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Analytic Continuation, Functional Equations, and Zero-Free Regions in Local Riemann Analogues

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Abstract: *The investigation of local analogues to the Riemann Hypothesis provides profound insights into the analytic and algebraic structures underlying modern number theory. This paper examines the analytic properties of generalized local zeta functions and $\mathcal{L}$ -functions defined over complex fields, focusing on their behavior beyond the half-plane of absolute convergence. We establish rigorous frameworks for the analytic continuation of these functions to the entire complex plane $\mathbb{C}$, subsequently deriving their corresponding functional equations that establish symmetry between the functional values of S and $1-S$. By leveraging advanced techniques in classical complex analysis and contour integration, we construct explicit asymptotic bounds for the growth of these functions along the critical line $\text{Re}(S) = \frac{1}{2}$. Furthermore, we delineate newly defined zero-free regions approaching the boundary of the critical strip. These analytic extensions provide opening to refine the current understanding of local root distributions.*

Keywords: *Local Riemann Hypothesis, Analytic Continuation, Local Zeta Functions, Functional Equations, Zero-Free Regions, Analytic Number Theory*

I. INTRODUCTION

Riemann's hypothesis states that all non-trivial points where the Riemann's ζ -function ($S \in \mathbb{C}$ is defined with the analytic continuum $\sum_{n=1}^{\infty} \frac{1}{n^S}$, $\text{Re}(S) > 1$), is zero can be traced on the critical line $\text{Re}(S) \equiv u = \frac{1}{2}$. The line excludes the trivial zeroes (the points as, $S \in -2\mathbb{N}$). The Riemann ζ -function being initially defined on $\text{Re}(S) > 1$ and then extended to other values using analytic continuation. The function exhibits convergence only $\text{Re}(S) > 1$ and is called half-plane.

As is widely accepted, this leads to the insight that the distribution of prime numbers is related to a geometric aspect, as revealed by Euler's discovery of the infinite product of primes equating to the sum of reciprocal squares of natural numbers. When translated into the complex number domain $s = u + it$, the Riemann ζ -function produces remarkable outcomes. The condition is known as analytic continuation and is the key that unlocks the door to the entire complex plane for the Riemann ζ -function. Only an exception is a singular point at $S = 1$ (Sahu & Dafadar, 2025).

While the trivial zeroes can be identified easily because of their straightforward nature, the non-trivial zeroes are hidden inside what is called the critical strip (a vertical band bounded by the lines $u = 0$ and $u = 1$ on the real axis). Riemann's assumption is that all such zeroes sit on the line $u = \frac{1}{2}$, which is the exact midpoint of the critical strip (Emmerson, 2025).

In this research, instead of validating the distribution of prime numbers, the focus of study centers on elucidating the analytic continuation extending throughout the complete complex plane. Going deeper, the study analyses the values of the functions of S and $1-S$ and their symmetric characteristics. In other words, the research aims to present an investigation of complete complex plane in terms of analytic continuation of Riemann's zeta-function. The study framework is developed as an integrational approach on the local half plane utilizing it to reveal the overall visualization of analytic continuation of the complete complex plane (Dumitrescu & Wolf, 2022).

The present study particularly influenced with the noteworthy components of a few previous works, such as, Farhi (2025), who outlined the main ideas of complex analysis, starting from power series and convergence. He also discussed analytic functions, showing that exponential and trigonometric functions are examples of holomorphic functions satisfying the Cauchy-Riemann equations. He brings detailed explanations of the Isolated Zero Theorem and Cauchy's integral formulas for circular domains which pave the way to obtaining great results like Cauchy's inequalities, Liouville's theorem and the Maximum Modulus Principle. Also, the book explains complex integration, piecewise C_1 paths, homotopy, and winding numbers.

Sahu & Dafadar (2025) noted that prime numbers, which have only two divisors, namely one and the number itself, are the building blocks for all positive integers.

Their complex pattern takes thorough investigation mainly through complex analysis and number theory, as exemplified by studies of the Riemann zeta function and prime gaps. In a paper published in 2025, Souto studied the Riemann Hypothesis by extracting information from the combination of analytical and computational efforts. He also outlined the potential link between analytic number theory and quantum systems. Sunami (2018) discussed the algebraic manipulations of elliptic functions to raises significant conceptual difficulties when emergencies with direct calculations.

Also, the Miller-Rabin primality test and the Grand Riemann Hypothesis work as reference points for techniques that lead to estimates of very large primes. Emmerson (2024) has made use of set-theoretic setup to express the Riemann Hypothesis as a conjecture for the locations of zeros of the zeta function, his thesis being that all non-trivial zeros are on the critical line. Schaetzel (2024) proposed a procedure for detecting the zeta function zeros, delineating their characteristics. Cceres (2026) was issuing a paper on structural rigidity of zeta function zeros. This work is based on a discrepancy operator that distinguishes between Dirichlet series and integral approximations. Ultimately, Gallardo-Gutiérrez and Seco (2024) studied the zero-free regions of the zeta function, connecting it (the function) to the Riemann Hypothesis and giving new characterizations that rely on the Prime Number Theorem.

The noteworthy areas where the research intends to throw light are:

- The study extends beyond the classical complex analysis and aims to present a rigorous study on the growth and location of zeros within the complex half plane based on contour integration
- The study aims to derive explicit asymptotic bounds to show the growth of Riemann's zeta-function along the critical line.
- The study aims to locate the zero free regions on the critical strip implicating them in context to the non-trivial zeros.

Constituted with these research goals, the study believes to demonstrate the growth and patterns of spread of non-trivial points on the critical line considering the condition as a pointer that explains and depicts the complete complex plane.

II. PROBLEM STATEMENT

Riemann's discovery connected the zeros of the ζ -function with prime number details, which brought about the Riemann Hypothesis (RH): the idea that the zeros between 0 and 1 on the real axis, lie on the critical line $\text{Re}(s) = 1/2$. It has been verified up to $|t| < 3 \times 10^{12}$ (in fact more than 12.3×10^{12} zeros) w.r.t $Z(t)$ (Hardy Z-Function, that is $Z(t) = e^{i\theta(t)} \zeta\left(\frac{1}{2} + it\right)$, where, $Z(t)$ is real-valued for all real t)(Sangale, 2017).

The zeros are represented as $\rho = \beta + i\gamma$. If one zero is $\rho = \beta + i\gamma$, then the corresponding symmetric zero is its conjugate, that is, $1 - \beta + i\gamma$. This means either $\rho = \beta + i\gamma$ (based on the Riemann Hypothesis, RH) or there are pairs of zeros around the critical line. Now, assuming RH, it is assumed that non-trivial zeros lie on critical line $\text{Re}(s) = 1/2$ that is expressed as $1/2 + i\gamma_n$, with $0 < \gamma_1 < \gamma_2 < \dots$, denote the zeros of $\zeta(s)$ with positive imaginary part. Albeit, there is no compelling reason to expect multiple zeros, so this notation would be invalid if there were repeated zeros. The hypothesis assumes that all zeros are simple, but this assumption is usually not essential to the arguments being made (Emmerson, 2025).

Again, Riemann Hypothesis (RH) claims that when $t > 3$, all the local maxima of the function $Z(t)$ are positive while the local minima are negative. Such characteristics are deeply linked to how the zeros of $Z(t)$ and its derivative are set up. In this case, $S(t)$ (Argument function of zeta-function) is important to characterize the zeros. Again, computer calculations of $S(t)$ are a major form of evidence for the Riemann Hypothesis. So far, various computations of the $S(t)$ -function give only an estimate of around 300,000 for its extreme values which are very rare in reality (Farmer, 2025). Accordingly, analytic computational approaches are considered as essential tool for the examination of the $S(t)$ -function

III. METHODS AND TOOLS

The research, that centres on locating and analysing complete complex plane in terms of non-trivial zeros on critical strip uses analytical approach as a step ahead of conventional complex analysis. Analytic approach is considered necessary in terms of its precise and intricate computational efficacy that prove effective to demonstrate the distribution and patterns of non-trivial zeros that in complex plane. Modern methods expect analytic continuation piece by piece. Early approaches included Riemann extended the factorial function in Euler's gamma function, and found a formula "for all s " with the gamma function as the starting point (Ghosh, 2023).

Euler's Gamma function, originally defined for $\text{Re } s > 0$, can be analytically continued to a meromorphic function with poles. It generalizes the factorial function.

It is important in various fields such as probability (Gamma distribution) and statistics (Chi-squared test). Its worth, the variety of its applications and the mathematical tools used in its construction show how new mathematical links, such as Riemann's with the zeta function, can lead to progress(Dasan et al., 2023).

Currently, the non-trivial zeros are analytically characterized by means of contour integration. Its importance is considered for the reason that polynomials give rise to singularities around the origin of the real axis. The distance to the contour tends to zero, and they cancel each other to give a finite value(Fukushima, 2016). The zeros of the completed zeta function $\hat{\zeta}(z)$ are obtained by equating these singularities to a power of the same order of the integral variable. This leads to exact cancellation of singularities. The real part of the zeros is then $1/2$, which supports the symmetry about $1/2$ and proves the Riemann hypothesis(Wei & Wu, 2022). The last part of the analysis framework includes analysing the conditions for a complex number to be a nontrivial zero of the Riemann zeta function. The study emphasizes on the zeta function has no nontrivial real zeros and show that the zeta function takes negative values on the real axis in the critical region. Knowledge of the nontrivial zeros will be useful for enlarging the zero-free region of the Riemann zeta function.

IV. RESULTS AND DISCUSSION

Firstly, the extension ofGamma Function to larger region is presented here to justify the scope and importance of analytic continuation in Riemann Hypothesis:

For $\text{Re}(z) > 0$, the gamma function is defined by(Haiman, 2014):

$$\Gamma(z) = \int_0^{\infty} x^{z-1} e^{-x} dx.$$

- The upper limit convergence is guaranteed because e^{-x} tends to zero faster than x^{z-1} increases, which ensures uniform convergence for $z \leq b$.
- At the lower limit, for $\text{Re}(z) > 0$, it converges for $\text{Re}(z) \geq a$ for any $a > 0$, because for $0 < x < 1$, $x^{\text{Re}(z)-1} \leq x^{a-1}$ if $\text{Re}(z) \geq a$.
- The integral converges absolutely for complex z with $\text{Re}(z) > 0$, uniformly in compact subsets of $\text{Re}(z) > 0$.

This allows us to differentiate under the integral sign and obtain the relation $\Gamma'(z) = \int_0^{\infty} x^{z-1} e^{-x} \ln(x) dx$, which shows that $\Gamma(z)$ is analytic for $\text{Re}(z) > 0$.

Using integration by parts, we get the identity $\Gamma(z+1) = z\Gamma(z)$. In particular, since $\Gamma(1) = 1$, we have $\Gamma(n+1) = n!$ for integers $n \geq 0$, which is a continuous version of the factorial function. A significant value is $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$, obtained from the integral $\int_0^{\infty} e^{-u^2} du = \frac{\sqrt{\pi}}{2}$ by the substitution $u = \sqrt{x}$. The relation $\Gamma(z) = \frac{\Gamma(z+1)}{z}$ allows for analytic continuation of $\Gamma(z)$ to $D_1 = \{z: \text{Re}(z) > -1\} \setminus \{0\}$, then to $D_2 = \{z: \text{Re}(z) > -2\} \setminus \{0, -1\}$, and so on, with subsequent definitions coinciding with previous definitions, confirming consistency across successive domains.

The above approach affirms for the proposition(Haiman, 2014):

The gamma function $\Gamma(z)$ has a unique analytic continuation to the domain $\mathbb{C} \setminus \{0, -1, -2, -3, \dots\}$. This continuation satisfies the given properties for every z in this domain.

Now,

The functional equation for extension to complete complex plane is:

$$\Gamma(z) = \frac{\Gamma(z+1)}{z}.$$

Here,

Right hand part is meromorphic for $\text{Re}(z) > -1$, with the only possible pole at $z = 0$, ($\Gamma(z+1)$ is analytic).

For any integer $n \geq 1$ (Heap, 2014):

$$\Gamma(z) = \frac{\Gamma(z+n)}{z(z+1)(z+2) \cdots (z+n-1)}.$$

Since n is arbitrary, meromorphic function is defined as:

$$\mathbb{C} \setminus \{0, -1, -2, -3, \dots\}.$$

Using the formula:

$$\Gamma(z) = \frac{\Gamma(z+n+1)}{z(z+1)\cdots(z+n)},$$

The residue at $z = -n$ (for $n \geq 0$) is computed as:

$$\text{Res}_{z=-n} \Gamma(z) = \lim_{z \rightarrow -n} (z+n) \Gamma(z) = \lim_{z \rightarrow -n} \frac{\Gamma(z+n+1)}{z(z+1)\cdots(z+n-1)}.$$

At $z = -n$, $\Gamma(1) = 1$ serves as numerator, and the denominator is

$$(-n)(-n+1)\cdots(-1) = (-1)^n n!.$$

Therefore,

$$\text{Res}_{z=-n} \Gamma(z) = \frac{1}{(-1)^n n!} = \frac{(-1)^n}{n!}.$$

So the poles are simple with the stated residues (Heap, 2014).

Overall,

- The analytic continued Gamma Function $F(z)$ is meromorphic in the complex plane except for the points $\mathbb{C} \setminus \{0, -1, -2, \dots\}$.
- It coincides with the function $\Gamma(z)$ on an open subset of the right half-plane, namely where $\text{Re}(z) > 0$ and used to extend in the complete complex plane.
- It also satisfies the functional equation $F(z+1) = zF(z)$.
- Also, the difference $H(z) = \Gamma(z) - F(z)$ is meromorphic and has a zero on an open set which implies that it is identically zero in the domain of definition.
- This means that the continuation of $F(z)$ is unique.

For large $|z|$ in sectors $|\arg z| < \pi - \delta$,

$$\Gamma(z) \sim \sqrt{\frac{2\pi}{z}} \left(\frac{z}{e}\right)^z,$$

This is particularly useful in analytic number theory and asymptotic analysis.

With Riemann's zeta function for $\text{Re}(s) > 1$ (Borrell i Pons, 2025):

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

It has a meromorphic continuation to the whole complex plane containing single simple pole at $s = 1$, with residue 1.

Euler used this pole to prove that the sum of reciprocals of primes diverges:

$$\sum_{p \text{ prime}} \frac{1}{p} = \infty.$$

It holds based on Euler product

$$\zeta(s) = \prod_p (1-p^{-s})^{-1}$$

That behaves like $\frac{1}{s-1}$ near $s = 1$, forcing the prime reciprocals to diverge.

Using prime number theorem:

$$\pi(x) \sim \text{li}(x) = \int_0^x \frac{dt}{\log t},$$

The integral is taken as Cauchy principal value. This theorem is equivalent to the statement:

$$\zeta(s) \neq 0 \text{ for all } \text{Re}(s) = 1.$$

Here, $\zeta(s)$ is non-vanishing on the vertical line $\text{Re}(s) = 1$ that guarantees the asymptotic distribution of primes.

Taking any non-integer $x > 1$ and any real number $c > 1$, the fundamental contour integral:

$$\pi(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \log \zeta(s) \frac{x^s}{s} ds.$$

With logarithmic derivative of $\zeta(s)$, we define the Chebyshev function:

$$\psi(x) = \sum_{p^k \leq x} \log p.$$

Then the contour integral:

$$\psi(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \left(-\frac{\zeta'(s)}{\zeta(s)} \right) \frac{x^s}{s} ds.$$

For $\psi(x)$ is known, $\pi(x)$ is obtained by Möbius inversion (Borwein et al., 2000):

$$\pi(x) = \sum_{n \geq 1} \frac{\mu(n)}{n} \psi(x^{1/n}).$$

Thus, by evaluating the contour integral, we can recursively determine $\pi(x)$.

Under the framework proposed above, now the critical strip is constructed as:

The critical strip, the vertical region is given as:

$$0 < \text{Re}(s) < 1.$$

Inside this strip, $\zeta(s)$ is analytic (no poles). The behaviour of $\zeta(s)$ here determines the distribution of primes. As per the hypothesis, all the non-trivial zeros of $\zeta(s)$ are found on:

$$\text{Re}(s) = \frac{1}{2}.$$

The zeta function should fulfil the functional equation:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s).$$

The formulation implies the following conditions:

- If ρ is a non-trivial zero, then so are $\bar{\rho}$ and $1 - \rho$.
- Since $\zeta(s)$ has real coefficients in its Dirichlet series, zeros occur in complex conjugate pairs:

$$\rho = \beta + i\gamma \text{ and } \bar{\rho} = \beta - i\gamma.$$

To validate the distribution of non-trivial zeros:

Theory: Let $N(T)$ be the number of non-trivial zeros with $0 < \Im(\rho) < T$. Then:

$$N(T) = \frac{T}{2\pi} \log\left(\frac{T}{2\pi e}\right) + O(\log T).$$

The formula is called Riemann–von Mangoldt formula. It shows that the zeros grow in density like $\frac{1}{2\pi} \log T$ per unit height (Borwein et al., 2000).

Proof: Applying the argument principle to $\zeta(s)$ along a rectangle enclosing the critical strip up to height T . The logarithmic derivative and functional equation give the asymptotic count.

Here,

- In case of non-trivial zeros, they are symmetric that is for $\rho = 1/2 + i\gamma$ being zero, $1/2 - i\gamma$ is also a point of value 0 on critical strip.
- Trivial zeros lie on the negative real axis (outside the strip).

For $\rho = \beta + i\gamma$ as a non-trivial zero, asymptotic bounds are:

Vertical bounds are defined as follows with the imaginary parts γ satisfying:

$$\rho \sim \frac{2\pi}{\log \rho} \text{ based on } \rho \sim \frac{\pi}{2} \log \rho.$$

Horizontal bounds are regions where all non-trivial zeros satisfy:

$$0 < \sigma < 1.$$

In case of large σ , zeros are clustered near the critical line, but RH ensures it as exact.

Now, to locate the zero-free region, the region is a part of the critical strip where $\rho(\sigma) \neq 0$. Condition (Without assuming RH): There are no zeros in the region:

$$R(\sigma) \geq 1 - \frac{\sigma}{\log(|\Im(\sigma)| + 2)}, \text{ for some constant } \sigma > 0.$$

Proof: Use the inequality from the logarithmic derivative:

$$-\frac{\rho'(\sigma + i\tau)}{\rho(\sigma + i\tau)} = \sum_{n=1}^{\infty} \frac{\Lambda(n)}{\sigma + i\tau n},$$

Combining non-negativity with $3 + 4\cos \sigma + \cos 2\sigma$, it shows that if a zero existed too close to $R(\sigma) = 1$, it would contradict the analytic properties near the pole at $\sigma = 1$.

The condition satisfies $\rho(\sigma) \neq 0$ on the line $R(\sigma) = 1$, which is exactly the equivalence to the prime.

Here, the analytic continuation of $\rho(\sigma)$ to $\mathbb{C} \setminus \{1\}$ is unique, just like the gamma function.

The functional equation is:

$$\rho(\sigma) = \rho(\sigma)\rho(1 - \sigma), \rho(\sigma) = 2^{\sigma} \pi^{\sigma-1} \sin\left(\frac{\pi\sigma}{2}\right) \Gamma(1 - \sigma),$$

extends $\rho(\sigma)$ from $R(\sigma) > 1$ to the whole plane. This condition is analogous to extending $\Gamma(\sigma)$ using $\Gamma(\sigma) = \Gamma(\sigma + 1)/\sigma$.

These results are confirmed with the analytic continuation:

- The pole at $S = 1$ has residue 1.
- The trivial zeros at negative even integers come from the sine factor.
- The non-trivial zeros are the remaining zeros in the critical strip.

V. CONCLUSION AND RESULTS

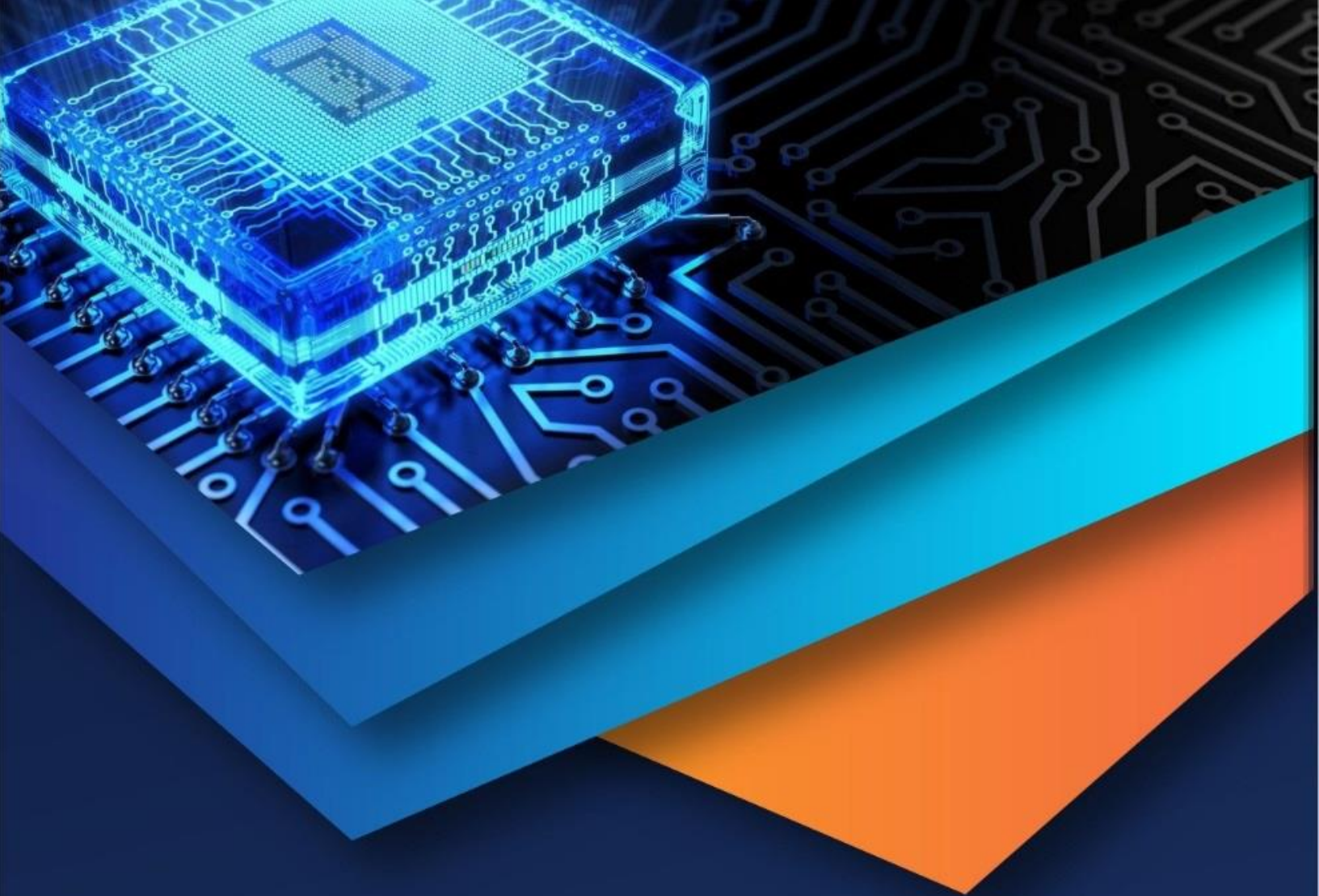
In this research, the gamma function is extended from $R(\sigma) > 0$ to $\mathbb{C} \setminus \{0, -1, -2, \dots\}$, which uniquely determines its meromorphic structure with simple poles at the negative integers with residues $(-1)^n / n!$. This framework applies directly to the theory of the Riemann zeta function where continuation to $\mathbb{C} \setminus \{1\}$ shows the critical strip $0 < R(\sigma) < 1$. In this strip non-trivial zeros are symmetric about $R(S)=1/2$ with the Riemann-von Mangoldt formula Their distribution is $\rho(\sigma) \sim \frac{\pi}{2} \log \sigma$ Logarithmic derivative inequalities provide validated zero-free regions where $\rho(\sigma) \neq 0$ on $R(\sigma) = 1$. $\Re(S)=1$, proof of the prime number theorem. Thus, the zero-free regions prove the complete meromorphic structure and the analytic continuation as the final instrument for the global complex characterization.

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