# Augmented Zagreb Index of Corona Product of Some Special Graphs 

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Abstract: Let $G(V, E)$ be a simple undirected graph. The Augmented Zagreb Index of a graph $G$ is defined as $A Z I(G)=\sum_{u v \in E(G)}\left(\frac{d u \cdot d v}{d u+d v-2}\right)^{3}$, Where $d u$ is the degree of the vertex $u$ in $G$. In this paper, the exact expression for Augmented Zagreb Index of different product of graphs like Comb, Wheel, Fan,andSun graph.
Keywords: Corona product, Augmented Zagreb index.

## I. INTRODUCTION

A graph $G$ is defined as a pair $G=(V, E)$, where $V$ is a non-empty set of vertices and $E$ is a set of edges. In this paper graphs are simple and connected. We will deal with finite graph,i.e., both $|V|$ and $|E|$ are finite sets. If $G$ is defined as the Augmented Zagreb index $A Z I(G)=\sum_{u v \in E(G)}\left(\frac{d u \cdot d v}{d u+d v-2}\right)^{3}$ of a connected graph.Augmented Zagreb index of corona product is expanded the result of Zagreb index of corona product [9,11].This research can be expanded to include domination of results and theorems have been verified by the Inverse sum indeg index[1,4,5,6,10,12,13,14]Some definitions and results are cited as follows [2,3,7,8,16]. This work can also be expanded upon in the context of automata theory $[17,18,19]$ which has a numerous applications. There are numerous application for graph labeling in both undirected [15,21,22,26,27] and directed graphs [20,23,24,25].

## II. MAIN RESULT

## Definition 2.1

The Augmented Zagreb index AZI $(G)$ of a connected graph $G$ is defined as $A Z I(G)=\sum_{u v \in E(G)}\left(\frac{d u \cdot d v}{d u+d v-2}\right)^{3}$, where u and $v$ are the degrees of the end-vertices of an edge $u v$, respectively.
Theorem 2.1
Let $G_{1}$ and $G_{2}$ be two simple connected graphs then the Augmented Zagreb index of corona product of $P_{n}$ and $K_{1}$ is

$$
\frac{216 m n+1026 m+729 n-3581}{64}
$$

Proof:
The structure of Corona product of $P_{n}$ and $K_{1}$ is shown in fig.2.1 the graph $P_{n} \circ K_{1}$ has $n(m+1)$ vertices and $n m+n-1$ edges.


Fig 2.1: $\operatorname{Comb} P_{n} \circ K_{1}$

Cleary, $P_{n} \circ K_{1}$ there are $(1,2)$ - edges, $(1,3)-$ edges, $(2,3)-$ edges, $(3,3)$ - edges, also

$$
\begin{aligned}
& \quad x_{1,2}=2, \quad x_{1,3}=m(n-2), \quad x_{2,3}=2, \quad x_{3,3}=2 m+n-5 \\
& \quad \operatorname{AZI}\left(P_{n} \circ K_{1}\right)=2\left(\frac{1 \cdot 2}{1+2-2}\right)^{3}+m(n-2)\left(\frac{1 \cdot 3}{1+3-2}\right)^{3}+2\left(\frac{2 \cdot 3}{2+3-2}\right)^{3}+2 m+n-5\left(\frac{3 \cdot 3}{3+3-2}\right)^{3} \\
& =2(2)^{3}+m n-2 m\left(\frac{3}{2}\right)^{3}+2(2)^{3}+2 m+n-5\left(\frac{9}{4}\right)^{3} \\
& =16+m n-2 m\left(\frac{27}{8}\right)+16+2 m+n-5\left(\frac{729}{64}\right) \\
& =32+\frac{216 m n-432 m+1458 m+729 n-3645}{64} \\
& = \\
& =\frac{64+216 m n+1026 m+729 n-3645}{64} \\
& =\frac{216 m n+1026 m+729-3581}{64}
\end{aligned}
$$

Theorem 2.2
Let $G_{1}$ and $G_{2}$ be two simple connected graphs then the Augmented Zagreb index of corona product of $K_{1}$ and $C_{m}$ is

$$
\frac{2457 m^{4} n+2187 m^{3} n+2187 m^{2} n+729 m n}{64 m^{3}+192 m^{2}+192 m+64}
$$

Proof:
The structure of corona product of $K_{1}$ and $C_{m}$ is shown in fig.2.2The graph $K_{1} \circ C_{m}$ has $n(m+1)$ vertices and $2 n m$ edges.


Fig 2.3: Wheel $K_{1} \circ C_{m}$

Clearly, $K_{1} \circ C_{m}$ there are $(3,3)-$ edges, $(3, m)-$ edges, also

$$
x_{3,3}=n m, \quad x_{3, m}=n m .
$$

$$
\operatorname{AZI}\left(K_{1} \circ C_{m}\right)=n m\left(\frac{3.3}{3+3-2}\right)^{3}+n m\left(\frac{3 m}{3+m-2}\right)^{3}
$$

$$
=n m\left(\frac{729}{64}\right)+n m\left(\frac{27 m^{3}}{(m+1)^{3}}\right)
$$

$$
=\frac{729 n m}{64}+\frac{27 n^{4}}{m^{3}+3 m^{2}+3 m+1}
$$

$$
=\frac{729 m n\left(m^{3}+3 m^{2}+3 m+1\right)+64\left(27 m^{4} n\right)}{64\left(m^{3}+3 m^{2}+3 m+1\right)}
$$

$$
=\frac{2457 m^{4} n+2187 m^{3} n+2187 m^{2} n+729 m n}{64 m^{3}+192 m^{2}+192 m+64}
$$

Theorem 2.3
Let $G_{1}$ and $G_{2}$ be two simple connected graphs then the Augmented Zareb index of corona product of $C_{n}$ and $K_{1}$ is

$$
\frac{1728 m n+5832 n}{512}
$$

Proof:
The structure of Corona Product of $C_{n}$ and $K_{1}$ is shown in fig.2.3 The graph $C_{n} \circ K_{1}$ has $n(m+1)$ vertices and $n+n m$ edges.


Fig 2.3:Sun $C_{n} \circ K_{1}$
Clearly, $C_{n} \circ K_{1}$ there are $(1,3)-$ edges, $(3,3)-$ edges, also
$x_{1,3}=n m, \quad x_{3,3}=n$
$A Z I\left(C_{n} \circ K_{1}\right)=n m\left(\frac{1 \cdot 3}{1+3-2}\right)^{3}+n\left(\frac{3 \cdot 3}{3+3-2}\right)^{3}$

$$
\begin{aligned}
& =n m\left(\frac{27}{8}\right)+n\left(\frac{729}{64}\right) \\
& =\frac{64(27 n m)+8(729 n)}{8 \cdot 64} \\
& =\frac{1728 m n+5832 n}{512}
\end{aligned}
$$

Theorem 2.4
Let $G_{1}$ and $G_{2}$ be two simple connected graphs then the Augmented Zagreb index of corona product of $K_{1}$ and $P_{m}$ is

$$
\frac{2457 m^{4} n+3456 m^{3} n-4374 m^{2} n-5832 m n-2187 n-4864 m^{3}+6144 m^{2}+6144 m+2048}{64 m^{3}+192 m^{2}+192 m+64}
$$

Proof:
The structure of corona product of $K_{1}$ and $P_{m}$ is shown in fig.2.4 The graph $K_{1} \circ P_{m}$ has $n(m+1)$ vertices and $2 n m-n$ edges.


Fig 2.4: Fan $K_{1} \circ P_{m}$
Clearly, $K_{1} \circ P_{m}$ there are $(2,3)-$ edges, $(2, m)-$ edges, $(3,3)-$ edges, $(3, m)-$ edges,

$$
\text { also } x_{2,3}=2, \quad x_{2, m}=2, \quad x_{3,3}=n(m-3), \quad x_{3, m}=(n m+2 n-4) .
$$

$$
\begin{aligned}
\operatorname{AZI}\left(K_{1}\right. & \left.\circ P_{m}\right)=2\left(\frac{2 \cdot 3}{2+3-2}\right)^{3}+2\left(\frac{2 m}{2+m-2}\right)^{3}+n(m-3)\left(\frac{3 \cdot 3}{3+3-2}\right)^{3}+(n m+2 n-4)\left(\frac{3 m}{3+m-2}\right)^{3} \\
& =2\left(\frac{6}{3}\right)^{3}+2\left(\frac{8 m^{3}}{m^{3}}\right)+(m n-3 n)\left(\frac{9}{4}\right)^{3}+(n m+2 n-4)\left(\frac{27 m^{3}}{(m+1)^{3}}\right) \\
& =2\left(\frac{216}{27}\right)+16+(m n-3 n)\left(\frac{729}{64}\right)+\frac{27 m^{4} n+54 m^{3} n-108 m^{3}}{m^{3}+3 m^{2}+3 m+1} \\
& =32+\frac{2457 m^{4} n+3456 m^{3} n-4374 m^{2} n-5832 m n-2187 n-6912 m^{3}}{64 m^{3}+192 m^{2}+192 m+64}
\end{aligned}
$$

$$
\begin{aligned}
& \quad=\frac{32\left(64 m^{3}+192 m^{2}+192 m+64\right)+2457 m^{4} n+3456 m^{3} n-4374 m^{2} n-5832 m n-2187 n-6912 m^{3}}{64 m^{3}+192 m^{2}+192 m+64} \\
& =\frac{2457 m^{4} n+3456 m^{3} n-4374 m^{2} n-5832 m n-2187 n-4864 m^{3}+6144 m^{2}+6144 m+2048}{64 m^{3}+192 m^{2}+192 m+64}
\end{aligned}
$$

## III. CONCLUSION

In this paper, Augmented Zagreb index of corona product of graphs are discussed. Some special graphs have been proved under Augmented Zagreb index of corona product of graphs. This index can be use as a numerical description with chemical, physical and biological parameters to study about its relationships.

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