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# Bipolar Vague Contra $\alpha$ Generalized Closed Mappings in Topological Spaces

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**Abstract:** This paper is dedicated to the learning of bipolar vague topological spaces. In this paper we present the bipolar vague contra  $\alpha$  generalized closed mappings and bipolar vague contra  $\alpha$  generalized open mappings. Some of their belongings of bipolar vague contra  $\alpha$  generalized closed mappings and bipolar vague contra  $\alpha$  generalized open mappings are discussed.

**Keywords:** Bipolar vague topology, bipolar vague  $\alpha$  generalized closed sets, bipolar vague contra  $\alpha$  generalized closed mappings and bipolar vague contra  $\alpha$  generalized open mappings.

## I. INTRODUCTION

Uncertain set was familiarized by L.A.Zadeh [12] in 1965. The thought of fuzzy topology was announced by C.L.Chang [3] in 1968. The generalized closed sets in general topology presented by N.Levine [9] in 1970. K.Atanassov [2] in 1986 announced the perception of intuitionistic fuzzy sets. The belief of vague set theory was familiarized by W.L.Gau and D.J.Buehrer [7] in 1993. D.Coker [6] in 1997 familiarized intuitionistic fuzzy topological spaces. Bipolar- valued fuzzy sets, which was announced by K.M.Lee [8] in 2000 is a postponement of fuzzy sets whose membership degree range is inflamed from the interval  $[0, 1]$  to  $[-1, 1]$ . A creative class of generalized bipolar vague sets obtainable by S.Cicily Flora and I.Arockiarani [4] in 2016. We have presented bipolar vague  $\alpha$  generalized closed sets [10] in 2024. In this paper we familiarize bipolar vague contra  $\alpha$  generalized closed mappings and bipolar vague contra  $\alpha$  generalized open mappings and premeditated its basic properties. We deliver some characterizations of bipolar vague contra  $\alpha$  generalized closed mappings and bipolar vague contra  $\alpha$  generalized open mappings

## II. PRELIMINARIES

At this point in this paper the bipolar vague topological spaces are designated by  $(G, BV_\tau)$ . Also, the bipolar vague interior, bipolar vague closure of a bipolar vague set  $F$  are denoted by  $BVInt(F)$  and  $BVCl(F)$ . The complement of a bipolar vague set  $F$  is denoted by  $F^c$  and the empty set and whole sets are denoted by  $0_-$  and  $1_-$  individually.

**Definition 2.1:** [8] Let  $G$  be the universe. Then a bipolar valued fuzzy sets,  $F$  on  $G$  is defined by positive membership function  $\mu_F^+$ , that is  $\mu_F^+: G \rightarrow [0, 1]$ , and a negative membership function  $\mu_F^-$ , that is  $\mu_F^-: G \rightarrow [-1, 0]$ . For the sake of simplicity, we shall use the symbol

$$F = \{ \langle g, \mu_F^+(g), \mu_F^-(g) \rangle : g \in G \}.$$

**Definition 2.2:** [8] Let  $F$  and  $E$  be two bipolar valued fuzzy sets then their union, intersection and complement are defined as follows:

- (i)  $\mu_{F \cup E}^+ = \max \{ \mu_F^+(g), \mu_E^+(g) \}$
- (ii)  $\mu_{F \cup E}^- = \min \{ \mu_F^-(g), \mu_E^-(g) \}$
- (iii)  $\mu_{F \cap E}^+ = \min \{ \mu_F^+(g), \mu_E^+(g) \}$
- (iv)  $\mu_{F \cap E}^- = \max \{ \mu_F^-(g), \mu_E^-(g) \}$
- (v)  $\mu_{F^c}^+(g) = 1 - \mu_F^+(g)$  and  $\mu_{F^c}^-(g) = -1 - \mu_F^-(g)$  for all  $g \in G$ .

**Definition 2.3:** [7] A vague set  $F$  in the universe of discourse  $L$  is a pair of  $(t_F, f_F)$  where  $t_F: L \rightarrow [0, 1]$ ,  $f_F: L \rightarrow [0, 1]$  are the mapping such that  $t_F + f_F \leq 1$  for all  $l \in L$ . The function  $t_F$  and  $f_F$  are called true membership function and false membership function respectively. The interval  $[t_F, 1 - f_F]$  is called the vague value of  $l$  in  $F$ , and denoted by  $v_F(l)$ , that is  $v_F(l) = [t_F(l), 1 - f_F(l)]$ .

**Definition 2.4:** [7] Let  $F$  be a non-empty set and the vague set  $F$  and  $E$  in the form

$F =$

$\{ \langle g, t_F(g), 1 - f_F(g) \rangle : g \in G \}$ ,  $E = \{ \langle g, t_E(g), 1 - f_E(g) \rangle : g \in G \}$ . Then

- (i)  $F \subseteq E$  if and only if  $t_F(g) \leq t_E(g)$  and  $1 - f_F(g) \leq 1 - f_E(g)$
- (ii)  $F \cup E = \{ \langle \max(t_F(g), t_E(g)), \max(1 - f_F(g), 1 - f_E(g)) \rangle : g \in G \}$ .

$$(iii) \quad F \cap E = \{ \langle \min(t_F(g), t_E(g)), \min(1 - f_F(g), 1 - f_E(g)) \rangle / g \in G \}.$$

$$(iv) \quad F^c = \{ \langle g, f_F(g), 1 - t_F(g) \rangle : g \in G \}.$$

Definition 2.5: [1] Let  $G$  be the universe of discourse. A bipolar-valued vague set  $F$  in  $G$  is an object having the form  $F = \{ \langle g, [t_F^+(g), 1 - f_F^+(g)], [-1 - f_F^-(g), t_F^-(g)] \rangle : g \in G \}$  where  $[t_F^+, 1 - f_F^+] : G \rightarrow [0, 1]$  and  $[-1 - f_F^-, t_F^-] : G \rightarrow [-1, 0]$  are the mapping such that

$$t_F^+(g) + f_F^+(g) \leq 1 \text{ and } -1 \leq t_F^-(g) + f_F^-(g).$$

The positive membership degree  $[t_F^+(g), 1 - f_F^+(g)]$  denotes the satisfaction region of an element  $g$  to the property corresponding to a bipolar-valued set  $F$  and the negative membership degree  $[-1 - f_F^-(g), t_F^-(g)]$  denotes the satisfaction region of  $g$  to some implicit counter property of  $F$ . For a sake of simplicity, we shall use the notion of bipolar vague set  $v_F^+ = [t_F^+, 1 - f_F^+]$  and  $v_F^- = [-1 - f_F^-, t_F^-]$ .

Definition 2.6: [5] A bipolar vague set  $F = [v_F^+, v_F^-]$  of a set  $L$  with  $v_F^+ = 0$  implies that  $t_F^+ = 0$ ,  $1 - f_F^+ = 0$  and  $v_F^- = 0$  implies that  $t_F^- = 0$ ,  $-1 - f_F^- = 0$  for all  $g \in L$  is called zero bipolar vague set and it is denoted by  $0$ .

Definition 2.7: [5] A bipolar vague set  $F = [v_F^+, v_F^-]$  of a set  $L$  with  $v_F^+ = 1$  implies that  $t_F^+ = 1$ ,  $1 - f_F^+ = 1$  and  $v_F^- = -1$  implies that  $t_F^- = -1$ ,  $-1 - f_F^- = -1$  for all  $g \in L$  is called unit bipolar vague set and it is denoted by  $1$ .

Definition 2.8: [4] Let  $F = \langle g, [t_F^+, 1 - f_F^+], [-1 - f_F^-, t_F^-] \rangle$  and  $E = \langle g, [t_E^+, 1 - f_E^+], [-1 - f_E^-, t_E^-] \rangle$  be two bipolar vague sets then their union, intersection and complement are defined as follows:

$$(i) \quad F \cup E = \{ \langle g, [t_{F \cup E}^+(g), 1 - f_{F \cup E}^+(g)], [-1 - f_{F \cup E}^-(g), t_{F \cup E}^-(g)] \rangle / g \in G \} \text{ where}$$

$$t_{F \cup E}^+(g) = \max \{ t_F^+(g), t_E^+(g) \}, t_{F \cup E}^-(g) = \min \{ t_F^-(g), t_E^-(g) \} \text{ and}$$

$$1 - f_{F \cup E}^+(g) = \max \{ 1 - f_F^+(g), 1 - f_E^+(g) \},$$

$$-1 - f_{F \cup E}^-(g) = \min \{ -1 - f_F^-(g), -1 - f_E^-(g) \}.$$

$$(ii) \quad F \cap E = \{ \langle g, [t_{F \cap E}^+(g), 1 - f_{F \cap E}^+(g)], [-1 - f_{F \cap E}^-(g), t_{F \cap E}^-(g)] \rangle / g \in G \} \text{ where}$$

$$t_{F \cap E}^+(g) = \min \{ t_F^+(g), t_E^+(g) \}, t_{F \cap E}^-(g) = \max \{ t_F^-(g), t_E^-(g) \} \text{ and}$$

$$1 - f_{F \cap E}^+(g) = \min \{ 1 - f_F^+(g), 1 - f_E^+(g) \},$$

$$-1 - f_{F \cap E}^-(g) = \max \{ -1 - f_F^-(g), -1 - f_E^-(g) \}.$$

$$(iii) \quad F^c = \{ \langle g, [f_F^+(g), 1 - t_F^+(g)], [-1 - t_F^-(g), f_F^-(g)] \rangle / g \in G \}.$$

Definition 2.9: [4] Let  $F$  and  $E$  be two bipolar vague sets defined over a universe of discourse  $G$ . We say that  $F \subseteq E$  if and only if  $t_F^+(g) \leq t_E^+(g)$ ,  $1 - f_F^+(g) \leq 1 - f_E^+(g)$  and  $t_F^-(g) \geq t_E^-(g)$ ,  $-1 - f_F^-(g) \geq -1 - f_E^-(g)$  for all  $g \in G$ .

Definition 2.10: [4] A bipolar vague topology (BVT) on a non-empty set  $G$  is a family  $BV_\tau$  of bipolar vague set in  $G$  satisfying the following axioms:

$$(i) \quad 0, 1 \in BV_\tau$$

$$(ii) \quad U_1 \cap U_2 \in BV_\tau, \text{ for any } U_1, U_2 \in BV_\tau$$

$$(iii) \quad \cup U_i \in BV_\tau, \text{ for any arbitrary family } \{U_i : U_i \in BV_\tau, i \in I\}.$$

In this case the pair  $(G, BV_\tau)$  is called a bipolar vague topological space and any bipolar vague set (BVS) in  $BV_\tau$  is known as bipolar vague open set in  $G$ . The complement  $F^c$  of a bipolar vague open set (BVOS)  $F$  in a bipolar vague topological space  $(G, BV_\tau)$  is called a bipolar vague closed set (BVCS) in  $G$ .

Definition 2.11: [4] Let  $(G, BV_\tau)$  be a bipolar vague topological space  $F = \langle g, [t_F^+, 1 - f_F^+], [-1 - f_F^-, t_F^-] \rangle$  be a bipolar vague set in  $G$ . Then the bipolar vague interior and bipolar vague closure of  $F$  are defined by,

$$BVInt(F) = \cup \{ U : U \text{ is a bipolar vague open set in } F \text{ and } U \subseteq F \},$$

$$BVCl(F) = \cap \{ I : I \text{ is a bipolar vague closed set in } F \text{ and } F \subseteq I \}.$$

Note that  $BVCl(F)$  is a bipolar vague closed set and  $BVInt(F)$  is a bipolar vague open set in  $G$ . Further,

$$(i) \quad A \text{ is a bipolar vague closed set in } G \text{ if and only if } BVCl(F) = F,$$

$$(ii) \quad A \text{ is a bipolar vague open set in } G \text{ if and only if } BVInt(F) = F.$$

Definition 2.12: [4] Let  $(G, BV_\tau)$  be a bipolar vague topological space. A bipolar vague set  $F$  in  $(G, BV_\tau)$  is said to be a generalized bipolar vague closed set if  $BVCl(F) \subseteq U$  whenever  $F \subseteq U$  and  $U$  is bipolar vague open. The complement of a generalized bipolar vague closed set is generalized bipolar vague open set.

Definition 2.13: [4] Let  $(G, BV_\tau)$  be a bipolar vague topological space and  $F$  be a bipolar vague set in  $G$ . Then the generalized bipolar vague closure and generalized bipolar vague interior of  $F$  are defined by,

$$GBVCl(F) = \cap \{ U : U \text{ is a generalized bipolar vague closed set in } G \text{ and } F \subseteq U \},$$

$$GBVInt(F) = \cup \{ U : U \text{ is a generalized bipolar vague open set in } G \text{ and } F \supseteq U \}.$$

Definition 2.14: [10] A bipolar vague set  $F$  of a bipolar vague topological space  $G$ , is said to be

$$(i) \quad \text{a bipolar vague } \alpha\text{-open set if } F \subseteq BVInt(BVCl(BVInt(F)))$$



- (ii) a bipolar vague pre-open set if  $F \subseteq BVInt(BVCl(F))$
- (iii) a bipolar vague semi-open set if  $F \subseteq BVCl(BVInt(F))$
- (iv) a bipolar vague semi- $\alpha$ -open set if  $F \subseteq BVCl(\alpha BVInt(F))$
- (v) a bipolar vague regular-open set  $BVInt(BVCl(F)) = F$
- (vi) a bipolar vague  $\beta$ -open set  $F \subseteq BVCl(BVInt(BVCl(F)))$ .

Definition 2.15: [10] A bipolar vague set  $F$  of a bipolar vague topological space  $G$ , is said to be

- (i) a bipolar vague  $\alpha$ -closed set if  $BVCl(BVInt(BVCl(F))) \subseteq F$
- (ii) a bipolar vague pre-closed set if  $BVCl(BVInt(F)) \subseteq F$
- (iii) a bipolar vague semi-closed set if  $BVInt(BVCl(F)) \subseteq F$
- (iv) a bipolar vague semi- $\alpha$ -closed set if  $BVInt(\alpha BVCl(F)) \subseteq F$
- (v) a bipolar vague regular-closed set if  $BVCl(BVInt(F)) = F$
- (vi) a bipolar vague  $\beta$ -closed set if  $BVInt(BVCl(BVInt(F))) \subseteq F$ .

Definition 2.16: [10] Let  $F$  be a bipolar vague set of a bipolar vague topological space  $(G, BV_\tau)$ . Then the bipolar vague  $\alpha$  interior and bipolar vague  $\alpha$  closure are defined as

$$BV_\alpha Int(F) = \cup \{U: U \text{ is a bipolar vague } \alpha\text{-open set in } G \text{ and } U \subseteq F\},$$

$$BV_\alpha Cl(A) = \cap \{T: T \text{ is a bipolar vague } \alpha\text{-closed set in } G \text{ and } F \subseteq T\}.$$

Definition 2.17: [10] A bipolar vague set  $F$  in a bipolar vague topological space  $G$ , is said to be a bipolar vague  $\alpha$  generalized closed set if  $BV_\alpha Cl(F) \subseteq L$  whenever  $A \subseteq L$  and  $L$  is a bipolar vague open set in  $G$ . The complement  $F^c$  of a bipolar vague  $\alpha$  generalized closed set  $F$  is a bipolar vague  $\alpha$  generalized open set in  $G$ .

Definition 2.18: [11] A bipolar vague topological space  $(G, BV_\tau)$  is said to be bipolar vague  $\alpha a T_{1/2}(BV_{aa} T_{1/2})$  space if every bipolar vague  $\alpha$  generalized closed set in  $G$  is a bipolar vague closed set in  $G$ .

Definition 2.19: [11] A bipolar vague topological space  $(G, BV_\tau)$  is said to be bipolar vague  $\alpha b T_{1/2}(BV_{ab} T_{1/2})$  space if every bipolar vague  $\alpha$  generalized closed set in  $G$  is a bipolar vague generalized closed set in  $G$ .

### III. BIPOLAR VAGUE CONTRA $\alpha$ GENERALIZED CLOSED MAPPINGS IN TOPOLOGICAL SPACES

In this sector, we present bipolar vague contra  $\alpha$  generalized closed mappings, bipolar vague contra  $\alpha$  generalized open mappings and study about of its belongings.

Definition 3.1: Let  $f$  be a mapping from a bipolar vague topological space  $(G, BV_\tau)$  into a bipolar vague topological space  $(R, BV_\sigma)$ . Then  $f$  is said to be a bipolar vague contra closed mapping if for every bipolar vague closed set  $F$  in  $G$ ,  $f(F)$  is a bipolar vague open set in  $R$ .

Definition 3.2: Let  $f$  be a mapping from a bipolar vague topological space  $(G, BV_\tau)$  into a bipolar vague topological space  $(R, BV_\sigma)$ . Then  $f$  is said to be a bipolar vague contra  $\alpha$ -closed mapping if for every bipolar vague closed set  $F$  in  $G$ ,  $f(F)$  is a bipolar vague  $\alpha$ -open set in  $R$ .

Definition 3.3: Let  $f$  be a mapping from a bipolar vague topological space  $(G, BV_\tau)$  into a bipolar vague topological space  $(R, BV_\sigma)$ . Then  $f$  is said to be a bipolar vague contra generalized closed mapping if for every bipolar vague closed set  $F$  in  $G$ ,  $f(F)$  is a bipolar vague generalized open set in  $R$ .

Definition 3.4: A mapping  $f: (G, BV_\tau) \rightarrow (R, BV_\sigma)$  is called a bipolar vague contra  $\alpha$  generalized closed mapping if for every bipolar vague closed set  $F$  of  $(G, BV_\tau)$ ,  $f(F)$  is a bipolar vague  $\alpha$  generalized open set in  $(R, BV_\sigma)$ .

Example 3.5: Let  $G = \{e, f\}$  and  $R = \{s, t\}$  and  $F = \langle g, [0.4, 0.4] [-0.4, -0.4], [0.2, 0.3] [-0.3, -0.2] \rangle$  and  $E = \langle r, [0.2, 0.4] [-0.4, -0.2], [0.2, 0.2] [-0.2, -0.2] \rangle$ . Then  $\tau = \{0_-, F, 1_-\}$  and  $\sigma = \{0_-, E, 1_-\}$  are bipolar vague topologies on  $G$  and  $R$  respectively. Define a mapping  $f: (G, BV_\tau) \rightarrow (R, BV_\sigma)$  by  $f(e) = s$  and  $f(f) = t$ . The bipolar vague set  $F^c = \langle g, [0.6, 0.6] [-0.6, -0.6], [0.7, 0.8] [-0.8, -0.7] \rangle$  is a bipolar vague closed set in  $G$ . Now,  $f(F^c) = \langle r, [0.6, 0.6] [-0.6, -0.6], [0.7, 0.8] [-0.8, -0.7] \rangle$  is a bipolar vague  $\alpha$  generalized open set in  $R$  as  $f(F^c) \cup BVInt(BVCl(BVInt(f(F^c)))) = f(F^c) \cup E = f(F^c)$  whenever  $f(F^c) \subseteq 1_-$ . Therefore  $f$  is a bipolar vague contra  $\alpha$  generalized closed mapping.

Definition 3.6: A mapping  $f: (G, BV_\tau) \rightarrow (R, BV_\sigma)$  is called a bipolar vague contra  $\alpha$  generalized open mapping if for every bipolar vague open set  $F$  of  $(G, BV_\tau)$ ,  $f(F)$  is a bipolar vague  $\alpha$  generalized closed set in  $(R, BV_\sigma)$ .

Example 3.7: Let  $G = \{e, f\}$  and  $R = \{s, t\}$  and  $F = \langle g, [0.3, 0.4] [-0.4, -0.3], [0.5, 0.5] [-0.6, -0.6] \rangle$  and  $E = \langle r, [0.1, 0.5] [-0.5, -0.1], [0.5, 0.5] [-0.6, -0.2] \rangle$ . Then  $\tau = \{0_-, F, 1_-\}$  and  $\sigma = \{0_-, E, 1_-\}$  are bipolar vague topologies on  $G$  and  $R$  respectively. Define a mapping  $f: (G, BV_\tau) \rightarrow (R, BV_\sigma)$  by  $f(e) = s$  and  $f(f) = t$ . The bipolar vague set  $F = \langle g, [0.3, 0.4] [-0.4, -0.3], [0.5, 0.5] [-0.6, -0.6] \rangle$  is a bipolar vague open set in  $G$ . Now,  $f(F) = \langle s, [0.3, 0.4] [-0.4, -0.3], [0.5, 0.5] [-0.6, -0.6] \rangle$  is a bipolar vague  $\alpha$  generalized closed set in  $R$  as  $f(F) \cup BVInt(BVCl(BVInt(f(F)))) = f(F) \cup E = f(F)$  whenever  $f(F) \subseteq 1_-$ . Therefore  $f$  is a bipolar vague contra  $\alpha$  generalized open mapping.

$[-0.4, -0.3], [0.5, 0.5] [-0.6, -0.6]$  is a bipolar vague open set in  $G$ . Now,  $f(F) = \langle r, [0.3, 0.4] [-0.4, -0.3], [0.5, 0.5] [-0.6, -0.6] \rangle$  is a bipolar vague  $\alpha$  generalized closed set in  $R$  as  $f(F) \cup BVCI(BVInt(BVCI(f(F)))) = f(F) \cup E^c = E^c \subseteq 1_-$  whenever  $f(F) \subseteq 1_-$ . Therefore  $f$  is a bipolar vague contra  $\alpha$  generalized open mapping.

**Proposition 3.8:** Every bipolar vague contra closed mapping is a bipolar vague contra  $\alpha$  generalized closed mapping but not conversely in general.

**Proof:** Let  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  be a bipolar vague contra closed mapping. Let  $F$  be a bipolar vague closed set in  $G$ . Since  $f$  is a bipolar vague contra closed mapping,  $f(F)$  is a bipolar vague open set in  $R$ . Since every bipolar vague open set is a bipolar vague  $\alpha$  generalized open set,  $f(F)$  is a bipolar vague  $\alpha$  generalized open set in  $R$ . Hence  $f$  is a bipolar vague contra  $\alpha$  generalized closed mapping.

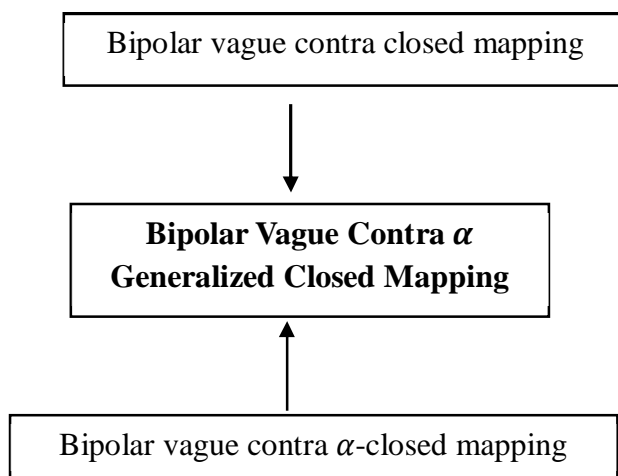
**Example 3.9:** Let  $G = \{e, f\}$  and  $R = \{s, t\}$  and  $F = \langle g, [0.6, 0.6] [-0.6, -0.6], [0.5, 0.8] [-0.8, -0.5] \rangle$  and  $E = \langle r, [0.5, 0.5] [-0.5, -0.5], [0.4, 0.4] [-0.4, -0.4] \rangle$ . Then  $\tau = \{0_-, F, 1_-\}$  and  $\sigma = \{0_-, E, 1_-\}$  are bipolar vague topologies on  $G$  and  $R$  respectively. Define a mapping  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  by  $f(e) = s$  and  $f(f) = t$ . Then  $f$  is a bipolar vague contra  $\alpha$  generalized closed mapping but not a bipolar vague contra closed mapping since  $D = \langle g, [0.7, 0.7] [-0.7, -0.7], [0.6, 0.9] [-0.9, -0.6] \rangle$  is a bipolar vague closed set in  $G$  but  $f(H) = \langle r, [0.7, 0.7] [-0.7, -0.7], [0.6, 0.9] [-0.9, -0.6] \rangle$  is not a bipolar vague open set in  $R$  as  $BVInt(f(H)) = E \neq f(H)$ .

**Proposition 3.10:** Every bipolar vague contra  $\alpha$ -closed mapping is a bipolar vague contra  $\alpha$  generalized closed mapping but not conversely in general.

**Proof:** Let  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  be a bipolar vague contra  $\alpha$ -closed mapping. Let  $F$  be a bipolar vague closed set in  $G$ . Then by hypothesis,  $f(F)$  is a bipolar vague  $\alpha$ -open set in  $R$ . Since every bipolar vague  $\alpha$ -open set is a bipolar vague  $\alpha$  generalized open set,  $f(F)$  is a bipolar vague  $\alpha$  generalized open set in  $R$ . Hence  $f$  is a bipolar vague contra  $\alpha$  generalized closed mapping.

**Example 3.11:** Let  $G = \{e, f\}$  and  $R = \{s, t\}$  and  $F = \langle g, [0.5, 0.5] [-0.5, -0.5], [0.5, 0.7] [-0.7, -0.4] \rangle$  and  $E = \langle r, [0.4, 0.4] [-0.4, -0.4], [0.3, 0.3] [-0.3, -0.3] \rangle$ . Then  $\tau = \{0_-, F, 1_-\}$  and  $\sigma = \{0_-, E, 1_-\}$  are bipolar vague topologies on  $G$  and  $R$  respectively. Define a mapping  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  by  $f(e) = s$  and  $f(f) = t$ . Then  $f$  is a bipolar vague contra  $\alpha$  generalized closed mapping but not a bipolar vague contra  $\alpha$ -closed mapping since  $H = \langle g, [0.6, 0.6] [-0.6, -0.6], [0.5, 0.8] [-0.8, -0.5] \rangle$  is a bipolar vague closed set in  $G$  but  $f(H) = \langle r, [0.6, 0.6] [-0.6, -0.6], [0.5, 0.8] [-0.8, -0.5] \rangle$  is not a bipolar vague  $\alpha$ -open set in  $R$  as  $BVInt(BVCI(BVInt(f(H)))) = E \neq f(H)$ .

The relations between various types of bipolar vague contra closed mappings are given in the following diagram:



**Proposition 3.12:** A mapping  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  is a bipolar vague contra  $\alpha$ -closed mapping if and only if the image of each bipolar vague open set in  $G$  is a bipolar vague  $\alpha$  generalized closed set in  $R$ .

**Proof:** Let  $F$  be a bipolar vague open set in  $G$ . This implies  $F^c$  is a bipolar vague closed set in  $G$ . Since  $f$  is a bipolar vague contra  $\alpha$  generalized closed mapping,  $f(F^c)$  is a bipolar vague  $\alpha$  generalized open set in  $R$ . Since  $f(F^c) = (f(F))^c$ ,  $f(F)$  is a bipolar vague  $\alpha$  generalized closed set in  $R$ .

**Proposition 3.13:** Let  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  be a bipolar vague contra  $\alpha$  generalized closed mapping, then  $f$  is a bipolar vague contra closed mapping if  $R$  is a  $BV_{\alpha\alpha}T_{1/2}$  space.

Proof: Let  $F$  be a bipolar vague closed set in  $G$ . Then  $f(F)$  is a bipolar vague  $\alpha$  generalized open set in  $R$ , by hypothesis. Since  $R$  is a  $BV_{aa}T_{1/2}$  space,  $f(F)$  is a bipolar vague open set in  $R$ . Hence  $f$  is a bipolar vague contra closed mapping.

Proposition 3.14: Let  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  be a bipolar vague contra  $\alpha$  generalized closed mapping. Then  $f$  is a bipolar vague contra generalized closed mapping if  $R$  is a  $BV_{ab}T_{1/2}$  space.

Proof: Let  $F$  be a bipolar vague closed set in  $G$ . Then  $f(F)$  is a bipolar vague  $\alpha$  generalized open set in  $R$ , by hypothesis. Since  $R$  is a  $BV_{ab}T_{1/2}$  space,  $f(F)$  is a bipolar vague generalized open set in  $R$ . Hence  $f$  is a bipolar vague contra generalized closed mapping.

Proposition 3.14: Let  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  be a mapping from a bipolar vague topological space  $G$  into a bipolar vague topological space  $R$ . Then the following conditions are equivalent if  $R$  is a  $BV_{aa}T_{1/2}$  space.

- (i)  $f$  is a bipolar vague contra  $\alpha$  generalized closed mapping.
- (ii) If  $F$  is a bipolar vague open set in  $G$ , then  $f(F)$  is a bipolar vague  $\alpha$  generalized closed set in  $R$ .
- (iii)  $BVCl(BVInt(BVCl(f(F)))) \subseteq f(F)$  for every bipolar vague set  $F$  in  $G$ .

Proof: (i)  $\Rightarrow$  (ii) is obviously true.

(ii)  $\Rightarrow$  (iii) Let  $F$  be any bipolar vague set in  $G$ . Then  $BVInt(F)$  is a bipolar vague open set in  $G$ . Thus  $f(BVInt(F))$  is a bipolar vague  $\alpha$  generalized closed set in  $R$ . Since  $R$  is a  $BV_{aa}T_{1/2}$  space,  $f(BVInt(F))$  is a bipolar vague closed set in  $R$ . Therefore  $BVCl(f(BVInt(F))) = f(BVInt(F))$ . This implies  $BVCl(BVInt(BVCl(f(F)))) \subseteq f(F)$ .

(iii)  $\Rightarrow$  (i) Let  $F$  be a bipolar vague closed set in  $G$ . Then its complement  $F^c$  is a bipolar vague open set in  $G$ . By hypothesis,  $BVCl(BVInt(BVCl(f(F^c)))) \subseteq f(F^c)$ . Hence  $f(F^c)$  is a bipolar vague  $\alpha$ -closed set in  $R$ . Since every bipolar vague  $\alpha$ -closed set is a bipolar vague  $\alpha$  generalized closed set,  $f(F^c)$  is a bipolar vague  $\alpha$  generalized closed set in  $G$ . Therefore  $f(F)$  is a bipolar vague  $\alpha$  generalized open set in  $G$ . Hence  $f$  is a bipolar vague contra  $\alpha$  generalized closed mapping.

Proposition 3.15: Let  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  be a bipolar vague closed mapping and  $g : (R, BV_\sigma) \rightarrow (C, BV_\delta)$  be a bipolar vague contra  $\alpha$  generalized closed mapping. Then  $g \circ f : (X, BV_\tau) \rightarrow (C, BV_\delta)$  is a bipolar vague contra  $\alpha$  generalized closed mapping.

Proof: Let  $F$  be a bipolar vague closed set in  $G$ . Then  $f(F)$  is a bipolar vague closed set in  $R$ , by hypothesis. Since  $g$  is a bipolar vague contra  $\alpha$  generalized closed mapping,  $g(f(F))$  is a bipolar vague  $\alpha$  generalized open set in  $C$ . Hence  $f \circ g$  is a bipolar vague contra  $\alpha$  generalized closed mapping.

Proposition 3.16: Let  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  be a bipolar vague contra  $\alpha$  generalized closed mapping and  $R$  is a  $BV_{ab}T_{1/2}$  space, then  $f(F)$  is a bipolar vague generalized open set in  $R$  for every bipolar vague closed set  $F$  in  $G$ .

Proof: Let  $f : (G, BV_\tau) \rightarrow (R, BV_\sigma)$  be a mapping and let  $F$  be a bipolar vague closed set in  $G$ . Then by hypothesis  $f(F)$  is a bipolar vague  $\alpha$  generalized open set in  $R$ . Since  $R$  is a  $BV_{ab}T_{1/2}$  space,  $f(F)$  is a bipolar vague generalized open set in  $R$ .

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