iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 10 Issue: VII Month of publication: July 2022

DOI: https://doi.org/10.22214/ijraset.2022.44310

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 10 Issue VII July 2022- Available at www.ijraset.com

Conditions for Inscribed Triangle to Be Similar to Given Triangle

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Abstract: In this research paper, I will derive the conditions for similarity between a triangle and its inscribed triangle. I will go through the inverse similarity conditions of an inscribed triangle for acute angled triangle and right angled triangle. I will use the basic laws of geometry to deduce the conclusion. Apart from mid-point triangle, we will analyze the conditions of similarity for other inscribed triangle.
Keywords: Inscribed triangle, Inverse Similarity, Mid-point triangle

I. INTRODUCTION

When we select one point on each sides of triangle and join all those points, the triangle thus formed is called inscribed triangle. We know that mid-point triangle is similar to the triangle in which it is inscribed. Here, I will explain the existence of an inscribed triangle similar to a given triangle except mid-point triangle.

A. Inverse Similarity

Inverse similarity means the inscribed triangle is similar to the given triangle where corresponding vertices are not collinear.
On the basis of number of mid-point included by inscribed triangle, I will further go into detail.
As shown in figure 1, P and R are the mid-points of AB and AC respectively while Q is not mid-point of BC.

Condition when inscribed triangle includes two mid-points of a side of triangle

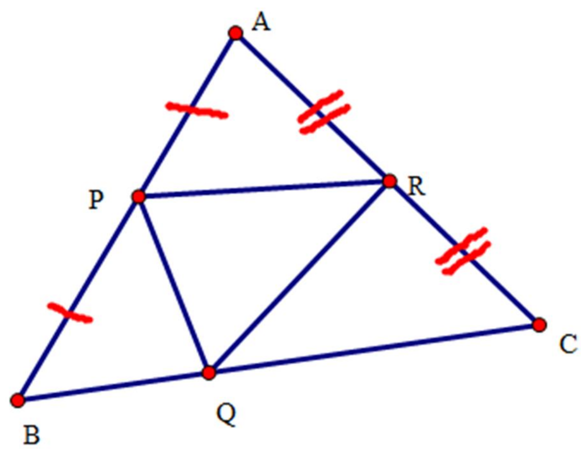

FIGURE 1:

As shown in figure 1, P and R are the mid-points of AB and AC respectively while Q is not mid-point of BC.
First case:
When **△ABC~ △QRP**
Given:
AP==PB & AR=RC
From mid-point theorem,
PR//BC
Now,

<ABC=<PRQ……… (1) (Corresponding angle of similar triangle)
<APR=<ABC……..... (2) (Being corresponding angle, since PR//BC)
From (1) and (2):
∵<PRQ=<APR
∴AB//QR
According to mid-point theorem,
If AR=RC and AB//QR, then BQ=QC which is contradicts our assumption that Q is not the mid-point of BC. Hence, it is not possible for an inscribed triangle being inversely similar to the given triangle when only two mid-points are included.

Condition when inscribed triangle includes only one mid-point of a side of a triangle

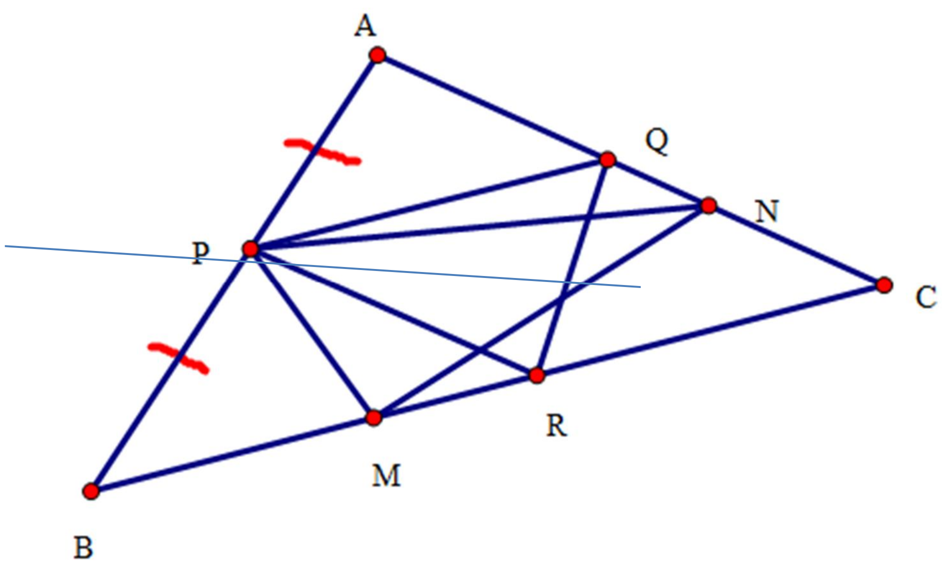

Figure 2:

As shown in the **figure 2**, P, Q and R are the mid-points of AB, AC and BC respectively. Similarly, M and N (both are not mid-points of BC and AC respectively) are the points on BC and AC respectively. Similarly, points C and P are joined.
We have,
△MNP~ △ABC
For the condition of inverse similarity, when M lies between B and R, Q had to lie between A and N and vice versa.
Proof:
△ABC ~ △MNP~ △RQP
Here:
<MPN=<RPQ (Corresponding angle of similar triangle)
<MPN-<RPN=<RPQ-<RPN (Subtracting common angle in both sides)
∴<MPR=<NPQ………….. (3)
∵ **△MNP~ △FED**
$\frac{PM}{PN} = \frac{PR}{PQ}$ (Ratio of corresponding sides of similar triangle is equal) …………… (4)
From equation (3) and (4), we can conclude that:
△PMR~ △PNQ
And, <PMR=<PNQ (Corresponding angle of similar triangle are equal)
∴ P, M, C and N are the cyclic points (Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle)
<PMN=<PCN ...…………… (5) (Angle subtended on the same arc chord of cyclic quadrilateral)
<PMN=<PAC ……………...... (6)(∵**△ABC ~ △MNP,** so corresponding angle of similar triangle are equal)

From (5) and (6), we have:
<PAC=<PCN
Hence, PCA is isosceles triangle PA=PC.
We have, PA=PB, since P is the mid-point of side AB
∴PA=PB=PC
Hence, **△ABC is right angle with <ACB=90°**
Similarly, <MPN=90° (Corresponding angle of congruent triangle)
From the above expression, we can conclude that if only one vertex of the inscribed triangle similar to given triangle incudes the mid-point of the side of given triangle, then both the inscribed triangle and given triangle are right-angled triangle.

II. CONVERSE PROOF

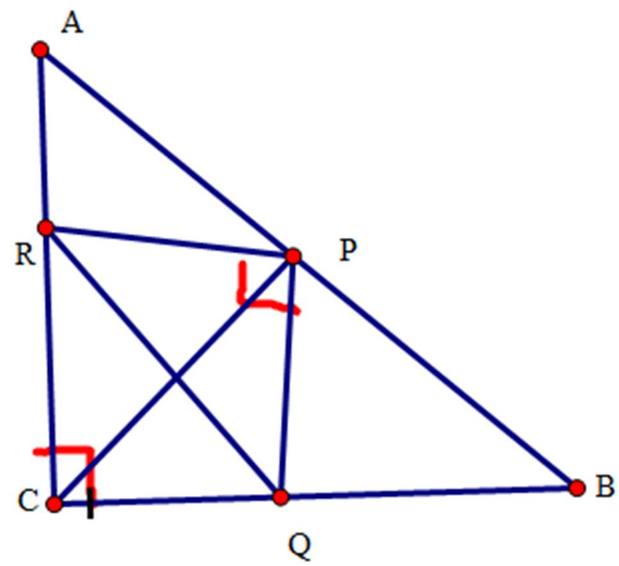

FIGURE 3:

In the **figure 3**, **△ABC~ △QRP**
And, points C and P are joined.
Similarly, both **△ABC and △QRP** are right angled triangle.
From the given figure:
<QCR+<QPR=180°
∴ Q, C, R and P are cyclic points.
Hence,
<PCR=<PQE............. (7) (Angle subtended on same chord of cyclic quadrilateral)
<PQR=<BAC............... (8) (Corresponding angle of similar triangle)
From (7) and (8):
<PAC=<PCA
△PAC is isosceles triangle.
∴ PA=PC......................... (I)
Again,
<QCP=<QRP.................... (9) (Angle subtended on same chord of cyclic quadrilateral)
<PRQ=<ABC....................... (10) (Corresponding angle of similar triangle)
From (9) and (10)
<PBC=<PCB
△PBC is isosceles triangle.
∴PB=PC...................... (II)
From (I) and (II):

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PA=PB=PC

Hence, we can conclude that if the inscribe triangle is inversely similar to the given triangle, where both the given triangle and inscribed triangle are right angled triangles, the vertex of the right angle of inscribed triangle will be mid-point of the hypotenuse of given triangle.

Condition when inscribed triangle does not include any mid-point of a side of a triangle

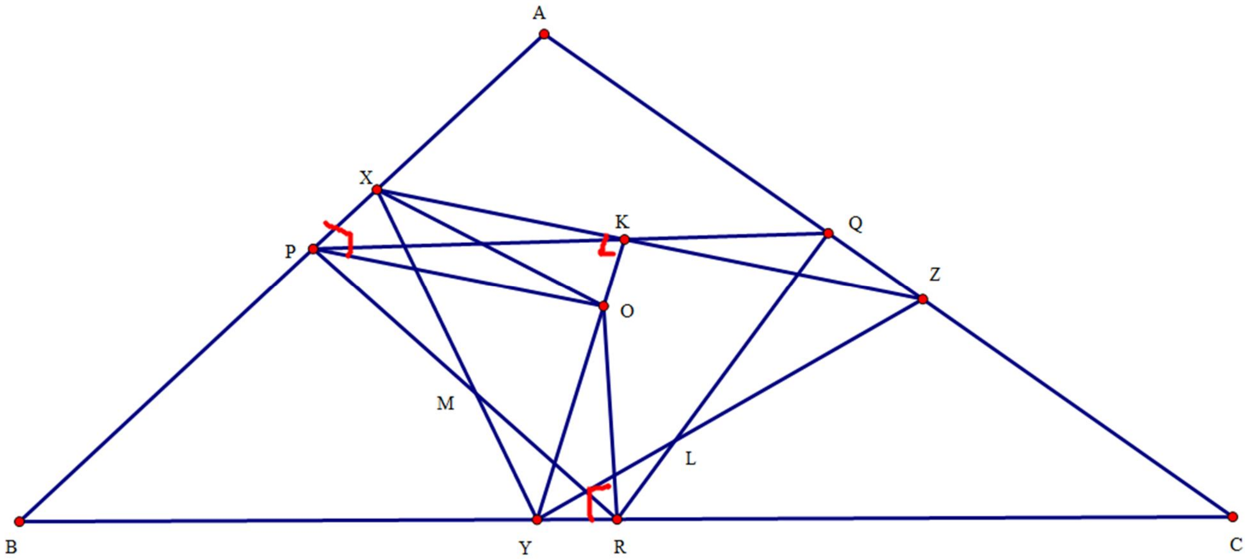


FIGURE 4

In the **figure 4**, K, L and M are the intersections points of corresponding sides of similar triangle. Similarly, O is the circumcenter of △**ABC** and YK⊥XZ. The reason behind choosing circumcentre O and YK⊥XZ is to find several pairs of perpendicular lines and cyclic points. P, Q and R are the mid-points of AB, AC and BC respectively.

Proof:

<ORB=90° (Perpendicular drawn from center to the chord bisects the chord)

∴<ORY+<OYR=90° (Being right-angled triangle)

PQ//BC (From mid-point theorem)

<PKY=<KYC (Being alternate angle)

Since, YK⊥XZ

<PKX+<PKO=90°

Similarly,

<OPX=90°.. (9)

<OPX+<OKX=90°+90°=180°

∴ O, P, X and K are cyclic points:

<XKP=<XCP... (10) (Angle subtended on same chord of cyclic quadrilateral)

From (9) and (10),

<XOP=<YOR

<OPX=<OYR=90°

∴△ORY~ △OPX

$\frac{OY}{OX} = \frac{OR}{OX}$ (Ratio of corresponding sides of similar triangle is equal)

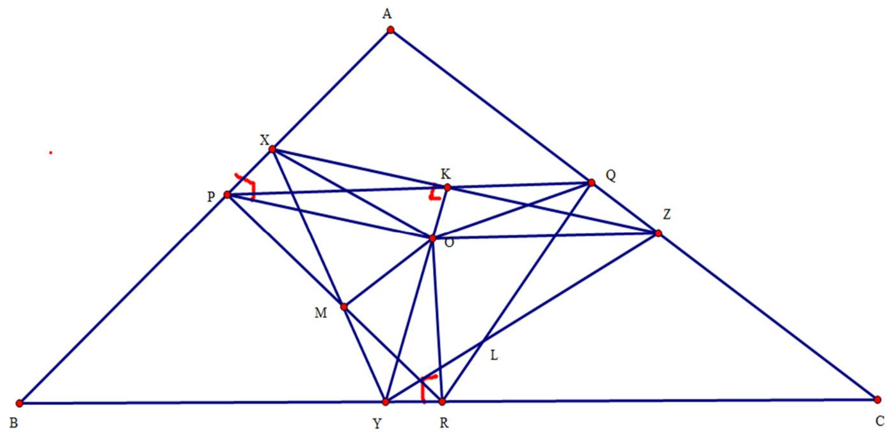
<YOR=<XOP

∴△OPR~ △OXY

<RPO=<YXO........................... (11) (Corresponding angle of similar triangle)

<OPK=<OXK............................ (12) (Angle subtended on same chord of cyclic quadrilateral)

Adding (11) and (12), we get:
<RPK=<YXK................................ (III)
In the next step, we will join OM, OQ and OZ.



<OQZ=90° (The perpendicular drawn from center to the chord bisects the chord)
<YKZ=90° (Given)
Since, <OKZ=<OQZ=90°
O, K, Q and Z are cyclic points.
<ZOQ=<ZKQ................... (13) (Angle subtended on same chord of cyclic quadrilateral)
<PKX=<ZKQ...................... (14) (Being vertically opposite angle)
From (13) and (14), we get:
<ZOQ=<PKZ............................. (15)
<YOR=<PKX................................. (16) (Already proved)
From (15) and (16), we get:
<YOR=<ZOQ..................................... (17)
<ORY=<OZQ=90° (Given)
∴△ORY~ △OQZ
Again,
$\frac{OZ}{OY} = \frac{OQ}{OR}$ (Ratio of corresponding sides of similar triangle is equal)
Adding <ROZ on both sides, we get:
<YOZ=<ROQ
So, **△YOZ~ △ROQ**
∴<OYZ=<ORQ............................ (18) (Corresponding sides of similar triangle)
∵△POR~ △XYO
<PRO=<XYO........................... (19)
On adding (18) and (19), we get:
<XYZ=<PRQ.................................... (IV)
From (III) and (IV), we have
∵△XZY~ △PQR
∴△YZX~ △ABC

Hence, for an acute angled triangle, the inscribed triangle is inversely similar to the given triangle when one altitude of an inscribed triangle passes through the circumcenter of given triangle and the perpendicular foot of altitude lies on the joining of two mid-points of the given triangle.

III. CONCLUSION

When the vertex of inscribed triangle includes the two mid points of sides of given triangle, then it is not possible for an inscribed triangle to be inversely similar to the given triangle without including the remaining mid points of a sides of given triangle. When the vertex of inscribed triangle includes only one midpoint of the sides of given triangle, then the inscribed triangle will be inversely similar to given triangle if and only if both inscribed triangle and given triangle are right angled triangle. For right angled triangle, inscribed triangle and given triangle are inversely similar if and only if the vertex of right angle of the inscribed triangle lies on the mid-point of the hypotenuse of given triangle. In case of acute angled triangle, inscribed triangle and given triangle are inversely similar if and only if one of the altitude of the inscribed triangle passes through the circumcenter of given triangle.

IV. FOLLOW-UP RESEARCH

1) For obtuse angled triangle, is the necessary for the altitude of inscribed triangle to pass through the circumcenter of given center to be inversely similar to the given triangle?
2) Under what conditions of obtuse angled triangle, inscribed triangle will be inversely similar to the given triangle?

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