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Color Class Dominating Sets on Regular Graphs of Degree 5

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Abstract: Let $G = (V, E)$ be a graph. A color class dominating set of G is a proper coloring \mathcal{C} of G with extra property that every color class in \mathcal{C} is dominated by a vertex in G . A color class dominating set is said to be minimal color class dominating set if no proper subset of \mathcal{C} is a color class dominating set of G . The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_{\chi}(G)$. Here we also obtain $\gamma_{\chi}(G)$ of regular graph degree 5.

Keywords: Chromatic number, Domination number, Color Class Dominating set, Color Class Domination number.

Mathematics subject classification: 05C15, 05C69

I. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definitions of graph theory [2]. Let $G = (V, E)$ be a graph of order p . The open neighborhood $N(v)$ of vertex $v \in V(G)$ consists of the set of all vertices adjacent to v . The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood $N(S)$ is defined to be $\bigcup_{v \in S} N(v)$, and the closed neighborhood of S is $N[S] = N(S) \cup S$ for any subset H of vertices of G , the induced subgraph $\langle H \rangle$ is the maximal subgraph of G with vertex set H . A subset S of V is called a dominating set if every vertex in $V - S$ is adjacent to some vertex in S . A dominating set S is called a minimal dominating set if no proper subset of S is a dominating set of G . The domination number $\gamma(G)$ is the minimum cardinality taken over all minimal dominating sets of G . A γ -set is any minimal dominating set with cardinality γ . A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$. A color class dominating set of G is a proper coloring \mathcal{C} of G with the extra property that every color class in \mathcal{C} is dominated by a vertex in G . A color class dominating set is said to be a minimal color class dominating set if no proper subset of \mathcal{C} is a color class dominating set of G . The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_{\chi}(G)$. This concept was introduced by Vijayalekshmi et al [2]. A graph G is said to be r -regular if degree of each vertex of G is r . A 3-regular graph is also called a cubic graph. In this paper we obtain color class domination of regular graphs of degree 5.

II. MAIN RESULTS

Theorem 2.1

Let G be a regular graph of degree 5 then $\gamma_{\chi}(G) = \frac{n}{2}$ or $\left(\frac{n}{2}\right) - 2$ **Proof:** Let G be the regular graph of degree 5 with order $n = 2p$ and

Let $V(G) = \{v_1, v_2, v_3, \dots, v_p, \dots, v_n\}$. We consider 2 cases

Case (i) Graph with triangles

$$N(v_1) = \{v_2, v_p, v_{p+1}, v_{p+2}, v_n\}, N(v_p) = \{v_1, v_{p-1}, v_{p+1}, v_{n-1}, v_n\},$$

$$N(v_{p+1}) = \{v_1, v_2, v_p, v_{p+2}, v_n\}, N(v_n) = \{v_1, v_{p-1}, v_p, v_{p+1}, v_{n-1}\}, N(v_i) = \{v_{i-1}, v_{i+1}, v_{i+p-1}, v_{i+p}, v_{i+p+1}\}, 2 \leq i \leq$$

$$p-1 \text{ and } N(v_j) = \{v_{j-1}, v_{j+1}, v_{j-p-1}, v_{j-p}, v_{j-p+1}\}, p+2 \leq j \leq n-1$$

Assign distinct colors say 1, 2 and 3 to the vertices $\{v_1, v_{n-1}\}$, $\{v_2, v_n\}$ and $\{v_3, v_{p+1}\}$ respectively. Also assign distinct colors, say i ($4 \leq i \leq \frac{n}{2}$) to the vertices $\{v_i, v_{p+(i-2)}\}$ we obtain the γ_{χ} coloring of G . Thus $\gamma_{\chi}(G) = \frac{n}{2}$

Illustration: (G_{26})

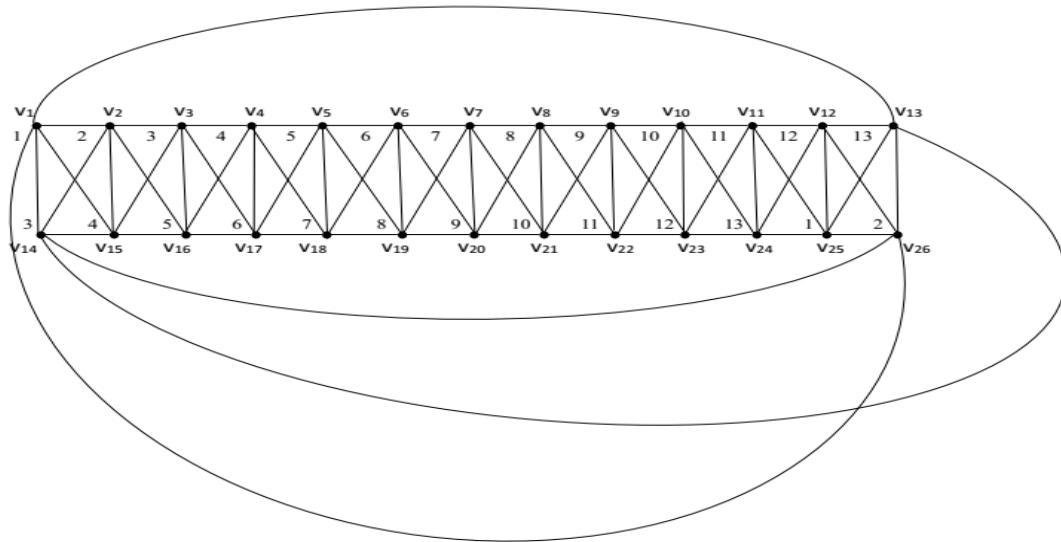


Figure 1

$$\gamma_\chi(G_{26}) = 13$$

Case (ii) A graph without triangles

Let $N(v_1) = \{v_2, v_3, v_{16}, v_{p+1}, v_{p-1}\}$

$N(v_i) = \{v_{i-2}, v_{i-1}, v_{i+1}, v_{i+2}, v_{p+(i-1)}\}$ for $i = \{3, 5, \dots, (n-1)\}$.

$N(v_i) = \{v_{i-2}, v_{i-1}, v_{i+1}, v_{i+2}, v_{p+i}\}$ for $3 \leq i \leq p$

and $N(v_i) = \{v_{i-2}, v_{i-1}, v_{i+1}, v_{i+2}, v_{i-p}\}$ for $p+1 \leq i \leq n$

Assign distinct colors i ($1 \leq i \leq 4$) to the vertices say $\{v_i, v_{i+4}, v_{i+p+2}\}$ respectively

Assign distinct colors i ($5 \leq i \leq \binom{n}{2} - 2$) to the vertices $\{v_{i+(p-1)}, v_{p+(i+2)}\}$ respectively. we get γ_χ coloring of G . Thus $\gamma_\chi(G) = \binom{n}{2} - 2$.

Illustration: (G_{16})

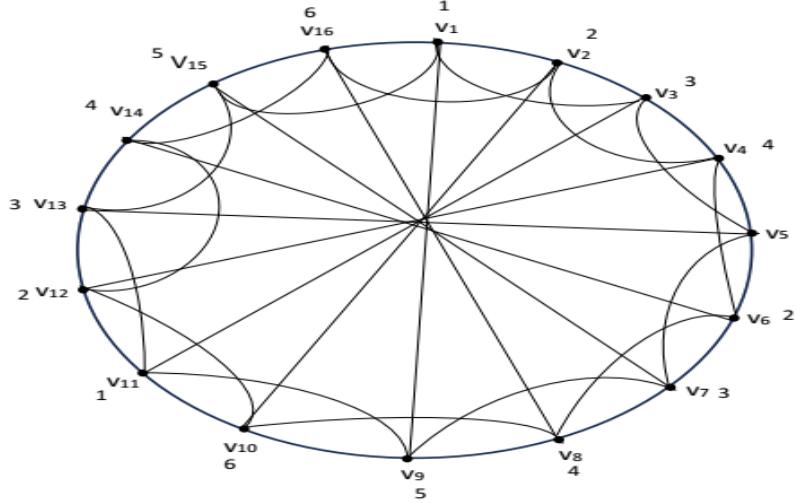


Figure 2
 $\gamma_\chi(G_{16}) = 6$

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