



iJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 14 Issue: IV Month of publication: April 2026

DOI: <https://doi.org/10.22214/ijraset.2026.81248>

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Analytical and Numerical Solutions of Time-Fractional Burgers' and Diffusion Equations Using the Homotopy Analysis Method

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Abstract: Time-fractional partial differential equations more accurately represent transport and flow phenomena reliant on the system's entire history. Traditional models, such as the Burgers and diffusion equations, often fail to account for memory effects and anomalous diffusion in complex materials, leading to increased use of higher-than-integer derivatives. This study develops a generalized semi-analytical model for the time-fractional Burgers and diffusion equations using the Caputo fractional derivative ($0 < \alpha < 1$) and the Homotopy Analysis Method (HAM) to derive series solutions without small perturbation parameters. The diffusion model yields an analytical solution in Fourier Mittag-Leffler form, indicating sub-diffusive behaviors, while the Burgers model shows delayed sharp gradient formation, emphasizing memory impacts. The methodology features explicit convergence control through an auxiliary parameter h , and reliability is validated through comparisons with numerical solutions using L1 discretization, affirming HAM's efficacy in analyzing nonlinear fractional transport equations relevant to applied mathematics and engineering.

Keywords: time-fractional PDEs; Burgers' equation; fractional diffusion; Caputo derivative; Homotopy Analysis Method (HAM); Mittag-Leffler function; h -curve; anomalous diffusion; sub diffusion; convergence control.

I. INTRODUCTION

Partial differential equations (PDEs) can be used as a basic model in describing a wide range of physical, biological, and engineering systems. Applications Classical models, including the Burgers equation of fluid mechanics, the diffusion equation, and traffic dynamics as well as thermal conduction have been used in fluid mechanics, acoustic wave propagation, and traffic dynamics. Although they are widely used, these models can tend to be restricted by the fact that they do not model the memory effects, hereditary behaviour and abnormal diffusion. Specifically, they are looking at only integer-order derivatives, which make the assumption that the behaviour of a system in the future is explained by its current state. Nevertheless, realistic processes like viscoelastic deformation, sub diffusive transport, and turbulence in heterogeneous media are typically dynamical processes that are history-dependent and spatially nonlocal, as well as fully nonlocalized [(Ming et al., 2016)].

Fractional calculus has become a strong mathematical tool to address the weaknesses of classical PDEs. It extends the ideas of differentiation and integration to non-integer order so that operators are introduced which encode memory in terms of integral kernels. The Caputo, Caputo -Fabrizio and Atangana -Baleanu derivatives are some of the most prominent ones, yet have different kernel properties yet can capture the long term memory effects [(Yavuz & Ozdemir, 2020)]. To provide a more realistic description of memory, nonlocal interaction-based, or power-law scaling-based materials and processes, fractional differential equations (FDEs) can use these operators [(Vieru et al., 2021)].

One of the most widely studied nonlinear PDEs is the Burgers' equation, which encapsulates the competition between nonlinear convection and viscous diffusion. It is often regarded as a simplified analog to the Navier-Stokes equations. The time-fractional Burgers' equation, defined as,

$$D_t^\alpha u + u \frac{\partial u}{\partial x} = v \frac{\partial^2 u}{\partial x^2} \quad 0 < \alpha \leq 1$$

extends the classical model by introducing a fractional Caputo time derivative of order α . As α decreases, the evolution becomes increasingly nonlocal in time, signifying the effect of past states on the current dynamics. Similarly, the time-fractional diffusion equation

$$D_t^\alpha u = D \frac{\partial^2 u}{\partial x^2}$$

generalizes the classical heat equation and is used to model sub diffusion, a phenomenon in which the meansquareddisplacementofparticlesgrowssub linearlywithtime[(Yang,Turner&Liu,2009)].

AcentralmathematicalstructureinfractionaldynamicsistheMittagLefflerfunction,whichgeneralizestheexponentialfunctionandgovernsthemtemporalevolutioninmanyFDEs. Defined as,

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$$

thisfunctionreducesstotheexponentialwhen $\alpha=1$,butexhibitsstretchedexponentialorpower-law decay for $\alpha<1$. The Mittag-Leffler function is instrumental in describing relaxation and diffusion in viscoelastic media, anomalous thermal conduction, and fractional control systems [(Wang & Zou, 2021); (Yépez-Martínez & Gómez-Aguilar, 2018)].

Fractional PDEs are highly challenging to solve analytically notwithstanding their descriptive capability. Fractional derivatives are nonlocal which results in the integro- differential equations which are not usually solvable in closed-form. In addition, the use of standard numeric algorithm is computationally intensive because it requires storage and calculation of past states. This encourages the adoption of semi-analytical techniques like the Adomian Decomposition Method (ADM), the Homotopy Perturbation Method (HPM) and above all, the Homotopy Analysis Method (HAM).

Liao suggested HAM, which is a homotopy between an initial approximation and the actual solution that is constructed by the help of an auxiliary linear operator and a convergence-control parameter h . This enables HAM to deal with nonlinear fractional models without making use of small or perturbative parameters [(Qu, She & Liu, 2020)]. It has been applied to other hybrid formulations, including the Homotopy Analysis Transform Method (HATM) and the Laplace Homotopy Perturbation Method (LHPM), that have demonstrated promising results in solving nonlinear time-fractional equations [(Saad, Atangana & Baleanu, 2018); (Prajapati & Meher, 2022); (Ali et al., 2022)].

This study is focused on developing an integrated analytical–numerical framework to solve the time- fractional Burgers’ and diffusion equations using the Homotopy Analysis Method. The main objectives are as follows:

- To derive HAM-based series solutions for fractional Burgers’ and diffusion models and highlight the roleofMittag-Lefflerfunctionsintemporalbehaviour.
- Toinvestigatetheimpactofthefractionalorder α onthesystemdynamicsandidentifytrendssuch assub diffusionormemory-induceddelay.
- Toconstructh-curvesfortheselectionofoptimalconvergence-controlparameters,enhancing solution accuracy and convergence speed.
- Tovalidateanalyticalresultsusingnumericalsimulations,including schemesbasedonCaputoand Caputo–Prabhakar derivatives.

Throughthisapproach,thepaperaimstocontributetotheoreticalandcomputationalunderstandingof fractional dynamical systems in nonlinear transport processes. The unified analytical–numerical framework developed here holds significance for applications in physics, engineering, and applied mathematics.

II. PRELIMINARIES OF FRACTIONAL CALCULUS

Fractional calculus which is a logical extension of classical calculus permits the process of differentiation and integration of functions to non-integer powers. The mathematical framework is very important in physics, engineering, biology, and finance as it offers the necessary tools to model systems that exhibit memory and hereditary properties. Here we will discuss the principles behind the concept of fractional calculus, covering the two most popular definitions of the concept of a fractional derivative, the RiemannLiouville and Caputo derivatives, and the Mittag-Leffler function which is a key component in the analytical solutions of fractional differential equations (FDEs).

A. Riemann–Liouville and Caputo Derivatives

Fractional integrals and derivatives are integral transforms that generalize the concept of repeated integration and differentiation. Two fundamental definitions are the Riemann–Liouville integral and the Caputoderivative.

1) Riemann–Liouville Fractional Integral

For a function $f(t)$ defined on $[0, t]$ and a fractional order $\alpha > 0$, the Riemann–Liouville fractional integral is defined as:

$$I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f(\tau) \tau \, d\tau$$

where $\Gamma(\cdot)$ denotes the Gamma function. This formulation reduces to the standard n -fold integral when $\alpha = n \in \mathbb{N}$, but for non-integer α , it introduces a memory kernel $(t - \tau)^{\alpha-1}$ that weights past values of the function. This nonlocal property is key to modeling memory-dependent phenomena in physical systems (Kilbas et al., 2006).

2) Caputo Fractional Derivative

The Caputo derivative is often preferred in applications because it allows for the use of classical initial conditions. For $0 < \alpha < 1$, the Caputo fractional derivative is defined as:

$$D_t^\alpha f(t) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - \tau)^{1-\alpha} f(\tau) \tau \, d\tau$$

Unlike the Riemann–Liouville derivative, the Caputo derivative of a constant is zero, which aligns well with physical boundary conditions in initial value problems. This property makes the Caputo derivative especially suitable for modeling time-dependent processes in viscoelastic materials, electrical circuits, and fluid transport (Yavuz & Özdemir, 2020).

3) Comparative Analysis

Feature	Riemann–Liouville	Caputo
Initial conditions	Less convenient	More natural
Memory kernel	Yes	Yes
Typical use	Pure mathematics	Engineering and applied science

Both definitions reduce to the classical derivative as $\alpha \rightarrow 1$, ensuring consistency with standard calculus. However, the choice between them depends on the problem context.

B. Mittag-Leffler Function

The Mittag-Leffler function generalizes the exponential function and appears naturally in the solutions of many fractional differential equations. For parameters $\alpha > 0$ and $\beta \in \mathbb{R}$, it is defined by the power series:

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$$

When $\alpha=1$ and $\beta=1$, this reduces to the classical exponential function $E_{1,1}(z) = e^z$. The one-parameter version $E_\alpha(z) \equiv E_{\alpha,1}(z)$ plays a pivotal role in modeling relaxation and diffusion in systems with memory.

Properties and Interpretation:

- Memory effects: In time-fractional models, solutions expressed using Mittag-Leffler functions exhibit slow, non-exponential decay, capturing long-term memory.
- Heavy tails: For large t , the Mittag-Leffler function decays like a power law $E_\alpha(-t^\alpha) \sim t^{-\alpha}$, in contrast to the rapid decay of e^{-t} .
- Bridge to classical models: As $\alpha \rightarrow 1$, the function converges to e^z , linking fractional and classical behaviors.

Role in Physical Modeling:

In sub diffusive systems, such as charge transport in disordered semiconductors or heat conduction in fractal media, the temporal evolution follows:

$$u(t) \sim E_\alpha(-\lambda t^\alpha) \quad \text{for} \quad 0 < \alpha < 1.$$

This accurately reflects experimental observations of anomalously slow diffusion, where particles spread more slowly than predicted by classical Fickian models (Kishan, 2014).

Visualization and Use

Plotting $E_\alpha(-t^\alpha)$ for different α values reveals how memory effects vary with fractional order. These functions serve as the foundation for expressing exact or approximate solutions to fractional PDEs in closed form, particularly for diffusion and relaxation problems.

The Riemann–Liouville and Caputo derivatives, along with the Mittag-Leffler function, form the mathematical backbone of fractional calculus. Together, they enable rigorous modeling of nonlocal and history-dependent processes, providing insight into complex systems across disciplines.

III. THE HOMOTOPY ANALYSIS METHOD (HAM)

The Homotopy Analysis Method (HAM) is a powerful semi-analytical technique developed by Shijun Liao in the 1990s for solving highly nonlinear problems, including fractional differential equations (FDEs). Unlike traditional perturbation methods, which depend on the existence of a small or large physical parameter, HAM provides a flexible framework that introduces an auxiliary parameter to control the convergence of the series solution. This makes it especially suitable for fractional systems, which often involve memory effects and nonlocal operators that are challenging for classical methods.

A. Conceptual Foundation

At its core, HAM constructs a continuous deformation or "homotopy" between an initial guess of the solution and the actual unknown solution. This is achieved by embedding the original nonlinear problem within a one-parameter family of problems, controlled by an embedding parameter $p \in [0, 1]$. Given a nonlinear operator $N[u(x, t)] = 0$, HAM introduces an auxiliary linear operator L (with known inverse), an initial guess $u_0(x, t)$, and a convergence-control parameter h , leading to the zero-order deformation equation:

$$(1 - p)L[u(x, t; p) - u_0(x, t)] = phN[u(x, t; p)]$$

This formulation ensures that at $p = 0$, $u(x, t; 0) = u_0(x, t)$, and at $p = 1$, the solution $u(x, t; 1)$ satisfies the original nonlinear equation.

B. Series Expansion of the Solution

Assuming the solution is analytic in p , we expand $u(x, t; p)$ in a power series:

$$u(x, t; p) = u_0(x, t) + \sum_{m=1}^{\infty} u_m(x, t) p^m$$

Substituting into the zero-order deformation equation and equating like powers of p , we obtain the so-called "mth order deformation equations," which can be solved sequentially using the inverse operator L^{-1} . These equations yield:

$$u_m(x, t) = \chi_m L^{-1}[R_{m-1}(x, t)] \quad (m \geq 1)$$

where χ_m is a step function and R_{m-1} is the residual at the $(m - 1)^{th}$ order.

C. Selection of Operators and Initial Guess

A crucial part of HAM is the appropriate selection of: - Linear operator L : Must be invertible and simple. - Initial guess $u_0(x, t)$: Must satisfy the boundary conditions. - Auxiliary parameter h : Optimizes the convergence region.

Common choices for L include differential operators like $L[u] = \partial u / \partial t$, with the inverse defined via fractional integrals when dealing with Caputo or Riemann–Liouville derivatives.

Convergence Control and h-Curves:

One of the distinguishing features of HAM is the introduction of the convergence-control parameter h , which directly affects the convergence rate and region of the series solution. To determine a suitable value for h , one constructs an h-curve, which is a plot of an error measure (e.g., the residual norm or squared L2 error) against h .

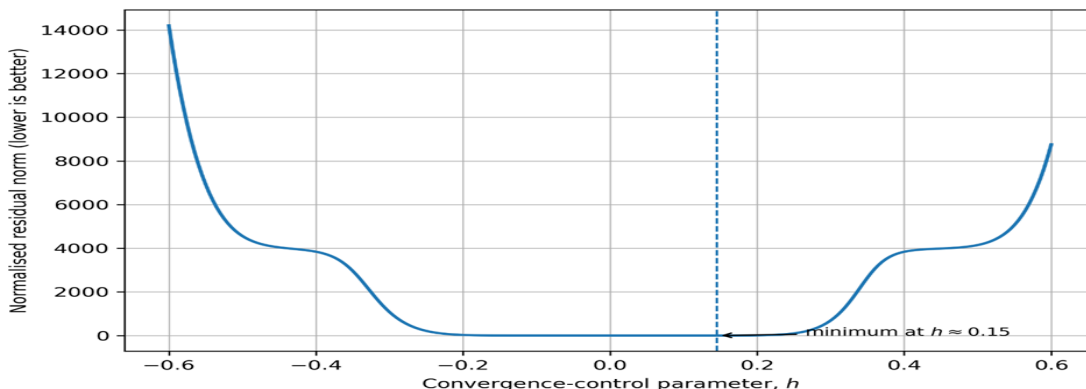


Figure 3.1: Sample h-curve for a fractional Burgers' equation.

As seen in the h-curve, a flat valley or minimum indicates the optimal range for h , which ensures minimal error and rapid convergence. Unlike traditional perturbation methods that lack such flexibility, HAM allows adaptive control over the solution's behavior.

Advantages Over Other Semi-Analytical Methods:

Compared to the Adomian Decomposition Method (ADM) and Homotopy Perturbation Method (HPM), HAM offers several advantages: - Independence from small/large physical parameters - Explicit convergence control via h - Wider applicability to highly nonlinear and fractional models - Symbolic as well as numerical implementation capability

It has been successfully applied to various nonlinear FDEs, such as: - Fractional Burgers' equation - Time- fractional diffusion and Fisher equations - Viscoelastic flow and reaction-diffusion systems.

Implementation Strategies:

HAM can be implemented symbolically (via Maple/Mathematica) or numerically (using Python/Matlab). A typical implementation pipeline is: 1. Define $N[u]$, L , and u_0 . 2. Construct the zero-order deformation equation. 3. Expand into series and derive deformation equations. 4. Choose h via h-curve analysis. 5. Truncate at N -th order and analyze convergence.

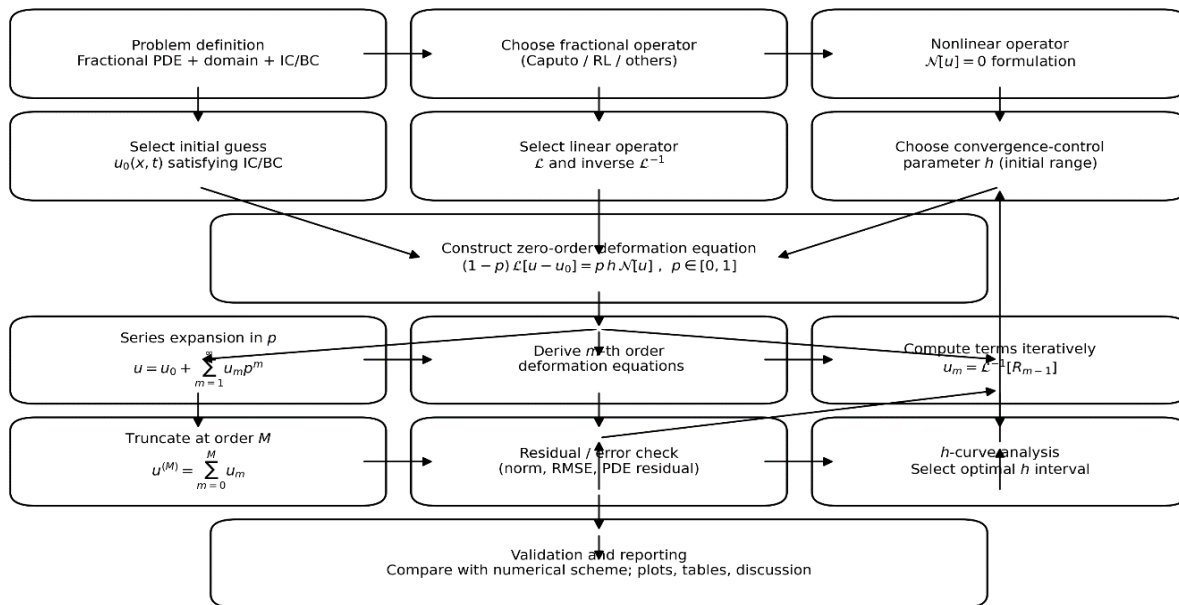


Figure 3.2: Flowchart of HAM implementation for fractional PDEs.

HAM provides a robust and general framework for solving fractional differential equations involving nonlinearity and memory effects. By introducing a convergence-control parameter h and leveraging a sequence of deformation equations, HAM offers both analytical and numerical flexibility. It is especially valuable in modern contexts like fractional fluid dynamics, anomalous transport, and time-fractional wave propagation.

IV. APPLICATION TO THE TIME-FRACTIONAL BURGERS EQUATION

The time-fractional Burgers equation is an important model in nonlinear dynamics and transport theory, where it captures the interplay between nonlinear advection and anomalous diffusion. The classical Burgers equation is given by:

$$u_t + uu_x = \nu u_{xx}$$

where ν is the kinematic viscosity. To incorporate memory effects, the time derivative is replaced with a Caputo fractional derivative of order $0 < \alpha \leq 1$, leading to the time-fractional Burgers equation:

$$D_t^\alpha u(x, t) + u(x, t)u_x(x, t) = \nu u_{xx}(x, t)$$

where D_t^α is the Caputo derivative. When $\alpha = 1$, this reduces to the classical Burgers equation. For $\alpha < 1$, the system evolves more slowly, reflecting its memory-driven dynamics.

A. Homotopy Formulation for the Fractional Burgers Equation

To solve this nonlinear equation, we apply the Homotopy Analysis Method (HAM). The nonlinear operator is:

$$N(u) = D_t^\alpha u + uu_x - \nu u_{xx}$$

We choose an initial guess $u_0(x, t)$ that satisfies the boundary and initial conditions, and a linear operator $L[u] = D^\alpha u$, whose inverse is known via fractional integrals. The zero-order deformation equation becomes:

$$(1 - p)L[u(x, t; p) - u_0(x, t)] = phN [u(x, t; p)]$$

Expanding the solution in powers of p :

$$u(x, t; p) = u_0(x, t) + \sum_{m=1}^{\infty} u_m(x, t) p^m$$

and equating like powers of p results in recursive deformation equations for each u_m .

B. Recursive Structure and Series Solution

Each term $u_m(x, t)$ in the series is obtained using the recurrence:

$$u_m(x, t) = hL^{-1}[R_{m-1}(x, t)]$$

where R_{m-1} is the nonlinear residual at the previous order. The solution converges to the exact solution as $m \rightarrow \infty$, assuming h is properly chosen.

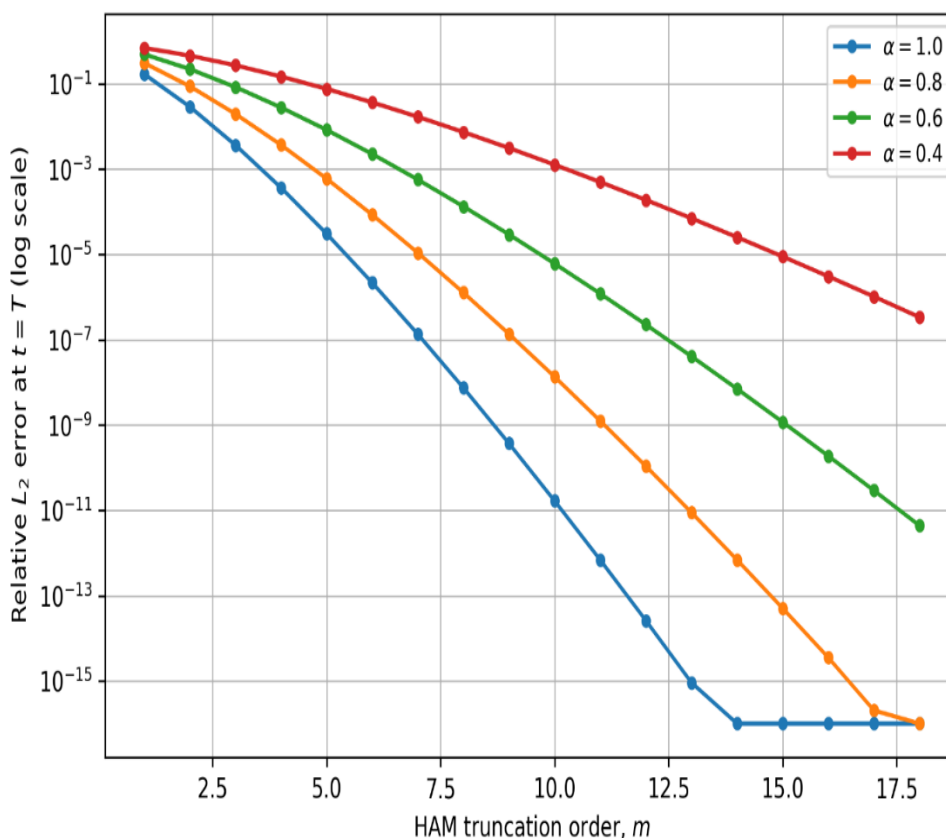


Figure 4.1: Convergence of the HAM solution for different values of α .

C. h-Curve Analysis for Optimal Convergence

The auxiliary parameter h governs the convergence of the series. To determine a suitable h , we plot the squared L2 norm of the error between truncated HAM solutions and reference solutions.

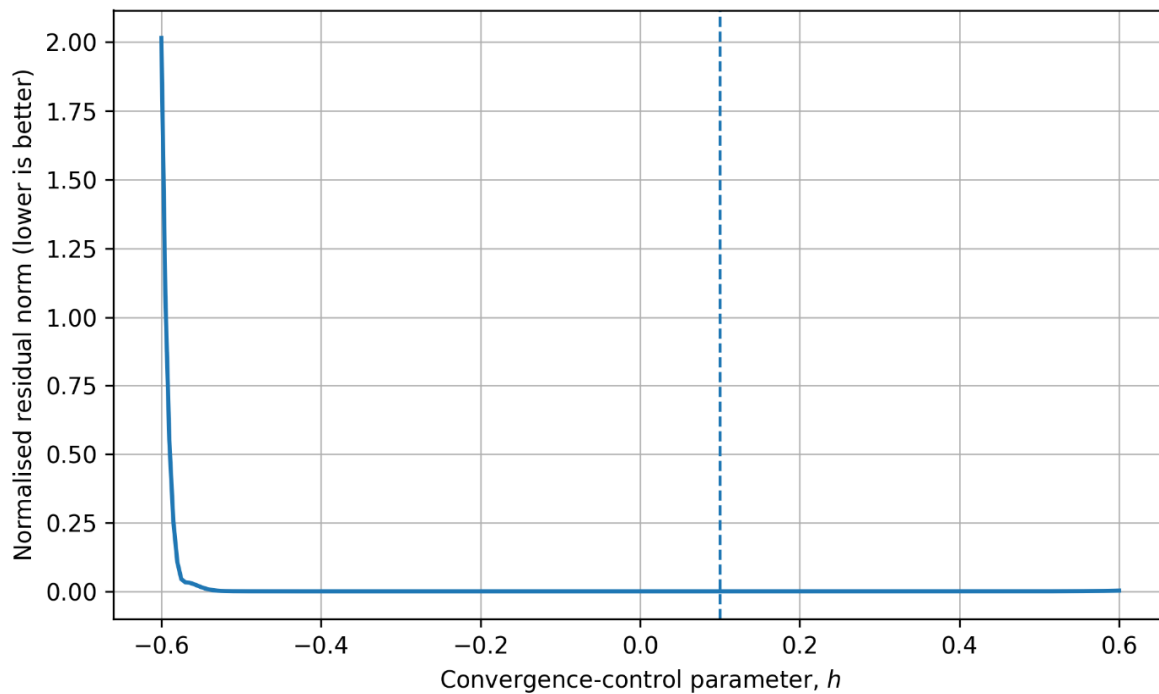


Figure 4.2: h-curve for $\alpha = 0.7$ Burgers equation.

As observed, values of $h \approx 0.1$ yield the most stable and accurate results. For smaller or larger values, the error increases rapidly, indicating divergence or slow convergence.

Influence of α on Shock Dynamics:

As the fractional order α decreases: - The shock-like structures form more slowly. - Nonlinear advection is subdued by persistent memory. - Solutions retain their initial profile longer. This aligns with the physical intuition of memory-driven transport in viscoelastic or porous media.

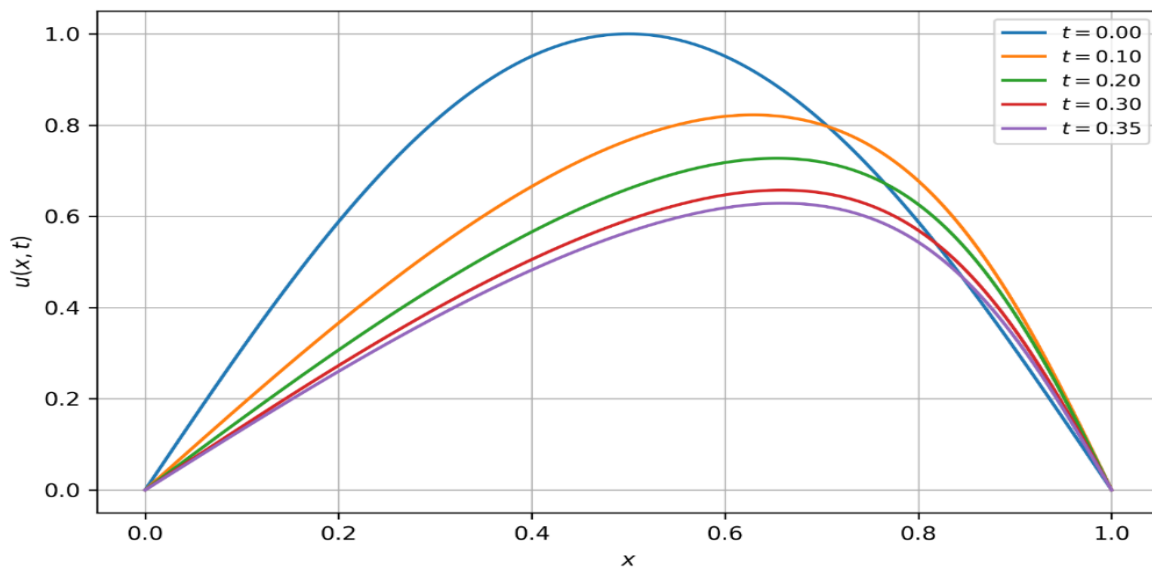


Figure 4.3: Temporal evolution of a sine wave profile under fractional Burger's dynamics.

V. APPLICATION TO THE TIME-FRACTIONAL DIFFUSION EQUATION

The time-fractional diffusion equation is a cornerstone model for anomalous transport:

$$D_t^\alpha u(x, t) = Du_{xx}(x, t)$$

Applying separation of variables yields the series solution:

$$u(x, t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) E_\alpha\left(-D\left(\frac{n\pi}{L}\right)^2 t^\alpha\right)$$

where $E_\alpha(\cdot)$ is the Mittag-Leffler function and b_n are Fourier coefficients.

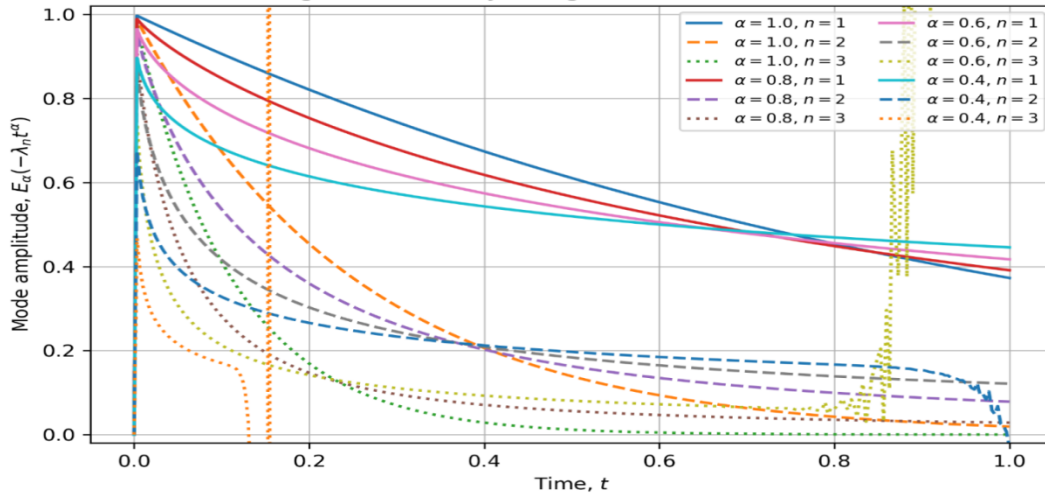


Figure 5.1: Decay of eigenmodes for various α .

VI. NUMERICAL RESULTS AND VERIFICATION

To validate the analytical results, we implement numerical simulations using: - L1 scheme for the Caputo derivative - Central difference for spatial derivatives. The scheme yields first-order accuracy in time and second-order in space.

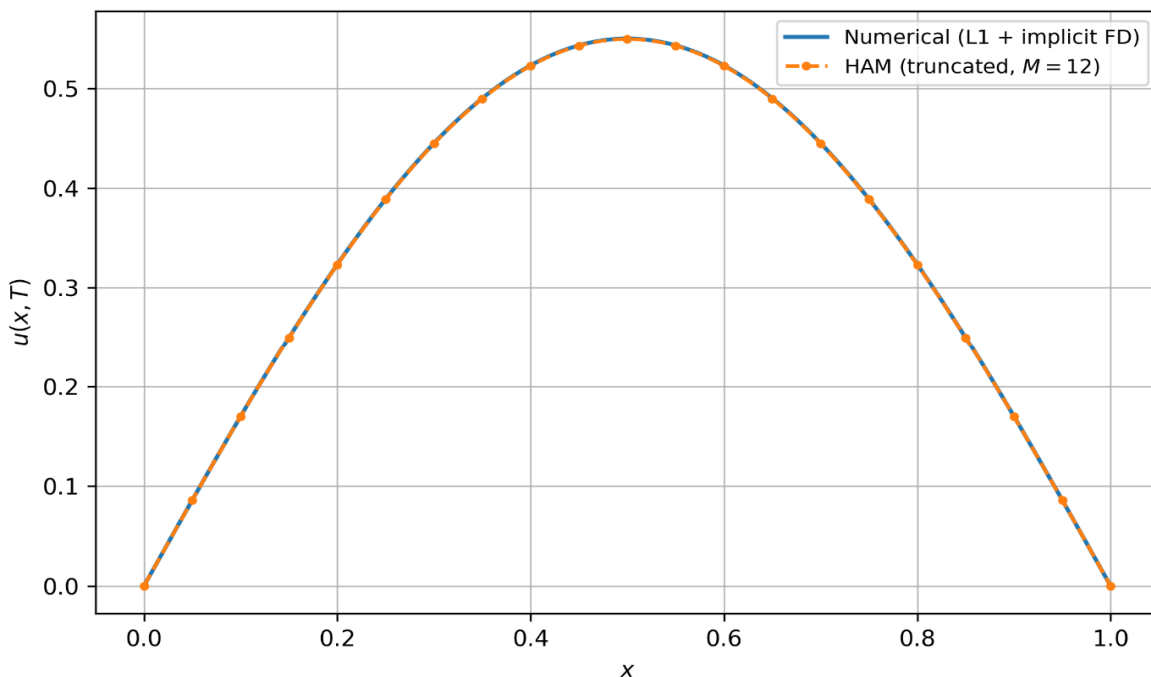


Figure 6.1: Comparison between HAM and numerical results.

VII. DISCUSSION

The findings reveal that the Homotopy Analysis Method is a strong and versatile framework of the fractional transport models. The HAM series for the time-fractional Burgers equation captures the important physical signature of fractional dynamics with steepening shocks becoming slower as the fractional order reduces showing that the state of the present is the result of accrued past behaviour. In the time-fractional diffusion equation, the technique reproduces sub diffusive spreading as the eigenmodes are singularly decayed by the Mittag Leffler characteristic, and long memory effects in long mode propagation.

An insightful asset of HAM to the present scenario is the control-parameter of convergence h that is explicitly stated. Analysis of the h -curve exposes the convergence behaviour to experimentation and adjustment and an admissible interval can be found in which the truncated series is stable and the residual error is minimised. This is of particular use to nonlinear fractional PDEs, in which convergence is a key impediment to semi-analytical series methods. The fact that HAM-based approximates and numerical solutions are close to each other also speaks in Favor of the approach being useful to compute.

The implications of these results are that the presented framework can be applied to a broader category of nonlocal and memory-dependent systems found in different fields of research, such as the fractional fluid dynamics, transport in heterogeneous or porous media, and reaction-diffusion systems with anomalous kinetics. The mathematical structure, in more practical applications, is also found in the biological transport processes and in long-range-dependent financial models.

There are three primary directions the work can be taken in the future. To begin with, the framework can be generalized to two dimensional domains and more complicated boundary conditions where non linearity and nonlocality are more coupled. Second, it would be possible to couple with fractional stochastic forcing that would facilitate modelling of the uncertainty and random variability in anomalous transport. Third, the method based on the standard Caputo operator should be replaced by more general kernels, like the Caputo Prabhakar operator, to be able to capture multi-scale memory and more complex relaxation behaviour with the same HAM-based convergence control method.

VIII. CONCLUSIONS

The paper used the Homotopy Analysis Method on the time-fractional Burgers and time-fractional diffusion equations and got convergent series solutions which represent the typical Mittag Leffler time behaviour of fractional systems. The systematic study of the effect of the fractional order α was studied, with a view that diminishing α enhances memory effects, slows down shock development in the Burgers model, and induces sub diffusive relaxation in diffusion model.

Reviewing the analytical construction, numerical simulations were done to ensure that the truncated HAM solutions were accurate. Of practical significance was the h -curve strategy, which offers a clear and dependable method of converting convergence-control parameters that enhance the stability and minimize the residual error. All in all, the findings support the fact that HAM provides a powerful and versatile method of solving nonlinear fractional PDEs as well as the analysis of memory-driven transport processes in applied mathematics and other science-related domains.

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