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Application of Numerical Nonlinear Optimization Problems

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Abstract: In this paper develops a method based on fuzzy logic for solving linear approximation relationship nonlinear optimization problems. In the proposed method, the nonlinear objective function as well as the constrained functions forming the feasible regions is linearly approximated about an initial feasible point by Taylors Series and then the nonlinear optimization problem is approximated to a linear optimization problem or a linear programming problem. The concept of fuzzy decision making is iteratively applied to find the fuzzy optimal solution of the problem.

Keywords: fuzzy open set, Fuzzy subset, Fuzzy Logic
Mathematics subject classification (2000): 54AO5

I. INTRODUCTION

In this paper, we develop a solution method for solving linear approximation relationship constrained nonlinear optimization problems by using the concept of fuzzy logic. We consider a constrained nonlinear optimization problem. The nonlinear objective function as well as the constrained functions forming the feasible regions is linearly approximated about an initial feasible point by Taylors Series and then the nonlinear optimization problem is approximated to a linear optimization problem or a linear programming problem. The concept of fuzzy decision making [e.g. see Bellman & Zadeh (1970), Warners (1987), Feng (1987), Zhang et.al. (2013), Chamani et.al. (2013)] is iteratively applied to find the fuzzy optimal solution of the problem.

In Sec. 5.2, we consider the procedure for obtaining the initial feasible solution. In Sec. 5.3, the solution method of the problem is investigated. An iteration algorithm is also developed. In Sec. 5.4, a physical numerical example is shown to illustrate the efficiency of the proposed problem.

A. Calculation of Initial Feasible Point

In this section, we consider a method of finding an initial feasible solution of a given system of constraints. A common method used in many practical situations is "trial and error" method [Walsh (1977)].

Let us consider a system of constraints:

$$\begin{aligned} g_i(x) &\le b_i \quad (i = 1,2,\ldots,u) \\ &\ge b_i \quad (i = u+1,\ u+2,\ldots,v) \\ &= b_i \quad (i = v+1, v+2,\ldots, p) \\ x &\ge 0,\quad x \in R^n \end{aligned} \qquad (5.2.1)$$

We write the constraints (5.2.1) in the form:

$$\begin{aligned} g_i(x) &\ge 0 \quad (i = 1,2,\ldots,v) \\ &= 0 \quad (i = v+1, v+2,\ldots, p) \end{aligned} \qquad (5.2.2)$$

Starting at some arbitrary point x_0, we find the feasible constraints and non feasible constraints at x_0. Then, we construct the unconstrained function Z which is to be minimized:

$$Z = -\sum_{i=1}^{v}{}' g_i(x) + \sum_{i=v+1}^{p} \left[g_i(x)\right]^2 . \qquad (5.2.3)$$

where Σ' indicates that the summation is taken over only those constraints that are violated at the current point x_0.

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The minimization of Z is otherwise unconstrained. When the value of Z has been reduced to zero, a feasible point has been found. An obvious way of dealing with equality constraints is to use them to eliminate one or more variables with caution, from the given problem.

An alternative method for finding the feasible starting point can be used asgiven below:

i) *Write the constraints of (5.2.1) in the form*

$g_i(x) \le 0, \quad i = 1,2,\dots, p$.

ii). Choose an arbitrary point x_1 and evaluate the constraints $g_i(x)$ at the point x_1. Since the point x_1 is arbitrary, it may violate some constraints.

If k out of p constraints are violated, we identify the constraints such thatthe last k constraints will become the violated ones that is

$$\left.\begin{aligned} &g_i(x_1) < 0 \quad i = 1,2,\dots, p-k \\ &g_i(x_1) \ge 0 \quad i = p-k+1, p-k+2,\dots, p \end{aligned}\right\} \qquad (5.2.4)$$

iii). We identify the constraints which is violated most at the point x_1, that is, we find the integer r such that

$$g_r(x_1) = \max\,[g_i(x_1)] \text{ for } i = p-k+1, p-k+2,\dots, p. \qquad (5.2.5)$$

iv). We now formulate a new optimization problem as:

$$\begin{aligned} &\text{Min } g_r(x) \\ &\text{subject to} \quad g_i(x) \le 0, \qquad i = 1,2,\dots, p-k. \\ &\qquad g_i(x) - g_r(x_1) \le 0, \qquad i = p-k+1, p-k+2,\dots, r-1, r+1,\dots, p. \end{aligned}$$

....(5.2.6)

v). Solve the optimization problem (5.2.6) by taking the point x_1 as a feasible starting point by using any unconstrained optimization technique preferably the interior penalty function method [see Rao (1992), Walsh (1977)]. It is noted that this optimization method can be terminated.

vi). If all the constraints are not satisfied at the point x^*, then set the new starting point as $x = x^*$ and renumber the constraints such that the last k constraints will be the unsatisfied ones, and go to step (*iii*).

The above procedure is repeated until all the constraints are satisfied and thus a point $x = x^*$ is obtained for which

$g_i(x_1) < 0 \quad i = 1, 2, \dots, m$.

Note that if the constraints are consistent, it should be possible to obtain, by applying the above procedure, a point x_1 that satisfies all the constraints.

However, the solution of the problem formulated in step (*ix*) gives a local minimum of $g_k(x)$ with a positive value. In such case, we are to start afresh with a new point x_1 from step (*i*) onwards.

B. Mathematical Formulation and Solution Procedure Based on FuzzyDecision Making Concepts

Consider a constrained nonlinear optimization problem:

$$\begin{aligned} &\text{Max } z = f(x) \\ &\text{subject to } g_i(x) \leq, =, \text{or} \geq b_i \quad i = 1.2, \dots, p \qquad (5.3.1) \\ &x \geq 0 \end{aligned}$$

where $x \in R^n$, b_i $(i=1,2,\dots,p)$ are constants. Suppose x_o is an initial feasible point of problem (5.3.1).

For finding x_1, we can use one of the methods discussed in Section 5.2. By using the linear approximation technique [Walsh (1997)] about the point x_1 and by change of variables, we obtain the following LP problem:

$$\begin{aligned} &Max\ \hat{z} = c^T (w - m) \\ &\text{subject to } a_i^T w \le, =, or \ge \hat{b}_i + a_i^T m \\ &w_j' \le w_j \le w_j'' \\ &i = 1,2,\ldots,p;\ j = 1,2,\ldots,n \end{aligned} \qquad (5.3.2)$$

where $w = [w_1, w_2, \ldots, w_n]$ (n-dimensional column vector)

$m = [m_1, m_2, \ldots, m_n]$, vector of step length

$$\left.\begin{aligned} &w_j' = \max\{x_j' - x_{oj} + m_j, 0\}, \\ &w_j'' = \min\{x_j'' - x_{oj} + m_j, 2m_j\}, \\ &x_j' \text{ and } x_j'' \text{ are the lower and upper bounds of } x_j \\ &(\text{the } j^{th} \text{component of } x). \\ &x_{oj} \text{ is the } j^{th} \text{component of } x_o \\ &a_i = \nabla g_i(x_o), \ (n\text{-dimensional column vector}) \\ &\hat{b}_i = b_i - g_i(x_o) \\ &c = \nabla f(x_o), \ (n\text{-dimensional column vector}) \\ &\hat{z} = z - f(x_o) \end{aligned}\right\} \qquad (5.3.3)$$

By omitting the constant termLP $-c^T m$ in problem (5.3.2), we obtain the following problem:

$$\begin{aligned} &\text{Max } \hat{z} = c^T w \\ &\text{subject to } a_i^T w \le, =, or \ge \hat{b}_i + a_i^T m \\ &w_j' \le w_j \le w_j'', w \ge 0 \\ &i = 1,2,\ldots,p;\ j = 1,2,\ldots,n \end{aligned} \qquad (5.3.4)$$

Now our problem is to construct a fuzzy LP model which is to be solvediteratively.

Before we construct a specific model of linear programming in a fuzzy environment it should have become clear that a fuzzy linear programming is not uniquely defined type of model but that many variations are possible, depending on the assumptions or features of the real situation to be modelled.

First of all we assume that the objective function should reach some aspiration levels which might not even be defined crisply. Thus, we might want to improve the present cost situation considerably and so on.

Secondly, we accept small violation of constraints but also attach different degrees of importance to violations of different constraints.

Here, we shall present a model that is particularly suitable for the type of linear programming model (5.3.4) in fuzzy environment which seem to have some advantages [Werners (1987)]. Werner (1987) suggested the following definition.

Definition 5.3.1[Werner (1987)]. Let $f: X \rightarrow R'$ be an objective function, R= fuzzy feasible region, $S(R)$ =support of R, R_1=1-level cut of R. The membership function of this goal (objective function) in the given solution space is then defined as

$$\mu_{G}(x) = \begin{cases} 0 & \text{if } f(x) \leq \sup_{R_1} f \\ \dfrac{f(x) - \sup_{R_1} f}{\sup_{S(R)} f - \sup_{R_1} f} & \text{if } \sup_{R_1} f < f(x) < \sup_{S(R)} f \\ 1 & \text{if } \sup_{S(R)} f \leq f(x) \end{cases}$$

The corresponding membership in functional space is then

$$\mu_{G}(r) = \begin{cases} \sup_{x \in f^{-1}(r)} \mu_{G}(r) & \text{if } r \in R, \ f^{-1}(r) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

We extend the above definition as follows:

Definition 5.3.2. Let $\hat{z}: W \rightarrow R^1$ be an objective function, R= fuzzy feasible region, $S(R)$ = support of R, R_1= 1-level cut of R and $\tilde{G}$= fuzzy goal. The membership function of this goal $\tilde{G}$ in the given solution space R is then defined as

$$\mu_{G}(w) = \begin{cases} 0 & \text{if } \hat{z}(w) \leq \sup_{R_1} \hat{z} \\ \dfrac{\hat{z} - \sup_{R_1} \hat{z}}{\sup_{S(R)} \hat{z} - \sup_{R_1} \hat{z}} & \text{if } \sup_{R_1} \hat{z} < \hat{z}(w) < \sup_{S(R)} \hat{z} \\ 1 & \text{if } \sup_{S(R)} \hat{z} \leq \hat{z}(w) \end{cases}$$

The corresponding membership in functional space is then

$$\mu_{G}(r) = \begin{cases} \sup_{w \in \hat{z}^{-1}(r)} \mu_{G}(w) & \text{if } r \in R, \ \hat{z}^{-1}(r) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

We assume that each of p rows of (5.3.4) shall be represented by a fuzzy set, themembership functions of which is given by $\mu_i(x)$, $i = 1, 2, \dots p$.

Let the membership functions of the fuzzy sets representing the fuzzy constraintsforming the fuzzy feasible region R $(a_i^T w \leq, =$ or $\geq \hat{b}_i + a_i^T m,\ i = 1, 2, \dots, p)$ be defined as

$$\mu_i(x) = \begin{cases} 1 & \text{if } a_i^T w \leq, = \text{ or} \geq \tilde{b}_i + a_i^T m \\ \dfrac{\hat{b}_i + a_i^T m + k_i - a_i^T w}{k_i} & \text{if } b_i + a_i^T m \leq a_i^T w \leq b_i + a_i^T m + k_i \\ 0 & \text{if } a_i^T w \geq \hat{b}_i + a_i^T m + k_i \end{cases} \quad (5.3.5)$$

where k_i is the j^{th} component of tolerance vector $k \in R^p$.

The membership function of objective function of fuzzy solution can bedetermined by solving the following LP problems:

$$\text{Maximize } \hat{z} = c^T w$$
$$\text{subject to } a_i^T w \leq, = \text{or} \geq \hat{b}_i + a_i^T m \quad (5.3.6)$$
$$w \geq 0$$

yielding $\sup_{\tilde{z}_1} \hat{z} = (c^T)_{w\ optimum} = \hat{z}_1$; and

$$\text{Maximize } \hat{z} = c^T w$$
$$\text{subject to } a_i^T w \leq, =, \geq \hat{b}_i + a_i^T m + k_i \quad (5.3.7)$$
$$w \geq 0$$

yielding $\sup_{\tilde{z}_1} \hat{z} = (c^T)_{w\ optimum} = \hat{z}_o$.

Therefore, the membership function of fuzzy goal G is given by

$$\mu_{\tilde{G}}(w) = \begin{cases} 1 & \text{if } \hat{z}_1 \leq c^T w \\ \dfrac{c^T w - \hat{z}_o}{\hat{z}_1 - \hat{z}_o} & \text{if } \hat{z}_o < c^T w < \hat{z}_1 \\ 0 & \text{if } c^T w \leq \hat{z}_o \end{cases} \quad (5.3.8)$$

By applying the concept of fuzzy decision set D [see Bellman & Zadeh (1970)], the membership function of D is defined as

$$\mu_D(w) = \min(\mu_G(w), \mu_i(w)) \quad (5.3.9)$$

Introducing a new variable, λ, which corresponds essentially to (5.3.8), we arriveat:

$$\text{Maximize } \lambda$$
$$\text{subject to } \lambda(\hat{z}_1 - \hat{z}_o) - c^T w \leq -\hat{z}_o \quad (5.3.10)$$
$$\lambda k_i + a_i^T w \leq, = or \geq \hat{b}_i + a_i^T m + k_i$$
$$\lambda \leq 1,\ \lambda, w \geq 0$$

Model (5.3.10) is to be solved for each iteration. If w^*_t is the optimal solution of problem (5.3.10) in thethe r^{th} iteration (which corresponds with trial point x_r), then, next trial point is:

$$x_r + 1 = x_r + w^*_t - m \qquad (5.3.11)$$

The iteration terminates when where is the value of obtained in the iteration and isa very small positive numerator.

The above procedure can be summarized as an algorithm as given below:

The above procedure can be summarized as an algorithm as given below:

II. ALGORITHM

Step 1. Choose an initial feasible solution x_1 of the given nonlinear optimization problem and a vector m of step length of the variables.

Step 2. Expand the objective function as well as the constraints about x_1 by Taylor Series which is truncated after the linear term.

Step3. Calculate $a, \hat{b}_t, c$ and $\hat{c}$ of (5.3.3) and then, construct the LP problem (5.3.4).

Step 4. Choose a tolerance vector $k \in R^p$ and solve the LPP problems (5.3.6) and (5.3.7) (the choice of vector k depends upon the nature of the problem).

Step 5. Calculate the membership function (5.3.8) and then solve the LPP Problem (5.3.10). If $(w^*, \lambda^*)_t$ is the solution of (5.3.10), then calculate $x_r + 1$ ($r = 0$) by using (5.3.11).

Step 6. Set $x_r + 1$ as new initial point, go to step 3.

Step 7. If $|\lambda^*_{t+1} - \lambda^*_t| < \delta$, stop. Then, the optimal solution is given by:

$x^*_t = x^*_{r-1} + w^*_{t-1} - m$ or $x^*_m = x^*_{t+1} = x^*_r + w^*_r -$ according as $f^*_t \leq f^*_{r+1}$ or $f^*_t \geq f^*_{r+1}$. Otherwise repeat the same procedure.

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III. NUMERICAL EXAMPLE

In order to illustrate the efficient of the above proposed model,

weconsider the following nonlinear optimization problem:

$$\begin{aligned} &\text{Maximize} && z = 2x_1^2 - x_1x_2 + 3x_2^2 \\ &\text{subject to} && 3x_1 + 4x_2 \le 12 \\ & && x_1^2 - x_2^2 \ge 1 \\ & && x_1, x_2 \ge 0 \end{aligned} \qquad (5.4.1)$$

Solution: We set $x_1 = [x_1,\ x_2] = [2,1]$(where the first subscript denotes the iteration number) as the initial feasible solution andcalculate $m = \begin{bmatrix} 1 & 1 \\ - & - \\ 2 & 2 \end{bmatrix}$ Using step 3, we

$a_1 = [3, 4],\ a_1 = [4, -2]$

$\hat{b}_1 = 2,\ \hat{b}_2 = -2$

$c = [7, 4]$

Then, construct the LPP (3.3.4) as

$$\begin{aligned} &\text{Max } \hat{z} = 7w_1 + 4w_2 \\ &\text{subject to } 3w_1 + 4w_2 \le 5.5 \\ &\quad 4w_1 - 2w_2 \ge -1 \\ &\quad 0 \le w_1 \le 1,\ 0 \le w_2 \le 1 \end{aligned} \qquad (5.4.2)$$

yielding $\hat{z} = 9.5,\ w_1 = 1,\ w_2 = 0.63.$

We set tolerance vector $k = [1.5, 3]$ and solve the LP problem (5.4.1) and

$$\begin{aligned} &\text{Max } \hat{z} = 7w_1 + 4w_2 \\ &\text{subject to } 3w_1 + 4w_2 \le 7 \\ &\quad 4w_1 - 2w_2 \ge 2 \\ &\quad 0 \le w_1 \le 1,\ 0 \le w_2 \le 1 \end{aligned} \qquad (5.4.3)$$

yielding $\hat{z} = 11,\ w_1 = 1,\ w_2 = 1.$

By calculating the membership function (see (5.3.8)), we have the parametric LPP:

$$\text{Max } \lambda = 7w_1 + 4w_2$$

$$\text{subject to } 1.5\lambda - 3w_1 - 4w_2 \le -9.5$$

$$1.5\lambda + 3w_1 + 4w_2 \le 7$$

$$3\lambda + 4w_1 - 2w_2 \ge 2$$

$$0 \le w_1 \le 1, \ 0 \le w_2 \le 1$$

yielding $\lambda^* = 0.5$, $w_1 = 1$, $w_2 = .81$. Thus, we get- $x_{11} = 2.5$, $x_{12} = 1.31$.

Setting $x_1 = [x_{11}, x_{12}] = [2.5, 1.31]$ as initial trial point, we repeat the same procedure. In this iteration, we get

$$\lambda_1^* = 0.5, \quad w_1 = 0.5, \ w_2 = 0.63$$

$$\text{yielding } x_{11} = 2.5, \ x_{12} = 1.44$$

Since $\left|\lambda_1^* - \lambda_\circ^*\right| < \delta, \delta = 0.00001$, therefore the required optimal solution of the given problem $x_1 = 2.5$, $x_2 = 1.44$, $z = 15.1208$…….

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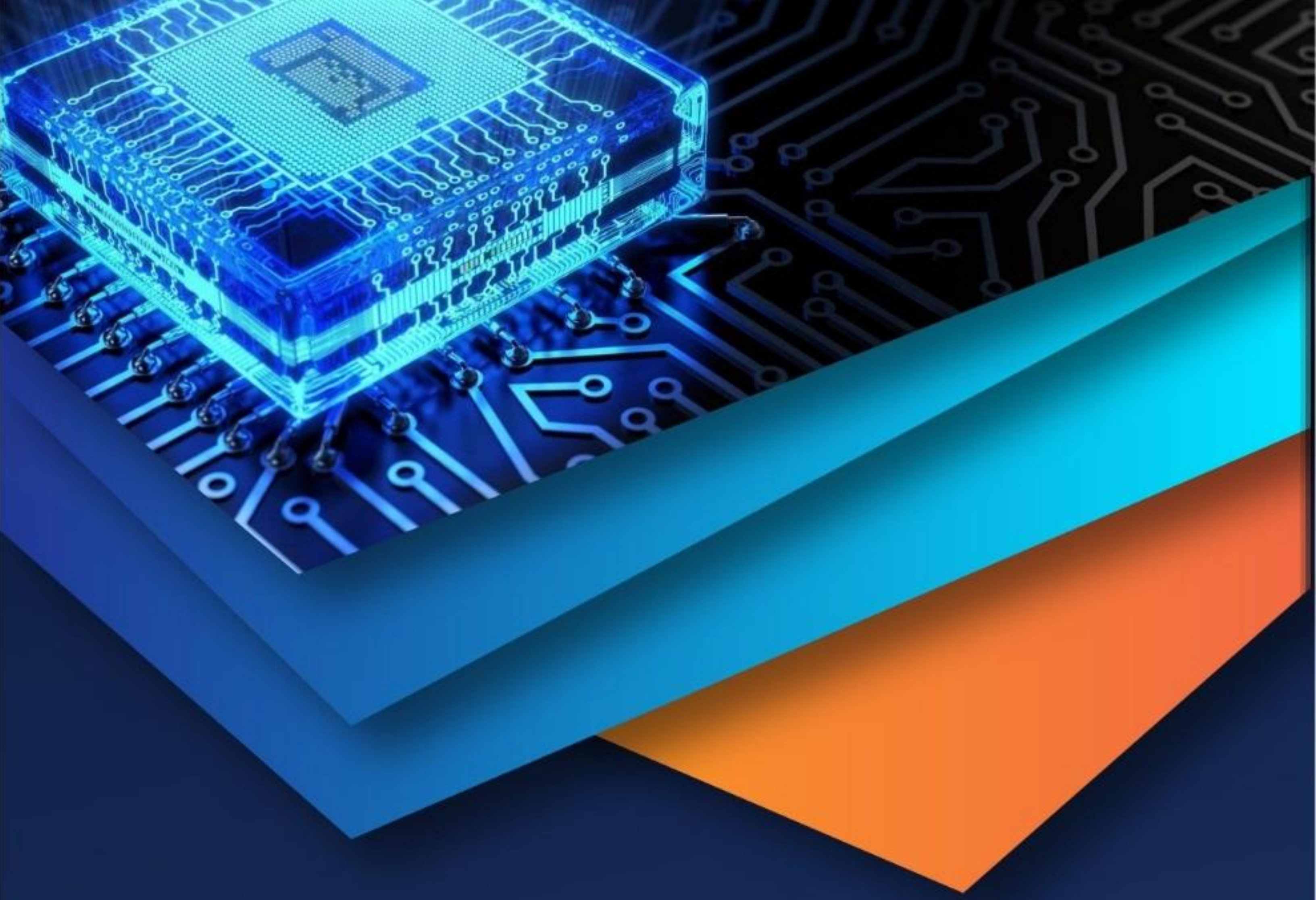

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