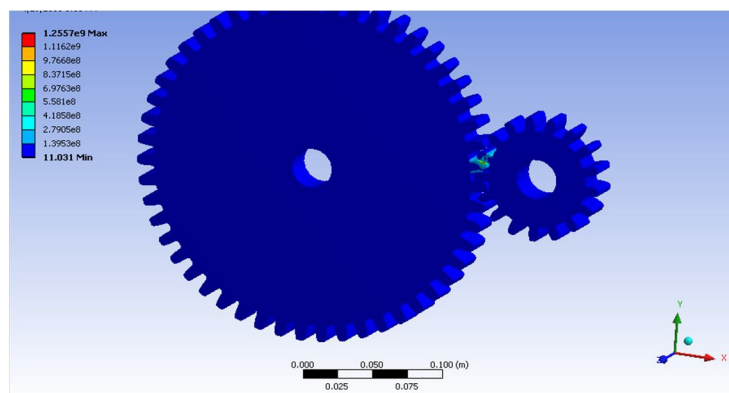7) Get the Results

Figure: Von misses stress

Figure :Maximum shear stress

A. *Dynamic Analysis of Contacting Spur Gear*
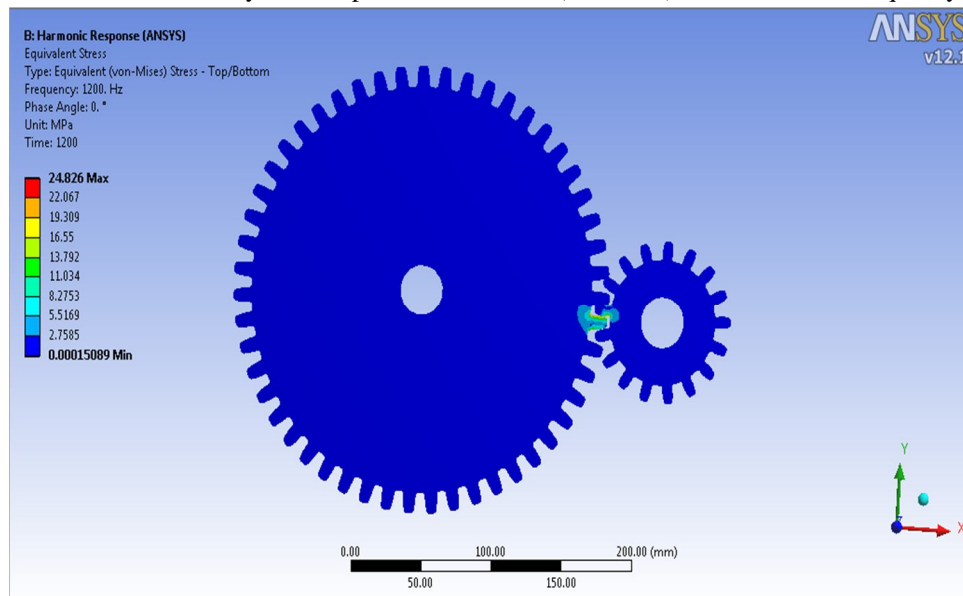This is the technique to determine the steady state response tosinusoidal (harmonic) loads of known frequency.
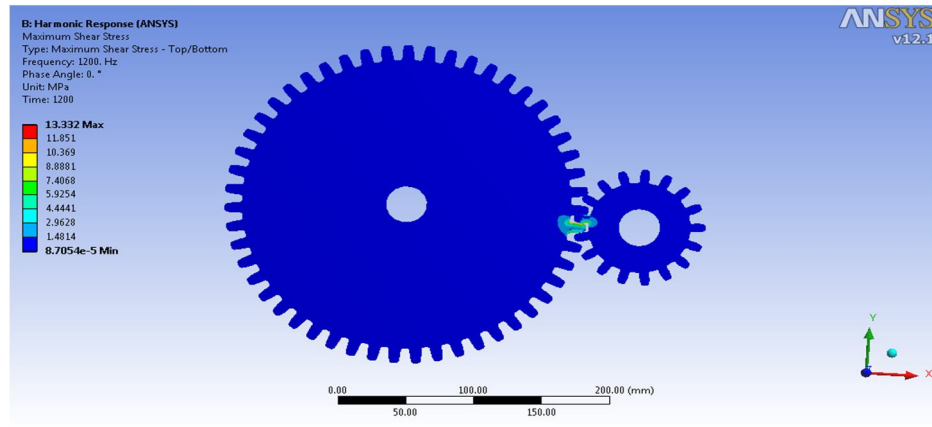

Figure :Von misses stress

Figure :Maximum shear stress
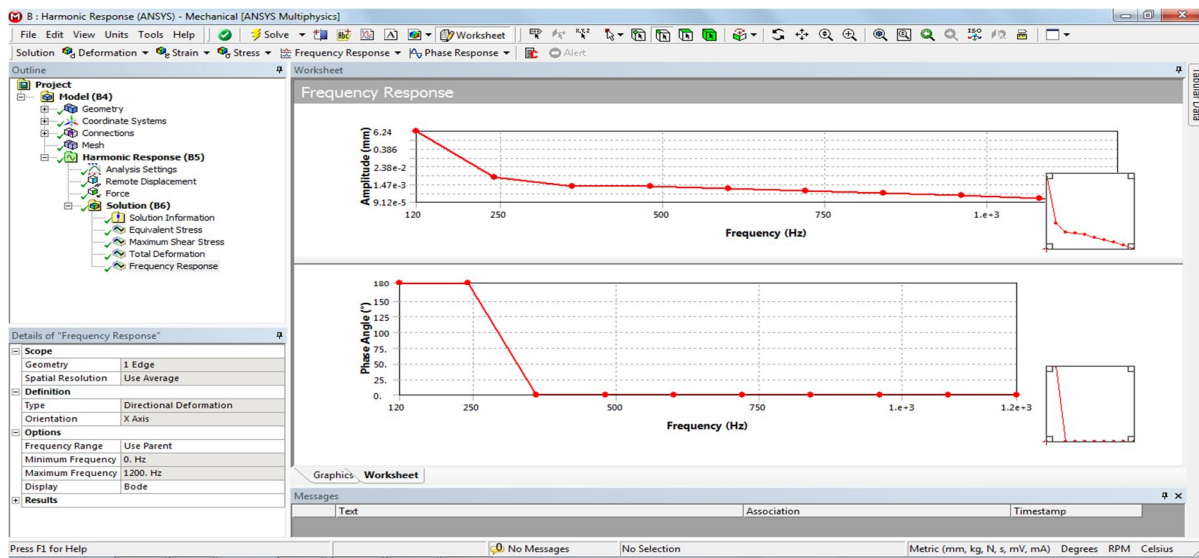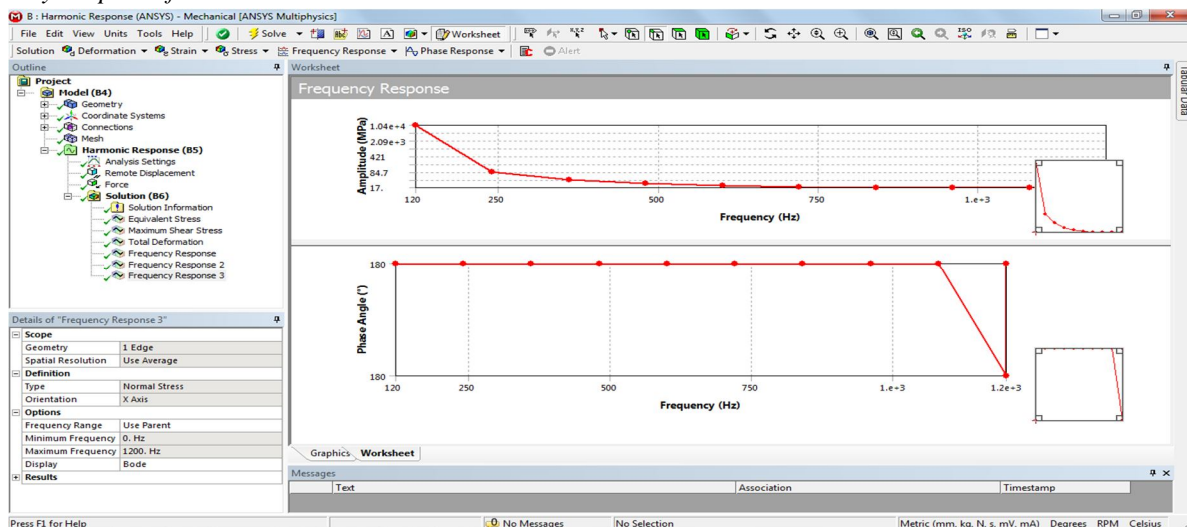
B. Frequency Response for Deformation

Figure :Frequency Vs Deformation
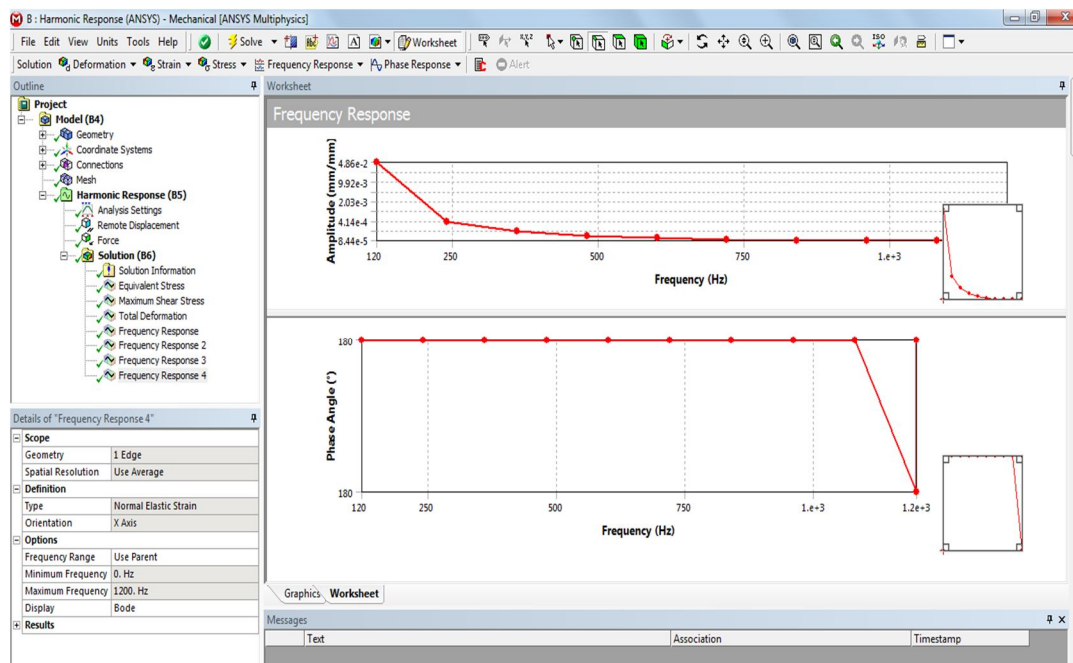
C. Frequency Response for stress

Figure : Frequency Response Vs Strain

IV. RESULTS AND DISCUSSION

Finite element modelling of the contact between two cylinders was examined in detail. The finite element method with special techniques, such as the incremental technique of applying the external load in the input file, the deformation of the stiffness matrix, and the introduction of the contact element were used.

The approximate results of the essential methods of the tooth contact stress using 2-Dimensional contact stress model and the root bending stresses using 2Dand3D FEA model should presented.

After static study we had Performed Dynamic analysis of the contacting Spur Gear which results shows more realistic whereas static analysis provides magnifying results.

It clearly shows that the Finite Element model can be used to replicate contact problems between two bodies specifically the contact stresses between two cylinders which are in contact and compare it with the theoretical value of Hertz equation

Contact stress values

Hertz equation	1221 .3 Mpa
FEA	1255.7 Mpa

Difference = 2.8%

Dynamic stress values

Equivalent stress von mises	24.826 Mpa
FEA	13.332 Mpa

A. *Future Scope*
1) By using modeling and simulation software's like PRO-E, ALTAIR-Hyper Mesh, ANSYS and others, gear models can be analyzed by simulating it under different operating conditions to obtain the desired results.
2) As FEM is easier in modelling of complex geometrical and irregular shapes and having good accuracy and adaptability so this will be used in automobile sectors and many more designs.

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