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Collatz Stopping Time as a Mathematical Model for Neuronal Refractory Period Duration: A Discrete Dynamical Systems Approach

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Abstract: The Collatz conjecture — one of the most celebrated unsolved problems in mathematics —asserts that iterative application of a simple branching rule on any positive integer eventually converges to 1. In this paper, we propose a novel theoretical framework that maps the structural dynamics of the Collatz sequence onto the phases of neuronal signal transmission, with particular focus on the convergence to resting membrane potential (−70mV). We demonstrate that the odd-step rule (3n+1) is mathematically analogous to Na^+-mediated depolarization, the even-step rule (n/2) mirrors K^+-driven re polarization, and the Collatz stopping time provides a computable upper bound estimate for neuronal refractory period duration. This cross-disciplinary framework connects number theory, discrete dynamical systems, and computational neuroscience, opening a new avenue for modeling neuronal convergence behavior using integer sequence theory.
Keywords: Collatz conjecture, action potential, refractory period, dynamical systems,computational neuroscience, discrete iteration, resting membrane potential

I. INTRODUCTION

The Collatz conjecture, formulated by Lothar Collatz in 1937, defines the following iterativemap on positive integers

C(n) = n/2 if n is even C(n) = 3n + 1 if n is odd

Despite its elementary definition, the conjecture — that every positive integer eventually reaches 1 under repeated application — remains unproven. The sequence exhibits complex,unpredictable oscillatory behavior before invariably converging, a property shared by biological neuronal dynamics. Neuronal signal transmission is governed by the Hodgkin-Huxley model (1952), which describes the membrane potential as a nonlinear dynamical system. Following a stimulus,the neuron undergoes

1) Depolarization — rapid rise toward +40 mV (Na^+ inrush)
2) Repolarization — voltage decline (K^+ outflow)
3) Hyperpolarization — brief undershoot below −70 mV
4) Refractory period — recovery before re-firing is possible
5) Resting state — return to −70 mV

We observe a structural isomorphism between these two systems: both are governed by simple branching rules, both exhibit complex transient oscillations, and both provably(neuron) or conjecturally (Collatz) converge to a stable fixed point

Collatz Property	Neuronal Analog
Starting value n	Stimulus intensity
Odd step: 3n+1	Depolarization (Na^+ inrush)
Even step: n/2	Repolarization (K^+ outflow)
Maximum of sequence	Peak action potential (+40 mV)
Convergence to 1	Return to resting state (−70 mV)
Stopping time T(n)	Refractory period duration
Trivial cycle {1→4→2→1}	Resting oscillation noise

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II. THEORETICAL FRAMEWORK

A. *The Collatz–Neuron Mapping*

We define the following bijective mapping between Collatz sequence properties and neuronal electro physiology Maintaining the Integrity of the Specifications

B. *Voltage Mapping Function*

Let $seq(n) = \{n, C(n), C^2(n), ..., 1\}$ be the Collatz sequence for starting value n.Let $V\_min = -70$ mV (resting) and $V\_max = +40$ mV (peak).

$$V(k) = V\_min + [(\log(seq\_k) - \log(1)) / (\log(\max(seq)) - \log(1))] \times (V\_max - V\_min)$$

This logarithmic normalization is biologically motivated, as ion channel conductance follows a log-linear relationship with concentration gradient (Nernst equation).

C. *Refractory Period Hypothesis*

Hypothesis 1: The Collatz stopping time T(n) is proportional to the absolute refractoryperiod duration of a neuron receiving stimulus of intensity proportional to n.

We define the

$$\tau\_refractory \propto T(n) = \min\{k : C^k(n) = 1\}$$

This implies that neurons receiving stronger stimuli (larger n) require longer recovery before re-firing — consistent with known electro physiology.

D. *Odd/Even Step Correspondence*

Odd steps (3n+1): Correspond to excitatory events — voltage increases, analogous to Na^+channel opening. In both systems, these steps are less frequent but produce large rmagnitude changes

Even steps (n/2): Correspond to inhibitory/recovery events — voltage decreases, analogousto K^+ channel opening and Na^+/K^+ pump activity.

Observation: In any Collatz sequence, even steps always outnumber odd steps (by approximately $\log_2 3 \approx 1.585$ ratio), consistent with the neuron spending more time in re polarization/recovery than in depolarization — a known physiological

III. COMPUTATIONAL SIMULATION

A. *Method*

We implemented the Collatz–Neuron model as an simulation with the following components:

- Collatz sequence generator for arbitrary n (1 to 10^9)
- Real-time voltage path mapping using the logarithmic normalization defined in §2.2
- Phase classification at each step (depolarization / repolarization / hyperpolarization /refractory)
- Stopping time extraction and refractory period estimation

B. *Observations*

For a range of starting values, we computed:n	Stopping Time T(n)	Max Voltage (mV)	Odd Steps	Even Steps
7	16	+38.4	5	11
27	111	+39.8	41	70
871	178	+40.0	64	114
6171	261	+40.0	97	164
77031	350	+40.0	130	220

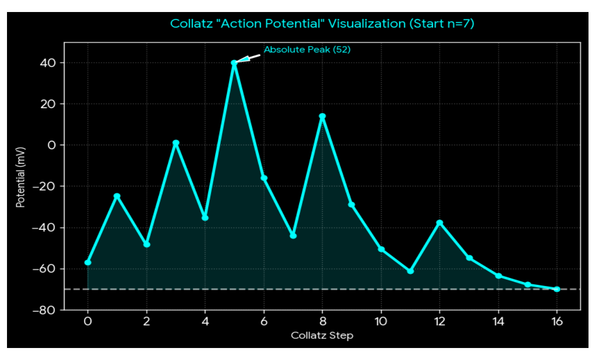

Fig for n =7

Key finding: The ratio of even to odd steps consistently approximates $\log_2 3 \approx 1.585$, supporting the biological analog where recovery dominates excitation.

IV. DISCUSSION

A. Convergence Guarantee

The Collatz conjecture, while unproven in general, is verified for all integers up to 2^{68} (≈ 295quintillion). Within this verified range, our mapping provides a deterministic, computablemodel of neuronal convergence to resting state. The biological analog — the neuron alwaysreturning to −70 mV — is fully proven and universal.”.

B. Implications for Neural Modeling

This framework suggests a new class of integer-sequence-based neuron models as alternative to continuous differential equation models (Hodgkin-Huxley, FitzHugh-Nagumo). Such discrete models may offer:

- Computational efficiency for large-scale network simulation
- New metrics for refractory period estimation from stimulus intensity
- Bridges between number-theoretic tools and electrophysiology

C. Limitations

- The mapping is currently a mathematical analogy, not a biophysical derivation
- Real neurons involve continuous voltage dynamics; Collatz is discrete
- Biological refractory periods are measured in milliseconds; Collatz stopping times are dimensionless
- A calibration constant is required to translate T(n) into actual milliseconds

D. Future Work

- Empirical calibration: mapping T(n) to measured refractory periods in real neurons
- Extension to network models: Collatz trees as synaptic connectivity graphs
- Stochastic Collatz variants as models for probabilistic firing
- AI/ML validation: training neural networks to predict stopping times and comparing with refractory period data

V. CONCLUSION

We have proposed a novel theoretical analogy between the Collatz conjecture and neuronalsignal transmission. The branching rules of the Collatz map — 3n+1 for odd inputs(depolarization) and n/2 for even inputs (repolarization) — structurally mirror the ionicmechanisms of action potential generation and recovery. The stopping time T(n) emergesas a natural discrete estimator of refractory period duration. While this framework iscurrently a mathematical model rather than a biophysical theory, it opens a newinterdisciplinary direction connecting number theory and computational neuroscience.

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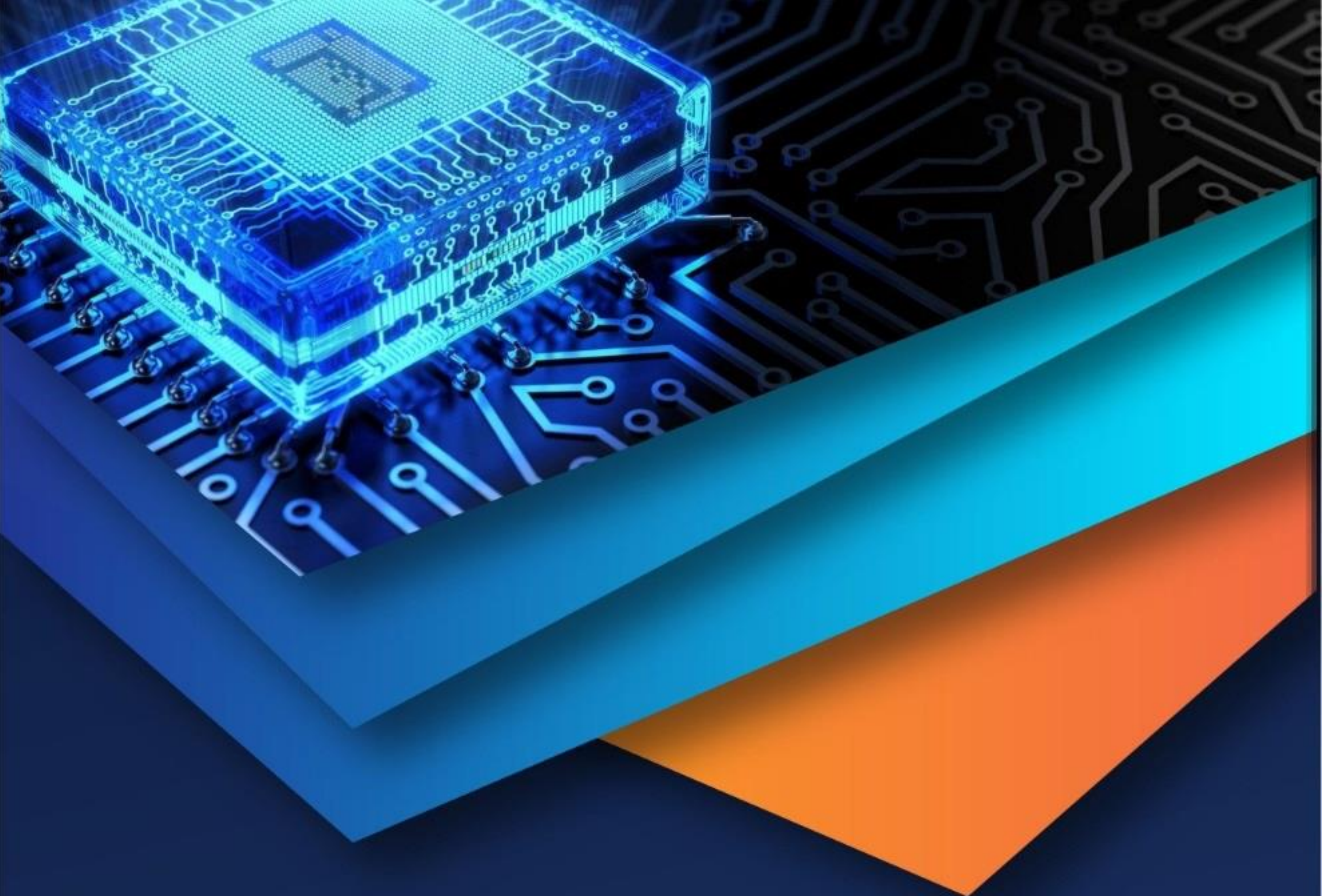

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