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Comparative Study of Structural Analysis Methods with Practical Application

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Abstract: This article intends to compare two classical methods of analyzing statically indeterminate structures: the force method and the displacement method. The analysis is based on the theoretical description of the principles, advantages, and limitations of each approach, considering different application contexts. The study explores the formulation of the main unknowns, the complexity of the solution procedures, and the suitability of each method for both simple and complex structures. The comparison is illustrated through the application of both methods to various example structures, demonstrating the differences in formulation and the results obtained. The results show that, although both methods remain relevant, the displacement method stands out due to its applicability in modern software, offering greater efficiency and flexibility in more complex analyses. It is concluded that a deep understanding of both methods remains essential for professional practice in civil engineering, enabling a careful choice of the most appropriate approach for each situation.
Keywords: Statically indeterminate structures, force method, displacement method.

I. INTRODUTION

Structural analysis is essential to ensure the safety, stability, and functionality of constructions, playing a crucial role in the development of civil engineering projects. Among the various approaches used to analyze statically indeterminate structures, two stand out: the force method and the displacement method. Both methods have been widely used over the years, each with its own characteristics and fields of application. The force method, which is based on determining internal forces through redundant forces, has been widely applied in various scenarios.

II. MATERIALS AND METHODS

This section describes the procedures adopted for the comparative analysis between the Force Method and the Displacement Method. Two representative structures were analyzed, each solved separately using one of the methods. The objective is to highlight the differences in the results obtained and to assess the applicability of each method to different types of structures.

A. Force Method: Definition

One of the classical methods used in structural analysis is the Force Method, also known as the compatibility method. In this method, the primary unknowns are the internal forces and support reactions, rather than the displacements. The approach involves formulating a system of equations based on equilibrium conditions, expressing all other variables in terms of the unknown forces. These expressions are then inserted into the deformation compatibility equations, which are solved to find the values of the forces.

The fundamental idea of the method is to find, among all force distributions that satisfy the equilibrium of the structure, the one that also fulfills the compatibility conditions imposed by the structure's geometry and material properties. The formulation of the Force Method follows a well-defined order: equilibrium equations are applied first, followed by constitutive material relations (e.g., Hooke's Law), and finally, geometric compatibility conditions (MARTHA, n.d., p. X).

B. Force Method: Applications

The Force Method is widely used to determine internal force diagrams in structural elements, such as bending moments and shear forces, especially in beams. This method stands out for its ability to produce these diagrams in a relatively simple and fast manner, making it highly effective for problems of lower complexity.

The force method is more suitable for simple structures, which have a limited number of supports and uniformly distributed loads, allowing for efficient resolution in a short period. However, in more complex structures, with a greater number of supports or concentrated loads, the Force Method presents certain limitations. In such cases, it is common for the method to be supplemented by automated software that employs more accurate and efficient approaches.

As stated in the study by Santos Duarte and Fernandes (2023), *"the force method proves sufficient to solve small problems that do not require great complexity in their resolution and calculation, being inadequate for structures with a large number of supports or in unconventional configurations, where software assistance is necessary."*

C. Force Method: Advantages and Disadvantages

The Force Method presents a clear and well-structured mathematical formulation, allowing for a direct analysis of internal forces in structural elements. Based on the principles of statics, this method offers transparency in calculations and greater control over the analytical process, which is especially useful for validating results obtained through computational means. Furthermore, the explicit application of equilibrium, constitutive, and compatibility conditions provides a deep understanding of structural behavior (Santos Duarte & Fernandes, 2023).

However, the method also presents significant limitations.

In structures with a large number of redundant supports, complex geometries, or unconventional loadings, the Force Method becomes more labor-intensive and less efficient. Solving systems with many force unknowns may involve a high computational effort or even modeling errors. For this reason, its practical application is generally limited to small-scale problems, often being replaced by more versatile methods such as the Displacement Method or by specialized software in the advanced stages of design.

D. Displacement Method: Definition

The Displacement Method is one of the fundamental approaches in structural analysis and is widely adopted in automated calculation programs due to its systematic algorithmic structure. In this method, the primary unknowns are the displacements and rotations at selected points of the structure, known as degrees of freedom. The total number of these degrees corresponds to the kinematic indeterminacy of the structure.

Unlike the Force Method, which solves for redundant internal forces, the Displacement Method formulates equations based on nodal displacements. These displacements are selected at points where the structure is not fully restrained, typically at the ends of beams. From the imposition of equilibrium conditions and the material properties, an algebraic system of equations is obtained, which allows for determining displacements and, subsequently, internal forces.

Each node in the structure may have up to three possible displacements—two translations and one rotation—depending on the constraints imposed by hinges, fixed supports, or joints. Displacements at fully fixed points are considered null, while free rotations, such as in hinges, are not included in the system as primary unknowns. This careful selection ensures that only the truly relevant displacements are considered, making the method efficient and compatible with advanced computational analysis. (SORIANO; LIMA, 2006, p. 47)

E. Displacement Method: Applications, Advantages, and Disadvantages

The Displacement Method is widely used in the analysis of statically indeterminate structures, where there are more constraints than required for equilibrium. In this method, the primary unknowns are displacements and rotations, which makes it particularly suitable for complex structures such as frames, space trusses, continuous beams, and multi-story buildings. It is also effective in the analysis of structures subjected to moving or dynamic loads, such as bridges. It forms the theoretical foundation of the Finite Element Method and is therefore extensively used in structural analysis software.

Among its main advantages are the ease of applying compatibility conditions and its systematic mathematical structure, which is ideal for computational implementation. However, in simpler (isostatic) structures, the Force Method may prove more efficient. Additionally, the Displacement Method can become complex in situations involving movable supports or nonlinear connections, requiring additional modeling techniques (Martha, 2006).

III. RESULTS

This chapter presents the analyses of two similar yet distinct structures using two classical structural analysis methods: the Displacement Method and the Force Method. The objective is to demonstrate the practical application of each method and highlight their differences and specific characteristics in the solution process.

In section 3.1, a structure will be analyzed using the displacement method, where the primary unknowns are the displacements of the structure. In section 3.2, a similar structure will be solved using the Force Method, focusing on the calculation of internal forces based on equilibrium conditions.

A. Analysis of a Structure Using the Displacement Method

This section presents the analysis of a statically indeterminate structure composed of three spans: the first, AB, is 4 meters long and includes one double support and one simple support; the second, BC, is 5 meters long with two simple supports; and the third, CD, is 3 meters long, also with two simple supports. The structure is analyzed using the Displacement Method, which is based on considering displacements as the primary unknowns, from which the internal forces are determined.

The nodes with degrees of freedom were identified, and the structure was classified as statically indeterminate. The relevant generalized displacements were determined—specifically, the rotations at the intermediate nodes of the beam.

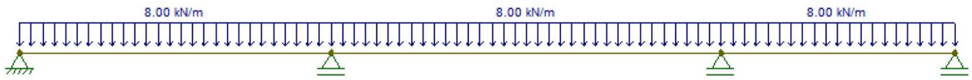

Figure 1 – Statically Indeterminate Structure

Unitary rotations were applied at the nodes with free displacement (Δ_1 and Δ_2), while maintaining the supports under their actual conditions. The unit hyperstatic bending moments generated in each section of the structure for $\Delta_1 = 1$ rad and $\Delta_2 = 1$ rad were determined using expressions derived from the elastic curve equation.

Figure 2- Segmented Structure

c

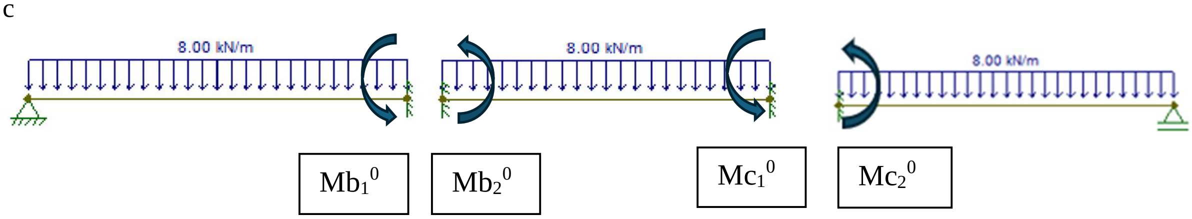

Figure 3 – Structure Divided for Loading

Unitary rotations were applied at the nodes with free displacement (Δ_1 and Δ_2), while maintaining the supports in their actual conditions.

$Mb_1^0 = -(q*l^2)/8 = -(8*4^2/8) = -16KN/m$

$Mb_2^0 = q*l^2/12 = 8*5^2/12 = 16,67$ KN/m

$Mc_1^0 = -(q*l^2)/12 = -16,67KN/m$

$Mc_2^0 = q*l^2/8 = 9KN/M$

The imbalance values were calculated using the following expression:

ß$_{10}$**=** $Mb_1^0 + Mb_1^0 = 0,67$ KN/m

ß$_{20}$**=** $Mc_1^0 + Mc_2^0 = -7,67$ KN/m

- $-\Theta_1=1$

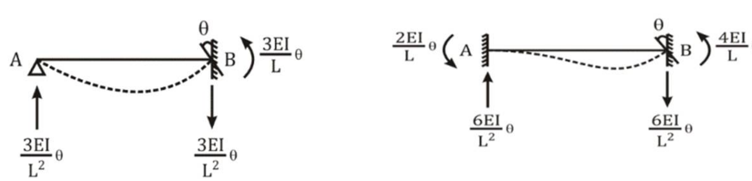

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Figure 4- Stiffness Coefficients

Mb_1^1=3*EI/L= 3*2000/4= 15000 KN/m
Mb_2^1=4*EI/L= 4*2000/5=16000 KN/m
Mc_1^1= 2*EI/l=8000 KN/m
Mc_2^1= 0
$K_{11=}$ Mb_1^1+ Mb_2^1= 31000KN/m
K_{21}= Mc_1^1+ Mc_2^1= 8000KN/m

- Θ_2=1 in the primary system

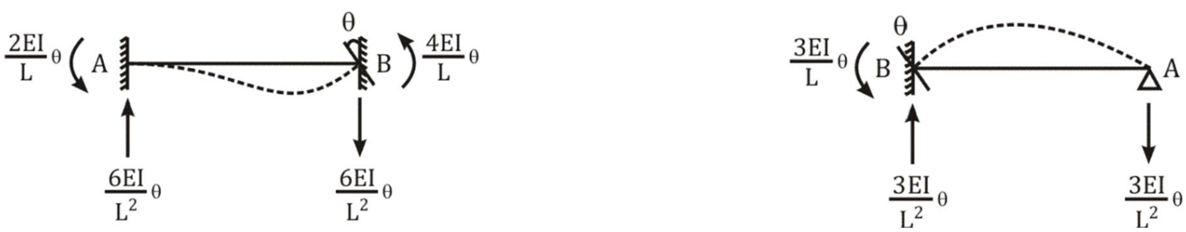

Figure 5- Stiffness Coefficients

Mb_1^2=0
Mb_2^2= 2*EI/L= 8000KN/m
Mc_1^2= 4*EI/L =16000KN/m
Mc_2^2= 3*EI/L= 2000KN/m
$K_{12=}$ Mb_1^2+ Mb_2^2= 8000 KN/m
K_{22}= Mc_1^2+ Mc_2^2=36000KN/m

The compatibility equation system was formulated based on the condition that the total rotation at each degree of freedom is zero (principle of superposition). The system has the following form:

$ß_{10}+K_{1*}\Theta_{1+}K_{12}*\Theta_2=0$
$ß_{20}+K_{12}*\Theta_1+K_{22}*\Theta_2=0$

The equations were solved to obtain the values of the generalized displacements:
↔ Θ_1=-8,112$*^{10-5}$ rad ; Θ_2= 2,310*10^{-4}rad
With the known values of θ_1 and θ_2, it was possible to calculate the final hyperstatic moments by summing the moments due to external actions with those induced by the displacements:
$E_f = E_0+\sum E_i*\Theta_1$
$M_{b1\,=}\,M_{b1}^{\,1}*\Theta_1+M_{b1}^{\,2}*\Theta_2$= -17,22 KN/m
$M_{b2\,=}\,M_{b2}^{\,1}*\Theta_1+M_{b2}^{\,2}*\Theta_2$= 17,22 KN/m
$M_{c1\,=}\,M_{c1}^{\,1}*\Theta_1+M_{c1}^{\,2}*\Theta_2$ =-13,62 KN/m
$M_{c2\,=}\,M_{c2}^{\,1}*\Theta_1+M_{c2}^{\,2}*\Theta_2$ =13,62 KN/m

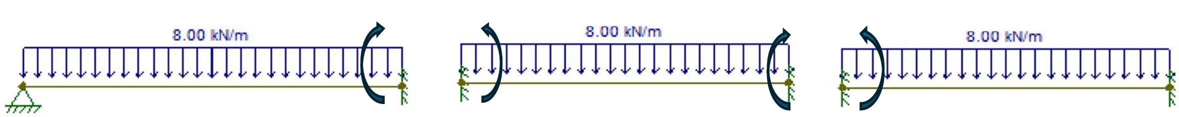

Figure 6- Structure Divided with the Corresponding Moments

With the final moments known, static equilibrium was applied to the structure to determine the support reactions and internal forces across the different spans.

$\sum M_D=0 \leftrightarrow 4*V_A-8*4*2+17{,}22=0 \leftrightarrow V_A=11{,}70$ KN/M

$\sum F_V=0 \leftrightarrow V_A+V_{B1}-8*4=0 \leftrightarrow V_{B1}= 20{,}32$ KN/m

$\sum M_c=0 \leftrightarrow 5*V_{B2}-17{,}22+13{,}62-8*5*2{,}5=0 \leftrightarrow V_{B2}=20{,}72$ KN/m

$V_B= V_{B1}+V_{B2}=41{,}02$ KN

$\sum F_V=0 \leftrightarrow V_{B2}*VC1-8*5=0 \leftrightarrow V_{C1}=19{,}28$ KN/m

$\sum M_D= \leftrightarrow V_{C2}*3-13{,}62-8*3*1{,}5=0 \leftrightarrow V_{C2}= 16{,}54$ KN/m

$V_C=V_{C1}+V_{C2} = 35{,}83$ KN/m

$\sum F_V=0 \leftrightarrow V_{C2}+V_D- 8*3=0 \leftrightarrow V_D= 7{,}46$ KN/m

$\sum F_H=0 \leftrightarrow H_A=0$

The bending moment (M) and shear force (V) diagrams were drawn based on the obtained values, ensuring consistency with the signs and conventions used throughout the analysis.

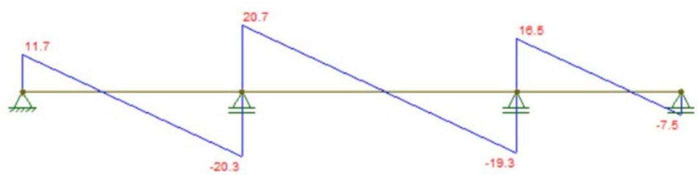

Figure 7- Shear Force Diagram

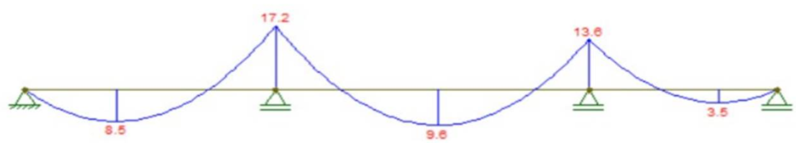

Figure 8- Bending Moment Diagram

B. Analysis of a Structure Using the Force Method

To illustrate the application of the Force Method, a statically indeterminate structure of degree 2 is considered—that is, a doubly indeterminate system. The structure under analysis has four simple supports, which results in four support reactions (four static unknowns), while the equilibrium conditions of the structure provide only two independent equations. This discrepancy between the number of unknowns and equations indicates the need to use a method suitable for analyzing indeterminate structures—in this case, the Force Method.

The structure is subjected to three vertical loads, with intensities of 20 kN/m, 15 kN/m, and 10 kN/m. The first span is 3 meters long, the second 4 meters, and the third 5 meters.

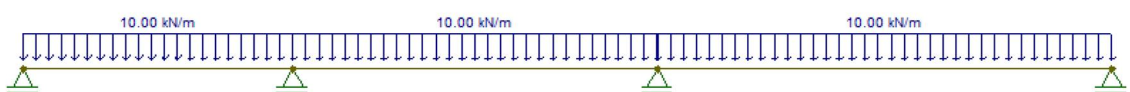

Figure 9- Statically Indeterminate Structure

The statically indeterminate structure was initially transformed into an isostatic one by adding two hinges, one at point B and another at point C, thereby making the moments at these points equal to zero, which simplifies the calculations later on.

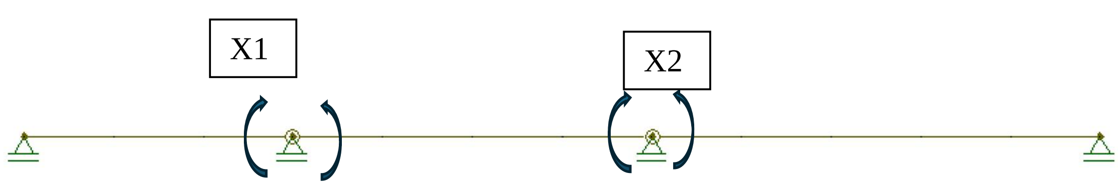

Figure 10- Isostatic Structure

By combining the following moment diagrams according to the Krut Beyer table, the equations were obtained:

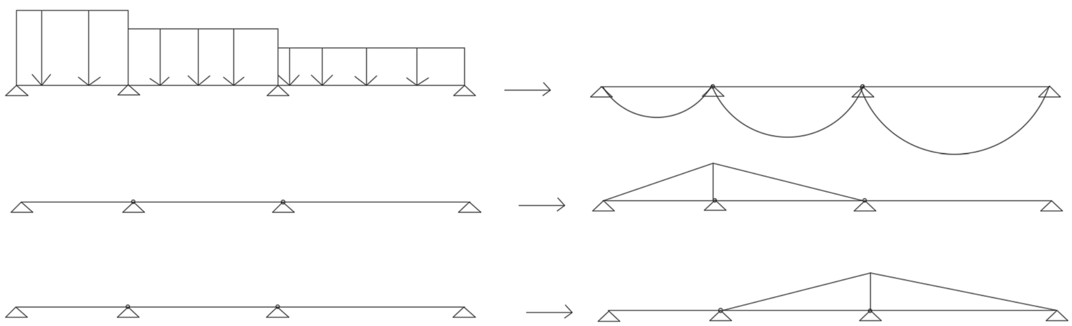

Figure 11- Moment Diagrams of the Isostatic Structures

δ_{10}=1/EI*(-1/3*3*22,5*1-1/3*4*30*1)=-62,5/EI
δ_{20}= 1/EI*(-1/3*4*30*1-1/3*5*31,25*1)=-1105/12*EI
δ_{11}=1/EI*(1/3*1*1*3+1/3*4*1*1)=7/3*EI
δ_{22}=1/EI*(1/3*4*1*1+1/3*5*1*1)=3/EI
δ_{12}=1/EI*(1/6*4*1*1)=2/3*EI

Substituting into the compatibility equation, we get:

$$\begin{cases} \delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 = 0 \\ \delta_{20} + \delta_{21} X_1 + \delta_{22} X_2 = 0 \end{cases}$$

Figure 12– Compatibility Equation

X_1=19,24KN.m
X_2= 26,42KN.m

Once the unknowns have been determined, the internal forces and displacements of the original structure can be calculated using:

$$\mathbf{E} = \mathbf{E}_0 + \sum_i X_i \cdot \mathbf{E}_i$$

Figure 13- Linear Combination

$V_A = V_A^{SP} + V_A^{1} * X_1 + V_A^{2} * X2 \leftrightarrow VA = 30 - 1/3 * 19,24 = 23,59$ KN
$\sum M_C = 0 \leftrightarrow V_D * 5 - 10 * 5 * 2,5 + 26,42 = 0 \leftrightarrow V_D = 19,72$KN
$V_B = V_B^{SP} + V_B^{1*} X_1 + V_B^{2} * X_2 \leftrightarrow V_B = 64,62$KN
$\sum F_Y = 0 \leftrightarrow -20*3 - 15*4 - 10*5 + 23,59 + 64,62 + 19,72 + V_C = 0 \leftrightarrow V_C = 62,07$ K

IV. DISCUSSION

The resolution of the structure using the Force Method demonstrated the applicability of this approach in statically indeterminate structures, such as the one analyzed, with four simple supports and three concentrated vertical loads. The static redundancy of degree two required the formulation of two compatibility equations, whose solution provided the values of the redundant forces—essential for obtaining the final internal force diagrams. The analysis of the results confirms that, despite the greater algebraic complexity associated with the Force Method when compared to the Displacement Method, it remains a robust approach, especially for structures with low degrees of redundancy. The appropriate selection of redundants, as recommended by Soriano (2015), was essential for simplifying the system of equations and for the physical interpretation of the results.

According to the literature (Soriano, 2015; Ghali, Neville & Brown, 2003; Timoshenko & Young, 1961), the Force Method presents both didactic and practical advantages in low-degree statically indeterminate structures, as it allows for a direct visualization of the consequences of redundancy on internal forces. The redistribution of bending moments across spans, as well as the presence of negative support reactions, are consistent with the theoretical behavior expected for this type of structural configuration (Salvadori & Heller, 1990).

Recent studies (e.g., Soares et al., 2020; Oliveira & Mendes, 2018) further emphasize the importance of the Force Method in validating computational models, as it is frequently used as a reference in finite element simulations. In the present case, the results obtained are consistent with theoretical expectations and demonstrate the coherence of the analytical process followed

V. CONCLUSIONS

The study successfully applied the Force Method to a statically indeterminate structure of degree two, composed of four simple supports and subjected to three concentrated loads. It was possible to determine the support reactions, internal forces, and the bending moment and shear force diagrams based on equilibrium conditions and deformation compatibility.

It was observed that, although the Force Method is more demanding in algebraic terms, it remains a relevant tool in structural analysis, especially for structures with low degrees of indeterminacy. The practical application of the method demonstrated its usefulness in both academic and professional contexts, corroborating the theoretical foundations described by Soriano (2015) and other classical authors such as Timoshenko (1961). From a scientific perspective, this work contributes to the consolidation of knowledge related to the analysis of statically indeterminate structures, providing a basis for future comparisons with the Displacement Method and with numerical methods. Pedagogically, it reinforces the importance of thoroughly understanding classical methods before relying on advanced computational tools.

VI. ACKNOWLEDGMENTS

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