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# Axially Symmetric Perfect Fluid Cosmological Model in Modified Theory of Gravity

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***Abstract:** With an appropriate choice of the function $f(R,T)$, an anisotropic Axially Symmetric Space – time filled with perfect fluid in general relativity and also in the framework of $f(R,T)$ gravity proposed by Harko et. al. (in arXiv: 1104. 2669 [gr-qc],2011) has been studied. The field equations have been solved by using the anisotropy features of the universe in Axially Symmetric Bianchi type-$I$ Space – time. We have been discussed some physical properties of the models. We observed that the involvement of new function $f(R,T)$ does not affect the geometry of the space-time but slightly changes the matter distribution.*

***Keywords:** $f(R,T)$ gravity, Perfect Fluid, Axially Symmetric Universe.*

## I. INTRODUCTION

Cosmological observations on expansion history of the universe indicate that current universe is not only expanding but also accelerating. This late time accelerated expansion of the universe has been confirmed by high red-shift supernovae experiments. Also, observations such as cosmic background radiation and large scale structure provide an indirect evidence for late time accelerated expansion of the universe.

Recently several modified theories of gravity have been developed and studied, in the view of the late time acceleration of the Universe and the existence of dark energy and dark matter. Noteworthy amongst them are the $f(R,T)$ theory of gravity proposed by Nojiri and Odintsov (2003a) and $f(R,T)$ theory of gravity formulated by Harko et al. (2011). Bertolami et al. (2007) proposed a generalization of $f(R)$ theory of gravity by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density $L_m$. Nojiri and Odintsov developed a general solution for the modified $f(R)$ gravity reconstruction from any realistic FRW Cosmology. They have showed that modified $f(R)$ gravity indeed represents a realistic alternative to general relativity, being more consistent in dark epoch. Nojiri et al. developed a general program for the unification of matter-dominated era with acceleration epoch for scalar-tensor theory or dark fluid. Shamir proposed a physically viable $f(R)$ gravity model, which showed the unification of early time inflation and late time acceleration.

Harko et al. (2011) developed $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar $R$ and the trace $T$ of the energy-momentum tensor. It is to be noted that the dependence of T may be induced by exotic imperfect fluid or quantum effects. They have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion of test particles, which follows from the covariant divergence of the stress-energy tensor. They have derived some particular models corresponding to specific choices of function $f(R,T)$. They have also demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of the function $f(R,T)$.

In $f(R,T)$ gravity, the field equations are obtained from a variational, Hilbert-Einstein type, principle.The action principle for this modified theory $f(R,T)$ gravity is given by

$$S=\frac{1}{16\pi G}\int f(R,T)\sqrt{-g}d^4x+\int L_m\sqrt{-g}d^4x \tag{1.1}$$

Where $f(R,T)$ is an arbitrary function of the Ricci scalar $R$, $T$ is the trace of stress energy tensor of matter, $T_{ij}$ and $L_m$ is the matter Lagrangian density.

We define the stress energy tensor of matter as

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\partial(\sqrt{-g})}{\partial g^{ij}} L_m , \qquad \Theta_{ij} = -2T_{ij} - p g_{ij} \tag{1.2}$$

By varying the action principle (1.1) with respect to metric tensor, the corresponding field equations of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T) g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j) f_R(R:T) \\ = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \tag{1.3}$$

Where

$$f_R = \frac{\delta f(R,T)}{\delta R}, \quad f_T = \frac{\delta f(R,T)}{\delta T} \text{ and } \Theta_{ij} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$$

Here $\nabla_i$ is the covariant derivation and $T_{ij}$ is standard matter energy-momentum tensor derived from the Lagrangian $L_m$.It can be observed that when $f(R,T) = f(R)$, then (1.3) yield the field equations of $f(R)$ gravity.

It is mentioned here that these field equations depend on physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et. al. gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases}$$

Assuming,

$$f(R,T) = R + 2f(T) \tag{1.4}$$

as a first choice, where $f(T)$ is an arbitrary function of trace of the stress energy tensor of matter

Then from (1.3) and (1.4), we get the gravitational field equation as

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} - f(T) g_{ij} \tag{1.5}$$

Where the overhead prime indicates differentiation with respect to the argument .

The Friedmann-Robertson-Walker models are the only globally acceptable perfect fluid space-times which are spatially homogenous and isotropic. The adequacy of isotropic cosmological models for describing the present state of the universe is no basis for expecting that they are equally suitable for describing the early stages of the evolution of the Universe. At the early stages of the evolution of Universe, it is, in general spatially homogenous and anisotropic. Bianchi spaces are useful tools for constructing spatially homogenous and anisotropic cosmological models in general relativity and scalar-tensor theories of gravitation. Reddy et. al. (2012a, 2012b) have obtained Kaluza-Klein cosmological model in the presence of perfect fluid source and Bianchi type $III$ cosmological model in $f(R,T)$ gravity using the assumption of law of variation for the Hubble parameter proposed by Bermann (1983), Shamir et al.(2012) obtained exact solution of Bianchi type-$I$ and type-$V$ cosmological model in $f(R,T)$ gravity. Chaubey and Shukla (2013) have obtained a new class of Bianchi cosmological models in $f(R,T)$ gravity. Reddy and Santi Kumar (2013) have presented some anisotropic cosmological models in this theory. Recently Rao and Neelima (2013) have discussed perfect fluid Einstein-Rosen universe in $f(R,T)$ gravity, Sahoo et al. (2014) have studied Axially symmetric cosmological model in $f(R,T)$ gravity. Pawar et al.(2014) have discussed Cosmological models filled with a perfect fluid source in the $f(R,T)$ theory of gravity

Sharif and Zubir (2012) investigated the anisotropic behavior of perfect fluid and massless scalar field for Bianchi type-$I$ space time in this theory. The negative constant deceleration parameter in presence of perfect fluid is studied in Bianchi type-$III$ cosmological model (Reddy et al. 2012). Bianchi type-$III$ dark energy model is derived in presence of perfect fluid using special law of variation for Hubble's parameter (Reddy et al. 2013). Yadav (2013) constructed Bianchi type-$V$ string cosmological model with power law expansion in this theory. Mishra and Sahoo (2014) solved the field equations of Bianchi type-$VI_h$ cosmological model in presence of perfect fluid in $f(R,T)$ gravity.

Sahoo and Mishra (2014) studied Kaluza–Klein dark energy model in form of wet dark fluid in this theory. Sahoo et al. (2014) constructed an axially symmetric cosmological model in $f(R,T)$ theory in the presence of a perfect fluid source. In particular Ahmed and Pradhan (2014) constructed Bianchi type-$V$ cosmological model for a specific choice of $f(R,T) = f_1(R) + f_2(T)$.

In this paper, We study anisotropic Axially symmetric models with perfect fluid matter source in $f(R,T)$ gravity. We present the explicit field equations in $f(R,T)$ gravity for Axially symmetric Bianchi type-$I$ model in presence of a perfect fluid for a particular choice of $f(R,T) = R + 2f(T)$.We obtained solution of field equations. We discuss some properties of the cosmological model.

## II. THE METRIC AND FIELD EQUATIONS

We consider axially symmetric space-time given by

$$ds^2 = dt^2 - A^2\left[d\chi^2 + f^2(\chi)d\phi^2\right] - B^2 dz^2 \tag{2.1}$$

Where $\alpha$ and $\beta$ are the functions of cosmic time $t$ only.

Since there is no unique definition of the matter Lagrangian ,the problem of perfect fluids described by an energy density $\rho$ , pressure $p$ and four velocity $u^i$ is complicated. Therefore, here, we assume that the stress energy tensor of matter is given by

$$T_j^i = (\rho + p)u_i u^j - \delta_j^i p \tag{2.2}$$

And the matter Lagrangian can be taken as $L_m = -p$

and we have

$$u^i \nabla_j u_i = 0 \quad , \quad u^i u_i = 1 \tag{2.3}$$

The matter tensor for perfect fluid is

$$\Theta_j^i = -2T_j^i - \delta_j^i p \tag{2.4}$$

The field equations in $f(R,T)$ theory of gravity for the function $f(R,T) = R + 2f(T)$

When the matter source is perfect fluid are given by

$$G_j^i = R_j^i - \frac{1}{2}\delta_j^i R$$
$$= 8\pi T_j^i + 2f'(T)T_j^i + \left[2pf'(T) + f(T)\right]\delta_j^i \tag{2.5}$$

where the prime indicates the derivative with respect to the argument.

Now, choose the function $f(T)$ as the trace of the stress energy tensor of the matter so that

$$f(T) = \lambda T \tag{2.6}$$

Where $\lambda$ is a constant.

Using commoving coordinate system, the field equations for the metric (2.1) with the help of (2.4) to (2.6) can be written as

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} = (8\pi + 3\lambda)p - \rho\lambda \tag{2.7}$$

$$\frac{2A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 - \frac{f_{11}}{A^2 f} = (8\pi + 3\lambda)p - \rho\lambda \tag{2.8}$$

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_4}{A}\frac{B_4}{B} - \frac{f_{11}}{A^2 f} = -(8\pi + 3\lambda)\rho + \lambda p \tag{2.9}$$

Here subscript 4 after variable denote differentiation with respect to $t$.

Here $k$ is the gravitational constant and subscript 4 and 1 after variable denotes differentiation with respect to t and $\chi$ respectively.

We can observe that it is possible to separate the terms of $f(\chi)$ to one side and the terms $A(t), B(t), \phi(t), \rho(t)$ and $\lambda(t)$ to another side. Hence, we can take that each part is equal to a constant. So,

$$\left(\frac{f_{11}}{f}\right) = k^2, k^2 = constant \quad (2.10)$$

If k=0 then $f(\chi) = c_1\chi + c_2,\ \chi > 0$ .where $c_1$ and $c_2$ are integrating constants.

Without loss of generality, by taking $c_1$=1 and $c_2$=0, we have $f(\chi) = \chi$ .

Now the field equations (2.7) to (2.9) take the form

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} = (8\pi + 3\lambda)p - \rho\lambda \quad (2.11)$$

$$\frac{2A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 = (8\pi + 3\lambda)p - \rho\lambda \quad (2.12)$$

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_4}{A}\frac{B_4}{B} = -(8\pi + 3\lambda)\rho + \lambda p \quad (2.13)$$

These are three linearly independent equations with four unknowns $A, B, \lambda$ and $P$ . In order to solve the system completely. We assume that the expansion scalar is proportional to shear scalar. This condition leads to

$$A = B^m \quad , m \neq 0 \quad (2.14)$$

Equation (2.12) and (2.11) implies that,

$$\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 - \frac{B_{44}}{B} - \frac{A_4}{A}\frac{B_4}{B} = 0 \quad (2.15)$$

Using equations (2.14) and (2.15), we get

$$B = \frac{1}{(n+2)}\log[(n+2)(ct+d)] \quad (2.12)$$

$$A = \frac{n}{(n+2)}\log[(n+2)(ct+d)] \quad (2.13)$$

Then metric (2.1) can now be written in the form

$$ds^2 = dt^2 - [(n+2)((ct+d)]^{\frac{2n}{(n+2)}} dx^2 - [(n+2)(ct+d)]^{\frac{2}{(n+2)}} (dy^2 + dz^2) \quad (2.14)$$

From equation (2.11) and (2.12), we obtained the pressure and density as

$$p = \frac{-(2n+1)C^2}{2(\lambda + 4\pi)[(n+2)(ct+d)]^2} \quad (2.15)$$

$$\rho = \frac{-(2n+1)C^2}{2(\lambda + 4\pi)[(n+2)(ct+d)]^2} \quad (2.16)$$

Respectively.

The metric (2.14) together with (2.15) and (2.16) represents an anisotropic Axially Symmetric Bianchi type $I$ perfect fluid cosmological model in $f(R,T)$ gravity.

## III. SOME PHYSICAL PROPERTIES OF THE MODEL

The volume element of model (2.14) is given by

$$V = \sqrt{-g} = [(n+2)(ct+d)] \quad (3.1)$$

The scalar expansion $\theta$, shear scalar $\sigma$ and average Hubble parameter are given by

$$\theta = \frac{c}{[ct+d]} \quad (3.2)$$

$$\sigma^2 = \frac{7c^2}{18[ct+d]^2} \tag{3.3}$$

$$H = \frac{c}{3[ct+d]} \tag{3.4}$$

The deceleration parameter $q$ is given by

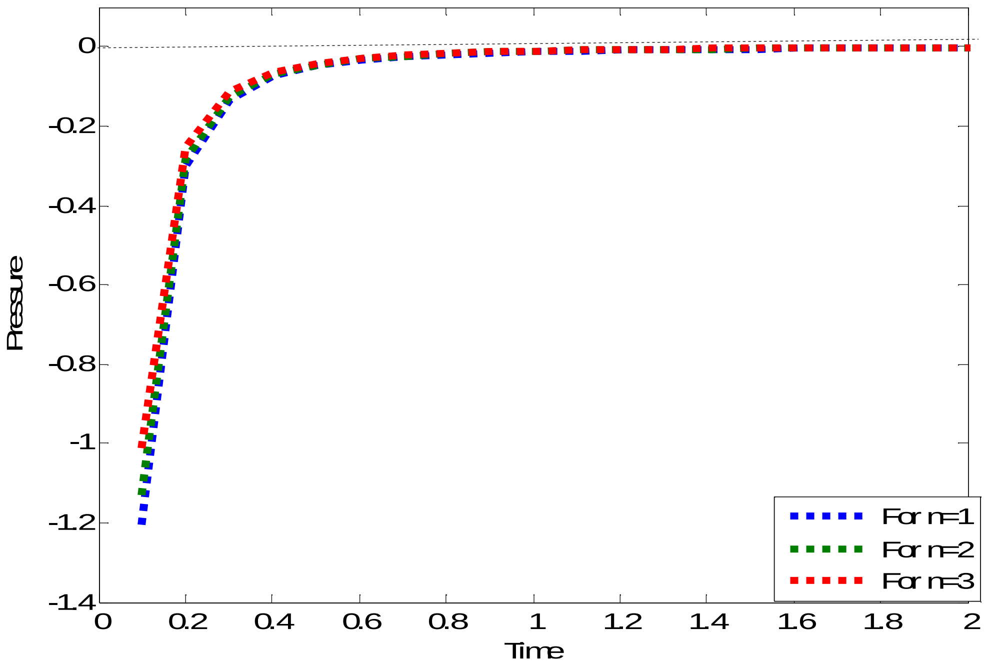
$$q = 2 \tag{3.5}$$

The average anisotropy parameter $A_m$ is given by

$$A_m = \frac{3(3n+2)(n-2)}{(n+2)^2} \tag{3.6}$$

The overall density parameter $\Omega$ is given by

$$\Omega = \frac{-3(2n+1)}{2(\lambda+4\pi)(n+2)^2} \tag{3.7}$$




## IV. CONCLUSIONS

In this paper we have presented an anisotropic Axially Symmetric space-time filled with perfect fluid in the framework of $f(R,T)$ gravity proposed by Harko et. al.(2011) and in general relativity. The model (2.1) has no initial singularity for positive values of m. The spatial volume increases with time. Since the mean anisotropy parameter $A_m \neq 0$, the models do not approach isotropy for $n \neq 2$. For $n = 2$, from field equations, we can easily see that we will get only isotropic Zeldovich universe. As $q = 2 > 0$, the model decelerates. The involvement of new function doesn't affect the geometry of the space-time but slightly changes the matter distribution. It is observed that the energy density and pressure tends to zero for large value of time t and spatial volume increases with time. For $t = \frac{-d}{c}$, the volume element of the model vanishes while all other parameters the scalar expansion $\theta$, shear scalar $\sigma$ and average Hubble parameter diverges. It is also observed that all the physical parameters are decreasing functions of time and they approach zero for large value of t.

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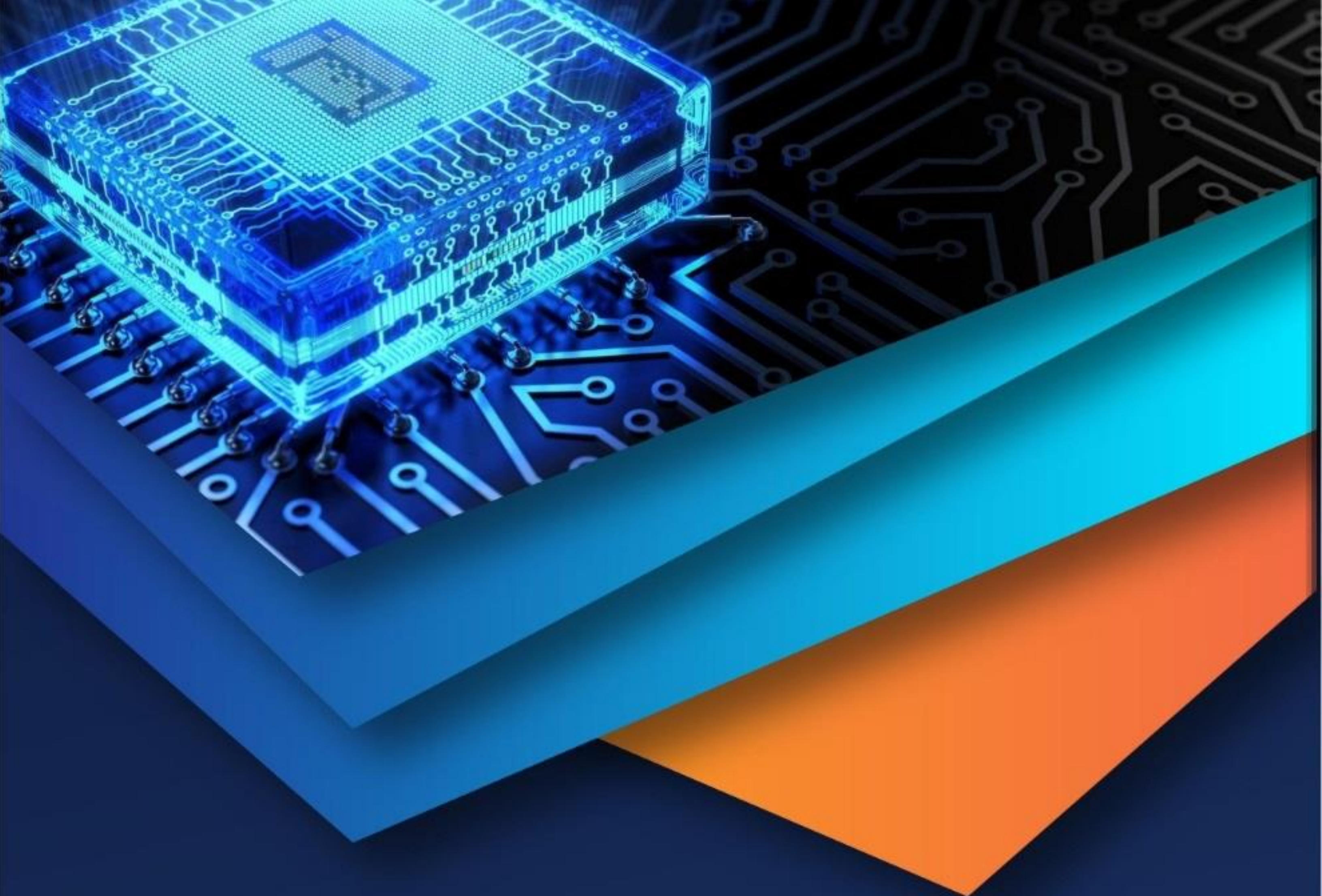


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