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Characterization of a Vertex Colouring of a Double Layered Complete Fuzzy Graph Using α – CUT

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***Abstract:** In this paper we defined a new fuzzy graph named Double layered complete fuzzy graph. (DLCFG). The double layered complete fuzzy graph gives a 3-D structure. Further we introduced vertex colouring of the double layered complete fuzzy graph using α-cut.*
***Keywords:** Graph theory, Fuzzy graph, colouring of graphs, double layered fuzzy graph, complete fuzzy graph, alpha cut, colouring of double layered fuzzy graph, colouring of double layered complete fuzzy using alpha cut.*
***AMS CLASSIFICATION:** 05Cxx, 05C63, 05C15.*

I. INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975.Graph theory is proved to be useful in modelling the essential features of systems with finite components. Graph theory plays a major role in colouring of various graphs. If the relation among accounts is to be measured as good or bad according to the frequency of contacts among the accounts, fuzziness should be added to representation. This and many another problem motivated to define fuzzy graphs Rosenfeld first introduced the concept of fuzzy graph. Fuzzy graph is a part of our life of graph theory which is involved in colouring, fuzzy logic, and some features of graphical concepts. After that fuzzy graph theory becomes a vast researched area. A fuzzy set is defined mathematically by assigning to each possible individual in the universe of discourse a value, representing its grade of membership, which corresponds to the degree, to which that individual is similar or compatible with the concept represented by the fuzzy set. Fuzzy graphs have many more applications in modelling real time systems where the level of information inherent in the systems varies with different level of precision. In this paper ,The notion of fuzzy set which is characterized by a membership function which assigns to each object a grade of membership which ranges from 0 to 1. The fuzzy colouring problem consists of determining the chromatic number of a fuzzy graph and an associated colouring function. For any level Alpha, the minimum number of colours needed to colour the crisp graph G α will be computed. In this way the fuzzy chromatic number is defined as fuzzy number through it's α– cuts.

A. Defenition: Fuzzy Set

A Fuzzy set A is defined on a non empty set X is the family A= $\{(x, \mu_A(x))/x Є X\}$ where $\mu_{A:}$ x →I is the membership function. In fuzzy set theory the set I is usually defined as the interval [0, 1] such that $\mu_A(x) = 0$ if x does not belong to A, $\mu_A(x) = 1$ if x strictly belongs to A and any immediate value represents the degree in which x could belong to A. The set I could be discrete set of the form I = {0, 1 ... k} where $\mu_A(x) < \mu_A(x I)$ indicates that the degree of the membership of x to A is lower than the degree of membership of X_I.

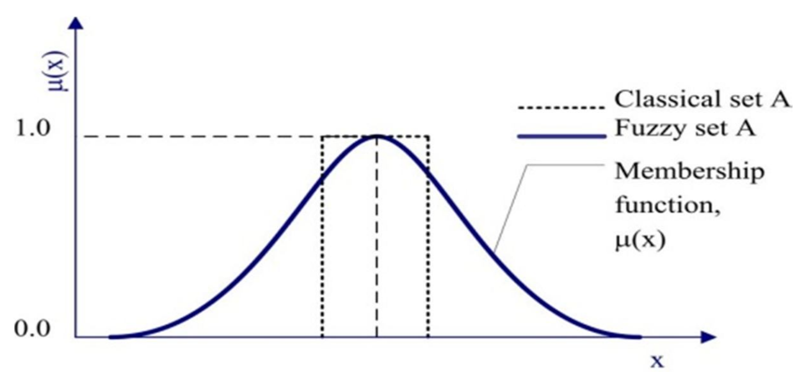

Fig 1; Sample fuzzy set of a membership function

B. Defenition: Fuzzy Graph

Formally, a fuzzy graph G = (V, μ, ρ) is a nonempty set V together with a pair of functions μ: V → [0, 1] and μ: V × V → [0, 1] such that for all x, y in V, ρ(x, y) ≤ μ(x) ∧ μ(y). We call μ the fuzzy vertex set of G and ρ the fuzzy edge set of G, respectively. Note that ρ is a fuzzy relation on μ.

Example: A fuzzy graph $G = (\sigma,\mu)$ is a pair of function $\sigma: V \rightarrow [0,1]$ and $\mu: V\times V \rightarrow [0,1]$, where for all $x,y \in V$, we have $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$.

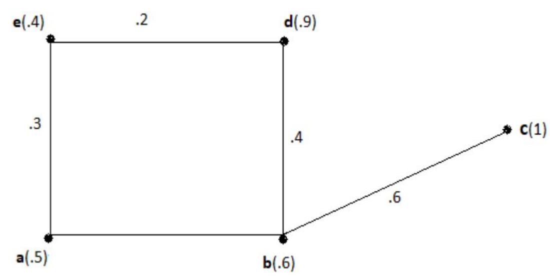

Fig 2; fuzzy graph

C. Defenition: Order And Size

The order and size of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $\sum_{x\in v} \sigma(X)$ and $\sum_{xy\in E} \mu(XY)$. It is also denoted as O (G) and S (G).

D. Defenition: Degree Of A Vertex

Let G = (σ, μ) be a fuzzy graph. The degree of a vertex u is defined as $d(u) = \sum_{uv\in G} \mu(u, v)$. It is also denoted as $d_G(u)$.

E. Defenition: Complete Fuzzy Graph

A fuzzy graph G: (σ, μ) is complete if μ(x, y) = σ(x) ∧ σ(y) for all x, y ∈ V. Next, Let G: (σ, μ) be a self-complementary fuzzy graph. Then $\sum_{x, y\in V} \mu(x, y) = (1/2) \sum_{x, y\in V} (\sigma(x) \wedge \sigma(y))$.

Example: Let $\sigma: V \rightarrow [0, 1]$ be a fuzzy subset of V then the complete fuzzy graph on σ is defined on $G = (\sigma, \mu)$ where $\mu(xy) = \sigma(x) \wedge \sigma(y)\ \forall x, y \in E$. It is denoted by k_σ.

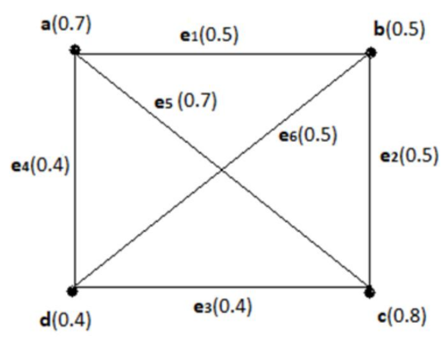

Fig 3; complete fuzzy graph

F. Defenition: A-CUT

It was continued by Arindam Dey, et al (2013) for the fuzzy set of vertices and edges. Alpha-cut on fuzzy graph is crisp graphs Gα that are induced from fuzzy graph by removing all vertex and edges in fuzzy graphs that have a degree of membership less than α, α∈[0,1].

The α cut of fuzzy graph defined as = G_α (Vα', Eα) where $V\alpha = \{v \in V / \sigma \geq \alpha\}$ and $E\alpha = \{e \in E / \mu \geq \alpha\}$.

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G. Defenition: K-Coloring Fuzzy Graph

A K-colouring problem for undirected graphs is an assignment of colours to the nodes of the graph such that no two adjacent vertices have the same colour, and at most K colours are used to complete colour the graph.

A family Γ = {γ1, γ2 ... γk} of fuzzy sets on V is called a $\boldsymbol{k}$-colouring of fuzzy graph $G = (V,,)$

- $\vee\Gamma = \sigma$,
- γi ∧ γj = 0
- For every strong edge xy of G, min {(γi (x), γi (y)} = 0 (1≤ $i \leq k$)

The least value of k for which G has a k-fuzzy colouring denoted by X_f (G) is called the fuzzy chromatic number of G.

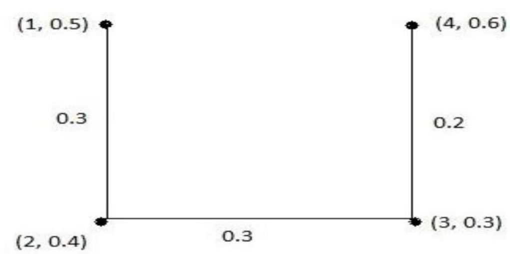


Fig 4; K Colouring of fuzzy graph

H. Double Layered Complete Fuzzy Graph

1) Defenition: Let σDL: V→ [0, 1] be a subset of V and μDL: V× V→ [0, 1] be a symmetric fuzzy relation on σDL. Any two vertex of the double layered complete fuzzy graph is adjacent [12]. The vertex set of complete double layered fuzzy graph be σ U μ and it's denoted by K σ U μ.

Example: Complete fuzzy graph with vertices 3 (K_3)

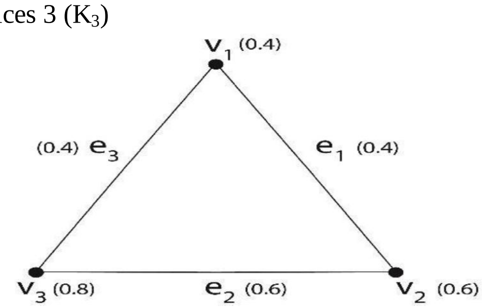


Fig 5; fuzzy graph with vertices 3

II. VERTEX COLORING OF DOUBLE LAYERED COMPLETE FUZZY GRAPH USING α- CUT.

In this DLCFG (fig 6), there are 3 Alpha -cut is presented, they are {0.4, 0.6, 0.8}.For every value of Alpha, we find DLCF (Gα) and find its fuzzy chromatic number. For **α=0.4** Double layered complete fuzzy graph

DLC (G) = (σ_{DL}, μ_{DL}) Where σ_{DLC} = {0.8, 0.6, 0.4}

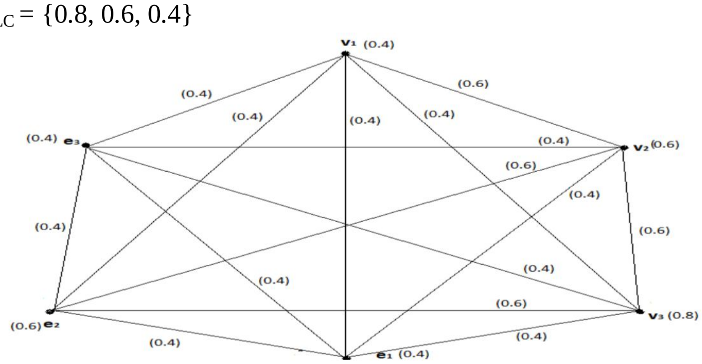


Fig 6; vertex colouring of double layered complete fuzzy graph using alpha cut

Here we need minimum 6 colours to proper colour all the vertices of the graph DLC ($G_{0.4}$).so the chromatic number of DLC ($G_{0.4}$) is 6

For α= 0.4, $X_{0.4} = X_{DL}$ (0.4) = 6. For **α=0.6** Double layered complete fuzzy graph DLC (G) = (σ_{DL}, μ_{DL}) Where σ_{DLC} = {0.8, 0.6}. (Fig 7)

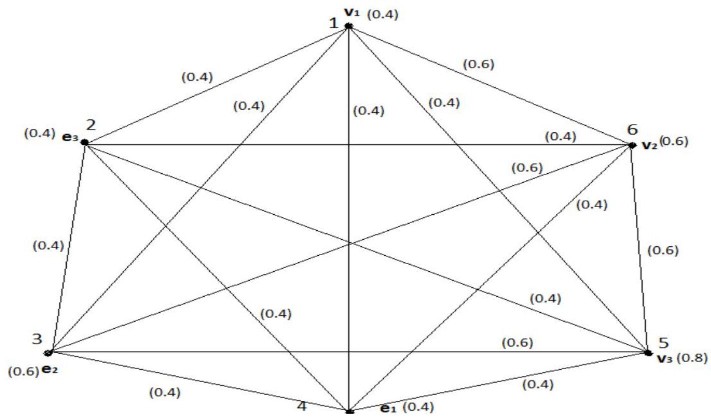


Fig 7;

Here we need minimum 2 colours to proper colour all the vertices of the graph DLC ($G_{0.6}$).so the chromatic number of DLC ($G_{0.6}$) is 2.

For α= 0.6, $X_{0.6} = X_{DL}$ (0.6) = 2. For **α=0.8** Double layered complete fuzzy graph
DLC (G) = (σ_{DL}, μ_{DL},) Where σ_{DLC} = {0.8}. (Fig 8)

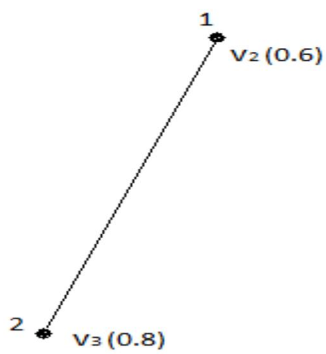


Fig 8;

Here we need minimum 1 colour to proper colour all the vertices of the graph DLC ($G_{0.8}$).so the chromatic number of DLC ($G_{0.8}$) is 1.

Example: Consider the complete fuzzy graph with vertex 4 (K_4)

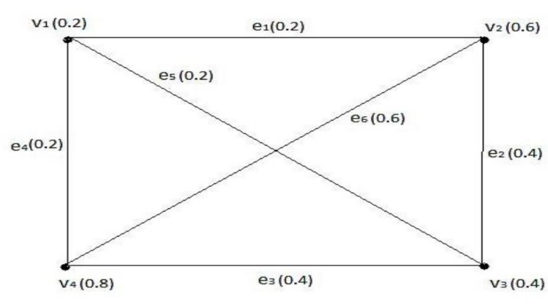


Fig 9;

In this DLCFG, there are 4 Alpha -cut is presented,
They are {0.2, 0.4, 0.6, and 0.8}. For every value of Alpha, we find DLCF (G_α) and find its fuzzy chromatic number. For **α=0.2** Double layered complete fuzzy graph DLC (G) = (σ_{DL}, μ_{DL}) Where
α_{DLC} = {0.8, 0.6, 0.4, 0.2}. Here, we need minimum 10colors to proper colour all the vertices of the graph
DLC ($G_{0.2}$).so the chromatic number of DLC ($G_{0.2}$) is 10.

III. THEORITICAL CONCEPTS

1) *Theorem 1:* The order of double layered complete fuzzy graph KσUμ is equal to the sum of the order and size of the complete graph.

Proof: Let X U Y be a node set of complete double layered fuzzy graph and the fuzzy subset XDL on X^* U Y^* is defined as,

$$XDL = \begin{cases} X(u)\ if\ u \in X^* \\ Y(uv)\ if\ uv \in Y^* \end{cases}$$

By the definition, order of the double layered fuzzy graph is,

$$O(DL(G)) = \sum_{u \in X \cup Y} XDL(u)$$
$$= \sum_{u \in X} XDL(u) + \sum_{u \in Y} XDL(u)$$
$$= \sum_{u \in X} X(u) + \sum_{u \in Y} X(u)$$

O (DL (G)) = Order (G) + size (G).

2) *Theorem 2:* Every double layered complete fuzzy graph is a strong fuzzy graph.

Proof: As the node set of DL (G) is X^* U Y^* and the fuzzy subset XDL on X^* U Y^* is defined as,

$$XDL = \begin{cases} X(u)\ if\ u \epsilon X^* \\ Y(uv)\ if\ uv \epsilon Y^* \end{cases}$$

By the definition of double layered complete fuzzy graph
Y (u, v) = X (u) ∧ X (v) ----------------------①
And also by the definition of strong fuzzy graph
Y (u, v) =min (X (u), X (v)) ---------------- ②
From equation① &②; we get

Every double layered complete fuzzy graph is a strong fuzzy graph.
Example: We choose DL (G) of K_3 graph,
v_1=0.4; v_2=0.6; v_3=0.8 and e_1=0.4; e_2=0.6; e_3=0.4

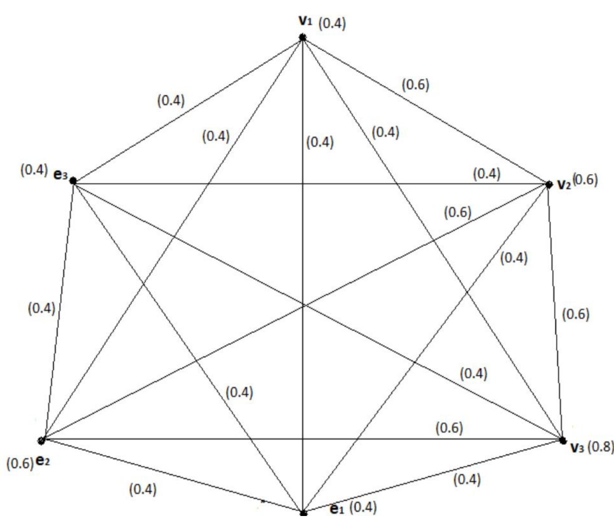


Fig 10;

(i) $\mu(v_1, v_2) = \sigma(v_1) \wedge \sigma(v_2)$
$=0.4 \wedge 0.6$
$=0.4$

(ii) $\mu(e_1, e_2) = \sigma(e_1) \wedge \sigma(e_2)$
$=0.4 \wedge 0.6$
$=0.4$

(iii) $\mu(v_1, e_1) = \sigma(v_1) \wedge \sigma(e_1)$
$=0.4 \wedge 0.4$
$=0.4$

Every double layered fuzzy graph is a strong fuzzy graph.

Example:

- DLCFG(K4) = K4 + DLCFG(K3)
  = K4 + K6
  = K10 DLCFG (K4) = CFG (K10)

- DLCFG(K5) = K5 + DLCFG(K4)
  = K5 + K10
  = K15 DLCFG (K5) = CFG (K15).

Table 1: Relation between complete fuzzy graph and Double layered complete fuzzy graph.

COMPLETE FUZZY GRAPH	**DOUBLE LAYERED COMPLETE FUZZY GRAPH**
K_3	DLCFG(K_3)= K_6
K_4	DLCFG(K_4)= K_{10}
K_5	DLCFG(K_5)=K_{15}
K_6	DLCFG(K_6)=K_{21}
K_7	DLCFG(K_7)=K_{28}
K_8	DLCFG(K_8)=K_{36}
K_9	DLCFG(K_9)=K_{45}
K_{10}	DLCFG(K_{10})=K_{55}

IV. CONCLUSION

In this paper we have briefly discussed about the vertex colouring of the double layered complete fuzzy graph using alpha cut. We conclude that the chromatic number is decrease when the value of alpha cut is increase. This concept will help not only in vertex colouring also in edge colouring of DLCFG using alpha cut. Also we have found a double layered complete fuzzy graph, and theoretical concepts based on double layered fuzzy graph and proved double layered fuzzy graph as strong fuzzy graph and illustrated with some examples and have given a relation between complete fuzzy graph and a double layered complete fuzzy graph. Further structures can be developed by increasing number of cycles. This structural pattern with the cycles gives further information into different patterns in networking models.

REFERENCES

[1] "Vertex colouring of double layered complete fuzzy graph using α – cut", International Journal of Mechanical Engineering ISSN: 0974-5823 Vol.7 No.4 (April, 2022).

[2] " Colouring in various graphs", Jaya priya.B, and Kamali.R, GEDRAG & ORGANISATIE REVIEW - ISSN:0921-5077 VOLUME 34 : ISSUE 03 - 2021 http://lemma-tijdschriften.com/

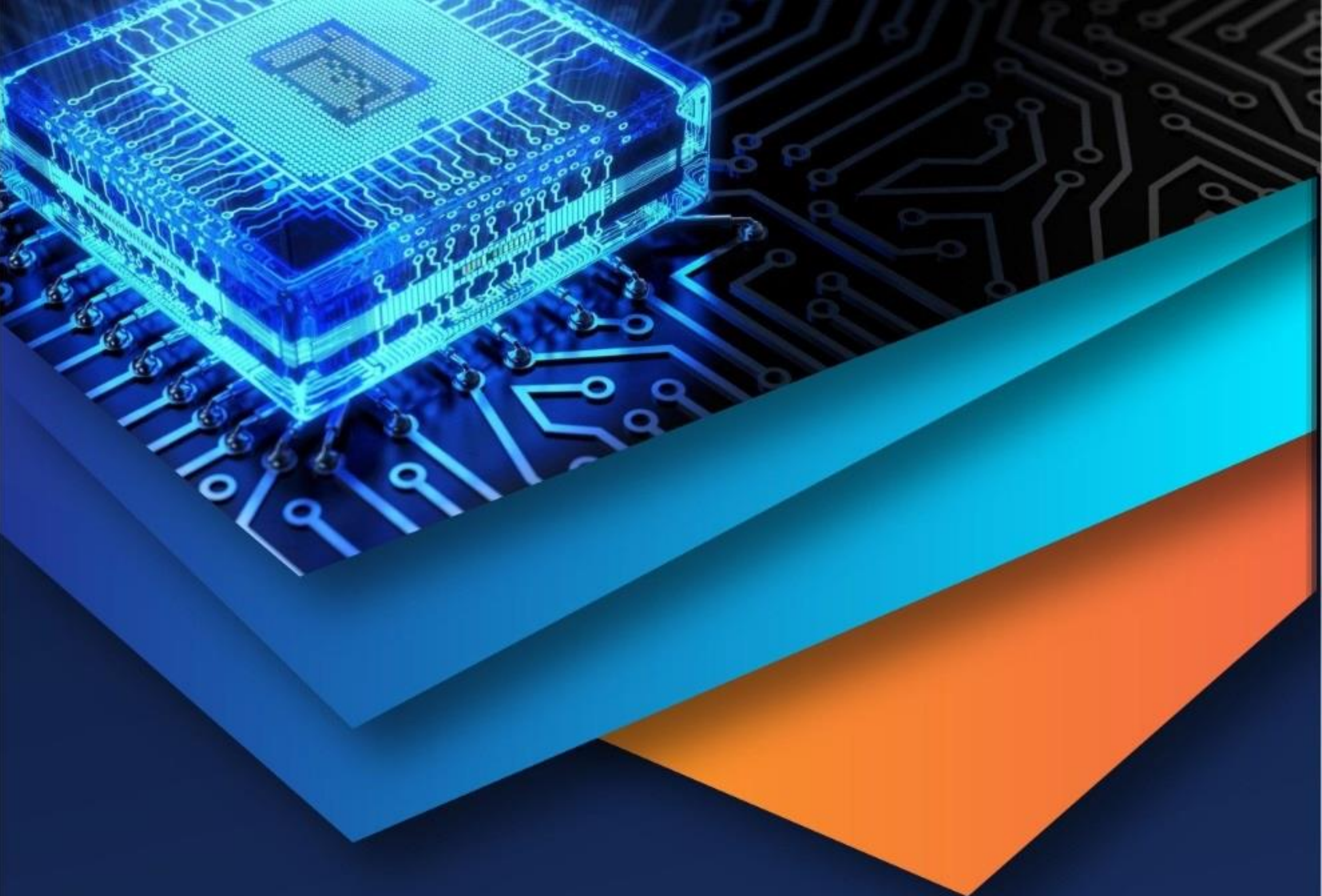
[3] "A study on vertex colouring", N. Malini, and R. Subramani, Advances and Applications in Mathematical Sciences Volume 21, Issue 3, January 2022, Mili Publications, India.

[4] Arindam Dey, Anita Pal, Vertex Colouring Of Fuzzy Graph Using α- Cut, IJMIE, Volume 2 Issue 8, August 2012.

[5] "Applications of Edge colouring of fuzzy graphs", Rupkumar Mahapatra, Sovan Samanta, and Madhumangal Pala. Article in Informatica · January 2020 DOI: 10.15388/20-INFOR403. https://www.researchgate.net/publication/340383318

[6] J.JonArockiaraj and V.Chandrasekaran, "Double layered complete Fuzzy Graph (DLCFG)". Global Journal of Pure and Applied Mathematics. ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 6633-6646 © Research India Publications http://www.ripublication.com

[7] Arindam Dey, Anita Pal, Vertex Colouring Of Fuzzy Graph Using α- Cut, IJMIE, Volume 2 Issue 8, August 2012.



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