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Comparison between Isostatic and Hyperstatic Structures

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Abstract: This article presents a comparison between isostatic and hyperstatic structures, based on the analysis of beams subjected to identical loads. A theoretical description of both types of structures is provided, followed by the practical analysis of two models – a simply supported beam (isostatic) and a continuous beam with three supports (hyperstatic). Through calculations and internal force diagrams, the differences in reactions, bending moments, and structural behavior are evaluated. The study aims to deepen the understanding of these concepts among civil engineering university students.
Keywords: Isostatic structure, hyperstatic structure, beam, bending moments, structural analysis, civil engineering

I. INTRODUCTION

The classification of structures according to their determinacy is one of the first fundamental concepts taught in structural analysis. Isostatic structures can be solved solely using equilibrium equations, whereas hyperstatic structures require additional deformation compatibility conditions. This distinction affects not only the calculation approach but also structural behavior in terms of stiffness, load redistribution, and sensitivity to load variations. In this article, an objective comparison is proposed between an isostatic and a hyperstatic structure through a practical and representative example.

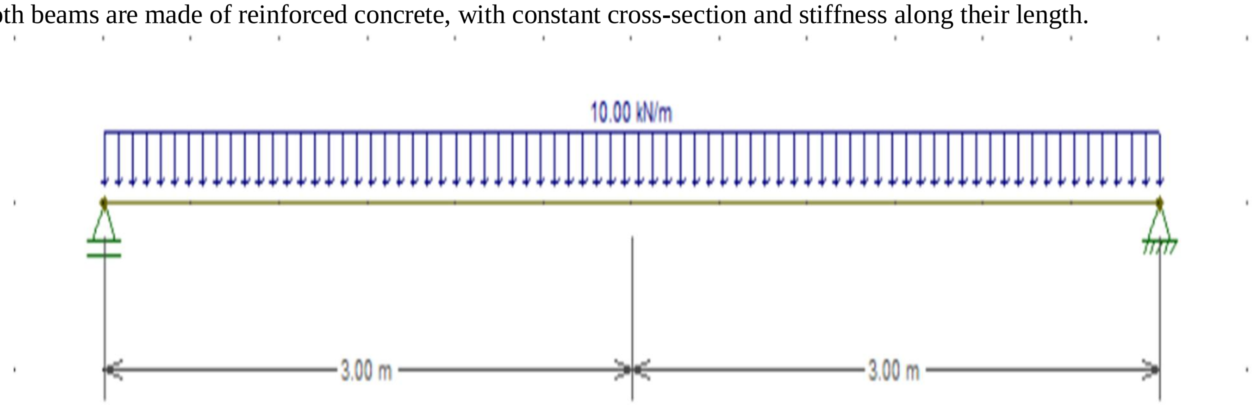
II. THEORETICAL CONCEPTS

A structure is said to be isostatic when the number of unknowns (reactions and internal forces) is equal to the number of available equilibrium equations. These structures are statically determinate and easily analyzed. On the other hand, a hyperstatic structure has more unknowns than equilibrium equations, requiring additional compatibility equations to be solved. Such structures exhibit greater stiffness and better performance under variable load conditions.

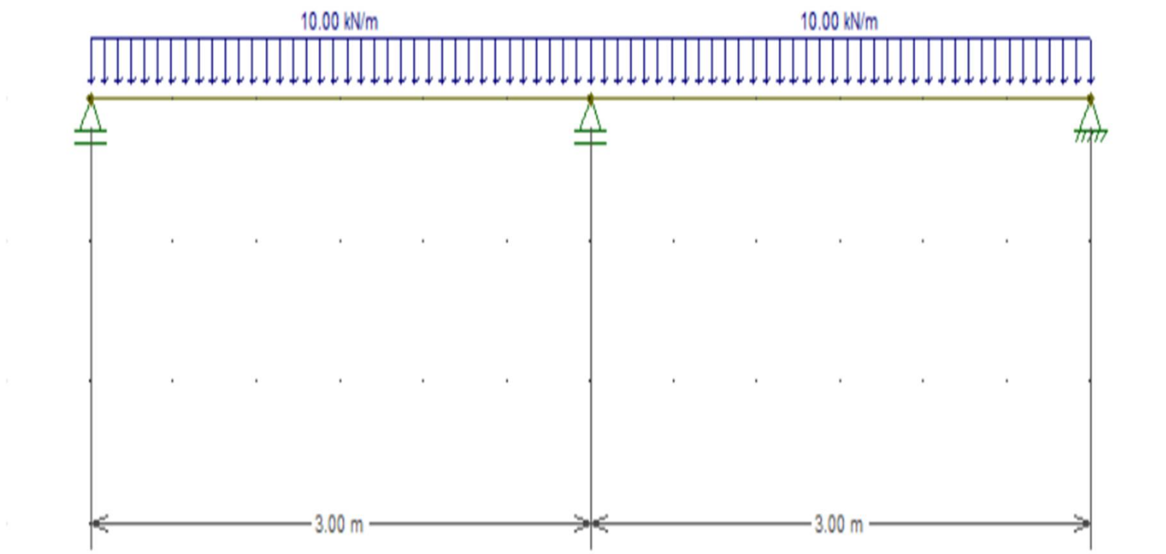
III. MODELS UNDER STUDY

For comparison purposes, two 6-meter beams are considered:

- Beam A (Isostatic): simply supported beam, loaded with q = 10 kN/m.
- Beam B (Hyperstatic): continuous beam with three supports (at 0 m, 3 m, and 6 m), with the same loading.
  Both beams are made of reinforced concrete, with constant cross-section and stiffness along their length.



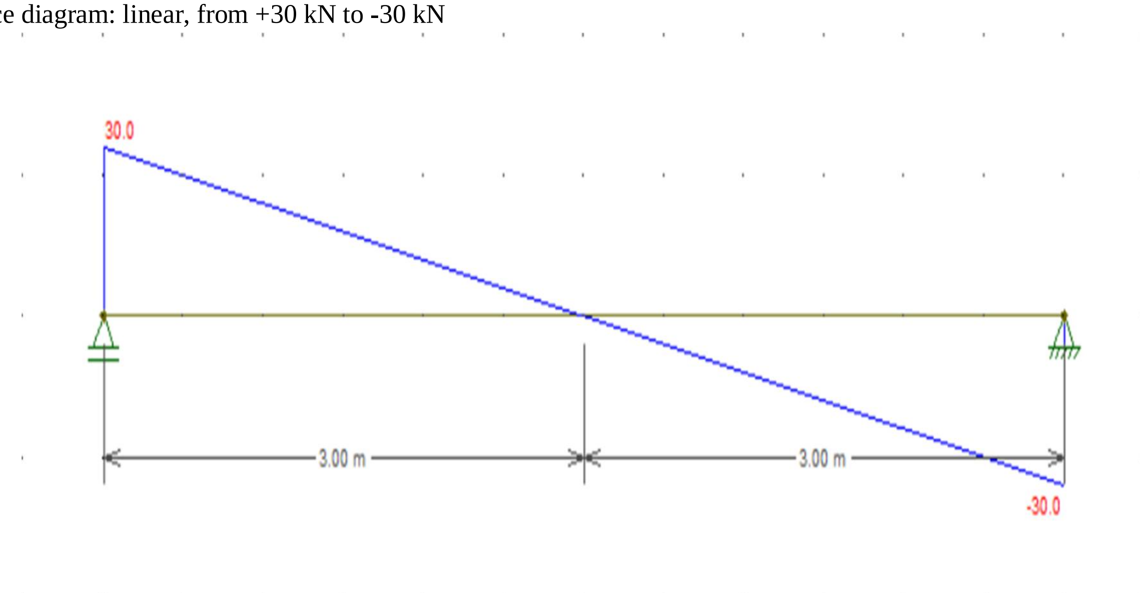
Beam A (Isostatic)



Beam B (Hyperstatic)

IV. ANALYSIS OF THE ISOSTATIC BEAM

• Support reactions: RA = 30 kN, RC = 30 Kn
• Shear force diagram: linear, from +30 kN to -30 kN



The figure shows the shear force diagram of a simply supported beam with a length of 6.00 m, subjected to a linearly varying distributed load. The shear force varies from +30 kN at the left support to -30 kN at the right support, forming a descending straight line.
This linear variation indicates that the applied load is triangular, acting downward from left to right. The shear force becomes zero at mid-span (3.00 m), which is typical for this type ofloading.
The diagram is essential for evaluating the shear strength of the beam and for properly designing its structural elements.
• Maximum bending moment at the center: M = 45 kN·m

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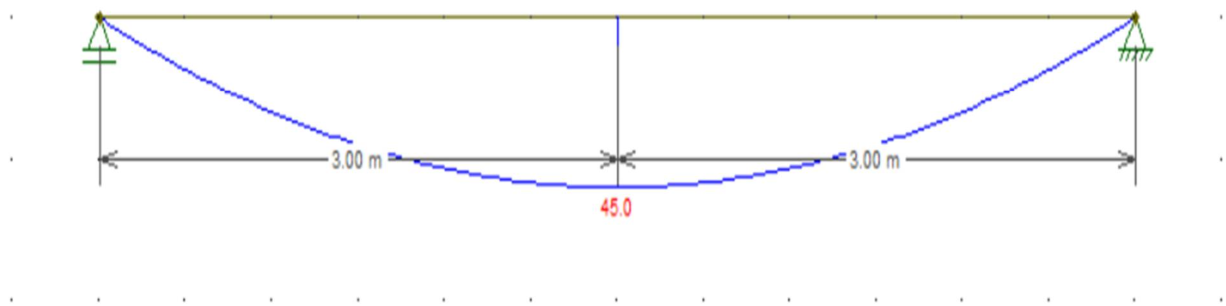
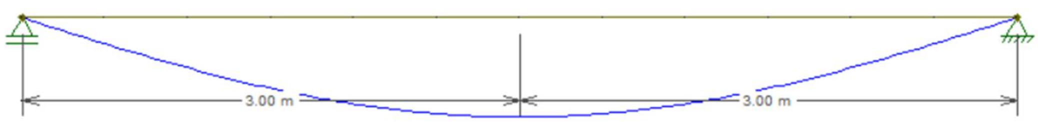


The bending moment reaches its maximum value at the center of the beam (3 m) due to the symmetry of the distributed load and the support reactions. Since the shear force varies linearly, the bending moment—which is the integral of the shear force—reaches its peak at mid-span. In this case, the value is 45 kN·m.

- Deflected shape



The figure shows the deflected shape of a simply supported beam subjected to a triangular distributed load. The beam exhibits a downward deflection, with the maximum displacement occurring at mid-span (3.00 m from each support), which is typical for this type of loading.

The deflected shape is symmetrical and continuous, indicating normal bending under load. This result allows for evaluating the structural behavior of the beam and verifying whether the displacements are within allowable limits.

• General analysis of the isostatic beam

The analyzed beam has one simple support and one double support, subjected to a distributed load of 10 kN/m over its entire 6 m span.

Support Reactions

The total applied load is 60 kN, resulting in the following reactions:

• Simple support: R_1 = 30 kN

• Double support: R_2 = 30 kN

Internal Forces

• Shear force: Varies linearly from +30 kN at the first support to -30 kN at the second.

• Bending moment: The maximum value occurs at mid-span, calculated as:

M_{max} = 45 kN·m

Structure's Deflection Profile

The beam shows a characteristic displacement profile:

• Maximum deflection at mid-span, with concave curvature.

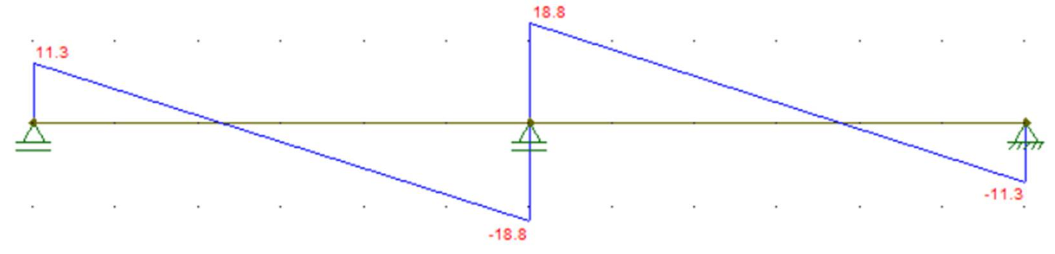
• Counter-deflection near the double support, due to imposed constraint.

• Curvature variation consistent with the calculated bending moments.

This analysis validates the structural behavior, ensuring that displacements and internal forces comply with safety and performance standards.

V. ANALYSIS OF THE HYPERSTATIC BEAM

1) Shear Force Diagram

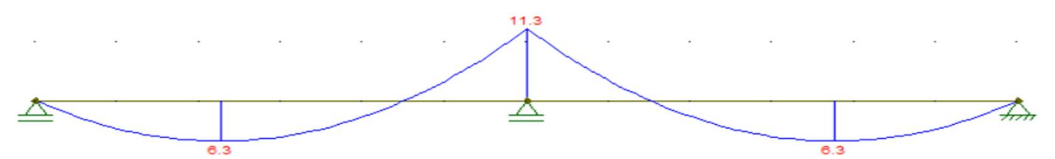


The figure presented shows the shear force diagram of a beam with two simple supports and one double (fixed) support. This configuration defines a first-degree hyperstatic structure, as the number of reactions exceeds the three static equilibrium equations ($\sum Fx = 0$, $\sum Fy = 0$, $\sum M = 0$).

The diagram was generated using Ftool software, which allows for the automatic analysis of hyperstatic structures by simultaneously solving equilibrium and compatibility equations.

The discontinuities (jumps) in the diagram represent vertical reactions at the supports, and the inclined ramps indicate the presence of a distributed load along the beam. The linear variation of shear force confirms this type of loading. Using Ftool is essential for this type of analysis, as manual calculation would require transforming the structure into an isostatic one by removing a redundancy and applying methods such as the force method or displacement method.

2) Bending Moment



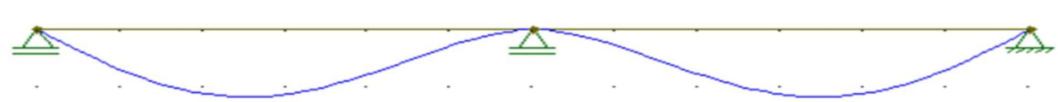
The bending moment diagram corresponds to a continuous beam with two simple supports and one fixed support, characterizing a first-degree hyperstatic structure. The analysis was carried out using Ftool software, which provides internal forces while accounting for deformation compatibility.

The diagram shows:

- Negative moments (−6.3 kN·m) near the supports, due to the beam's continuity.
- Maximum positive moment (+11.3 kN·m) at mid-span.
- Parabolic curvature, typical of a uniformly distributed load.

This type of moment distribution is common in continuous beams and must be considered in reinforcement design, ensuring sufficient strength in the most highly stressed regions.

3) Deflected Shape



The deflected shape of the beam represents how it deforms under the applied loads, taking into account the types of supports present.

Deflection behavior:
• At the simple supports, the beam can rotate freely but cannot move vertically at those points. As a result, the beam's curvature forms a smooth arch, with maximum downward deflection at the center of the free span.
• At the fixed (double) support, there is significant restriction to movement, limiting both rotation and displacement. This creates a zone of greater stiffness, where the beam tends to exhibit counter-deflection due to the negative moments observed.
• The positive moment in the central span reflects the maximum downward curvature, while the negative moments near the supports indicate inverse curvature (upward), resulting in a wave-like shape.

General Analysis of the Hyperstatic Structure
The beam analyzed has two simple supports and one fixed double support, characterizing it as a first-degree hyperstatic structure. It is subjected to a distributed load of 10 kN/m along its entire 6 m length.

Support Reactions
The total applied load is 60 kN. Due to the hyperstatic nature, determining the reactions requires applying deformation compatibility methods, such as the force method or the flexibility method.

Internal Forces
• Shear force: Shows linear variation along the span.
• Bending moment: Displays negative values near the supports due to the beam's continuity, with a reduced positive moment at mid-span, influenced by the additional stiffness of the fixed support.

Structure's Deflection Profile
• Maximum deflection at the central span, where the beam's curvature is most prominent.
• Counter-deflection at the ends, associated with negative moments near the supports.
• Increased stiffness at the fixed end, limiting the beam's displacement and rotation.
This analysis enables the evaluation of the beam's structural performance and ensures that internal forces and displacements comply with applicable safety and serviceability standards.

VI. COMPARISON BETWEEN THE MODELS

The comparison between the two structures reveals that, while the isostatic beam presents a concentrated maximum bending moment of 45 kNm, the hyperstatic beam distributes the internal forces, resulting in both positive and negative moments of 22.5 kNm. This distribution highlights the greater efficiency of the hyperstatic structure in redistributing loads and reducing peak stresses.
Despite the more complex analysis and the need for greater precision in constructing intermediate supports for hyperstatic structures, they offer higher structural efficiency and safety, making them particularly advantageous for permanent structures or those subject to variable loads. Isostatic structures, on the other hand, are simpler to analyze. This study emphasizes the importance of mastering these fundamental concepts for civil engineering students.

A. Detailed Advantages and Disadvantages

1) Isostatic Structures:

a) Advantages:

- Ease of Analysis and Calculation: The main advantage is that reactions and internal forces can be determined solely using the static equilibrium equations ($\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$). This simplifies the design process and understanding of structural behavior.
- Lower Construction Cost (in some cases): Due to the simplicity of the supports and the lack of need for strict deformation control between spans, execution can be more straightforward and therefore more cost-effective for certain structure types.
- Predictable Behavior: Under loading, the structural behavior is more direct and easier to predict, as there is no redistribution of internal forces in the event of a support failure.

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b) Disadvantages:

- Lack of Structural Redundancy: Failure of a single support or connection may lead to total structural collapse, as there are no alternative load paths. This makes them less safe in certain scenarios.
- Larger Displacements and Maximum Forces: As shown in the comparison, maximum bending moments may be significantly higher in isostatic structures (45 kNm in Beam A) compared to hyperstatic ones (with distributed forces, resulting in 22.5 kNm), requiring larger structural sections to resist those forces.

2) Hyperstatic Structures:

a) Advantages:

- Greater Stiffness and Performance: These structures have greater rigidity, resulting in lower deflections and displacements under the same loads, which improves structural performance.
- Structural Redundancy and Higher Safety: They can redistribute internal forces in case of failure of an element or support, offering a "backup system" and reducing the risk of total collapse. This makes them safer and more robust
- Section Optimization: The distribution of internal forces (positive and negative moments in different regions) may allow for the use of smaller cross-sections compared to isostatic structures, enabling more efficient material usage.

b) Disadvantages:

- More Complex Analysis and Calculation: Solving these structures requires the application of additional deformation compatibility equations along with equilibrium equations. Methods such as the force method, displacement method, or structural software (like Ftool) are essential.
- Greater Precision in Execution and Detailing: There is higher sensitivity to support settlement or execution errors. Intermediate supports demand stricter construction quality to ensure boundary conditions and load distribution match the design.

B. Typical Applications in Civil Engineering

1) *Isostatic Structures:* Commonly used where construction simplicity and cost are decisive factors, and structural redundancy is not a critical requirement or can be managed in other ways.

Examples: Small road bridges and walkways, simple warehouse roofs, cantilever beams in canopies, temporary structures (scaffolding, formwork), or secondary elements in larger structures.

2) *Hyperstatic Structures:* The preferred choice in large constructions and infrastructure due to their safety, durability, and capacity to support variable or unexpected loads.

Examples: Multi-story buildings (continuous beams, frames), large bridges (slabs, continuous beams), prestressed concrete structures, large-scale foundations (combined footings, pile caps), and elements in complex industrial structures.

C. Impact of Deformation Compatibility in Hyperstatic Structures

1) *Explanation*: In hyperstatic structures, the number of unknowns (support reactions and internal forces) exceeds the number of static equilibrium equations available. To solve these additional unknowns, deformation compatibility equations must be introduced.
2) *Practical Meaning:* This means that the displacements (deflections) and rotations of nodes and structural elements must be compatible with the support conditions (e.g., a fixed support does not allow rotation or displacement, while a simple support allows rotation but not vertical movement). The structure deforms in a way that is consistent with the constraints imposed by its supports and element continuity, and these deformations directly affect the distribution of internal forces.

D. Design Considerations

This section links structural analysis to the element design and sizing phase.

1) Project Optimization: When designing hyperstatic structures, the engineer must consider not only the absolute maximum values of internal forces (like bending moment), but also their distribution along the structure. The existence of negative moments at supports and positive moments at mid-span means that reinforcement (in reinforced concrete) or section size (in steel structures) must be designed to resist both types of forces in their respective regions.

2) Material Efficiency: The ability to redistribute stresses allows for more efficient use of materials, since peak forces can be "absorbed" by the structure's capacity to transfer loads to other areas. This can lead to a more economical and optimized design in terms of material consumption.

E. Importance of Structural Analysis Software

1) Calculation Complexity: For hyperstatic structures with multiple spans, complex loads, or irregular geometries, manual calculations become impractical and extremely time-consuming.
2) Speed and Accuracy: Software like Ftool allows engineers to perform complex analyses quickly and accurately, automatically solving both equilibrium and deformation compatibility equations. This speeds up the design process, allows exploration of multiple alternatives, and minimizes human error.
3) Visualization: These tools also provide visual outputs of internal force diagrams (shear force, bending moment) and deflections, facilitating the understanding of structural behavior and the identification of critical points.

VII. CONCLUSION

This article aimed to compare and deepen the understanding of isostatic and hyperstatic structures—fundamental concepts in structural analysis for civil engineering students. To this end, two beams subjected to identical loads were analyzed: a simply supported beam (isostatic) and a continuous beam with three supports (hyperstatic).

The analysis of the isostatic beam (Beam A) showed that it can be solved solely using equilibrium equations. The support reactions were RA = 30 kN and RC = 30 kN. The shear force diagram varied linearly from +30 kN to −30 kN, becoming zero at mid-span. The maximum bending moment, 45 kNm, occurred at the center. The deflected shape showed a maximum downward deflection at mid-span and a counter-deflection near the double support.

On the other hand, the hyperstatic beam (Beam B), due to having more unknowns than equilibrium equations, required the use of additional deformation compatibility equations and was analyzed with the aid of Ftool software. The shear force diagram showed discontinuities at the supports, indicating vertical reactions, and inclined ramps due to the distributed load. The bending moment diagram of the hyperstatic beam showed negative moments (−6.3 kNm) near the supports and a maximum positive moment (+11.3 kNm) at mid-span. The deflected shape, with its wave-like form, indicated a maximum downward deflection in the central span and counter-deflections at the ends associated with the negative moments.

In summary, the comparison between the two structures revealed that, while the isostatic beam concentrated the maximum bending moment at 45 kNm, the hyperstatic beam distributed the internal forces, resulting in positive and negative moments of 22.5 kNm. This redistribution of loads in the hyperstatic structure demonstrates its greater efficiency and ability to reduce peak stresses. Although the analysis of hyperstatic structures is more complex and requires greater precision in the execution of intermediate supports, they offer higher structural efficiency and safety, being particularly advantageous for permanent structures or those subject to variable loads. In contrast, isostatic structures are simpler to analyze. This practical study reinforces the importance of a solid understanding of these fundamental concepts in the training of future civil engineers.

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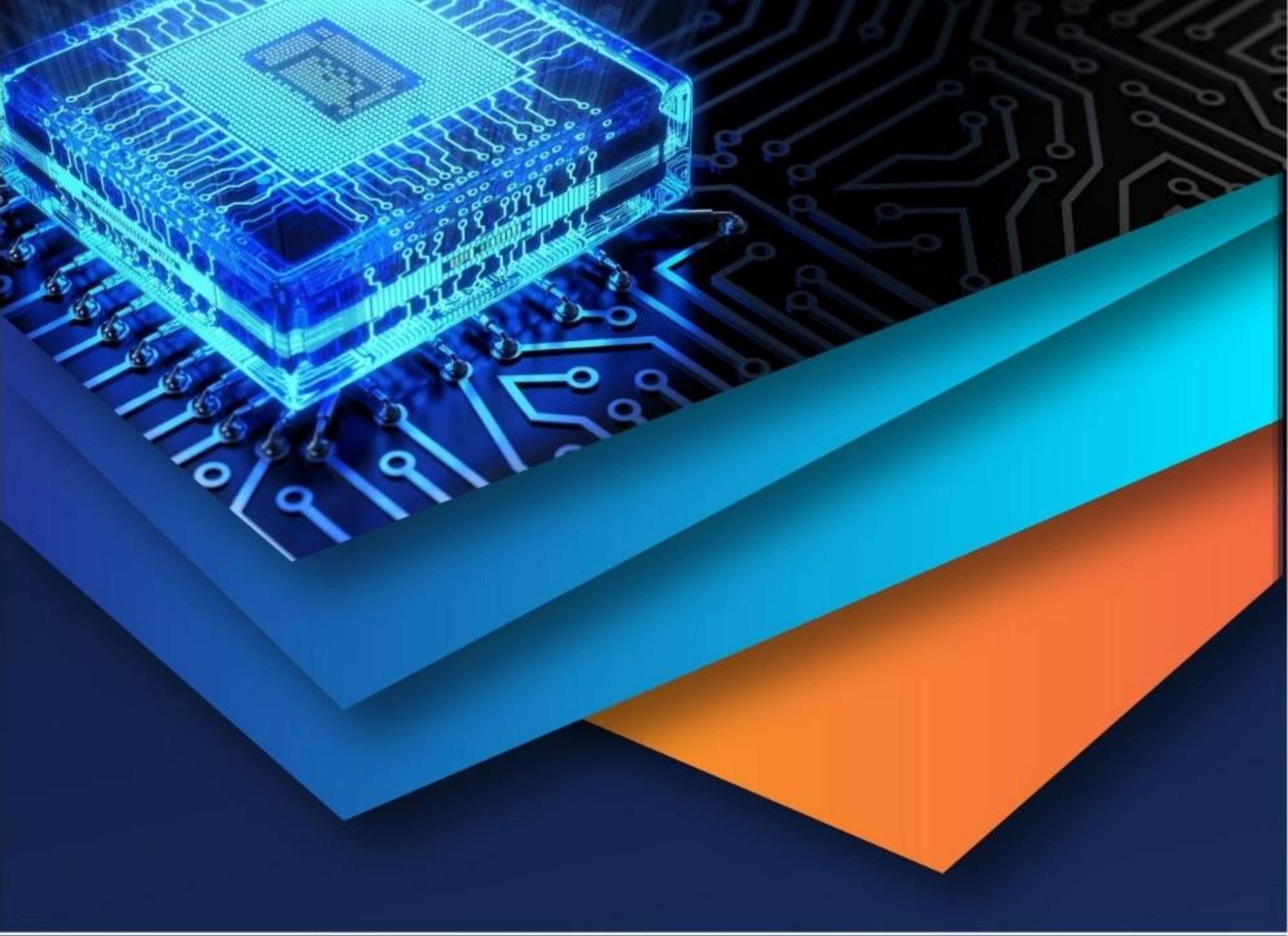

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