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Cosmic String Isotropic Model in Modified Theory of Gravity

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Abstract: The spatially homogeneous Bianchi type-I in Kasner form have been considered in modified theory of gravitation. The exact solutions of the field equations in $f(R,T)$ gravity theory for Bianchi type- I(kasner form) have been obtained. The physical parameters of the models are also mentioned.
Keywords: Modified gravity, Bianchi type-I space-time in Kasner form, string model.

I. INTRODUCTION

The standard model of cosmology, grounded in Einstein's General Theory of Relativity (GR), has been instrumental in explaining the large-scale structure and dynamics of the Universe. However, observational evidence such as the accelerated expansion of the Universe and the presence of dark energy suggests that GR may not be sufficient to fully describe cosmic evolution in [1] and [2]. This has led to the exploration of various modified theories of gravity, among which gravity has received considerable attention.

The gravity theory proposed by Harko in [3], extends the Einstein-Hilbert action by considering an arbitrary function of the Ricci scalar and the trace of the energy-momentum tensor. The inclusion of allows for matter-geometry coupling, potentially capturing quantum effects or interactions between matter and curvature not accounted for in GR. This coupling leads to additional terms in the field equations, enriching the cosmological dynamics and providing a framework to model the Universe's accelerated expansion without invoking dark energy.

Anisotropic cosmological models are of particular interest in the early Universe, where isotropy cannot be assumed at all scales. Bianchi type-I models are the simplest class of anisotropic, spatially homogeneous space-times. These models generalize the flat FLRW metric by allowing directional dependence in the expansion rates, making them ideal for studying early-universe anisotropies. When specialized to the Kasner form, the Bianchi type-I metric admits a vacuum solution in GR characterized by power-law scale factors constrained by the Kasner conditions in [4]

In this chapter, the Bianchi type-I space-time in Kasner form within the framework of $f(R,T)$ gravity has been explored. The spatially homogeneous Bianchi type-I in Kasner form have been considered in modified theory of gravitation. The exact solutions of the field equations in $f(R,T)$ gravity theory for Bianchi type- I (kasner form) have been obtained. The physical parameters of the models are also mentioned.

This chapter is structured as follows: **Section (2)** presents the formulation of the $f(R,T)$ gravity and the description of the Bianchi type-I metric in Kasner form. In **Section (3)**, we derive the field equations and obtain exact solutions. **Section (4)** analyzes the physical behavior of the solutions and concludes with a summary of the main results and potential future work.

II. FIELD EQUATIONS

We consider anisotropic Bianchi type-I metric in Kasner form as

$$ds^2 = dt^2 - t^{2p_1}dx^2 - t^{2p_2}dy^2 - t^{2p_3}dz^2 \tag{1}$$

where, p_1, p_2 and p_3 are three parameters that we shall requires to be constant, satisfying the two relations $p_1+p_2+p_3 = s$ and $p_1{}^2+p_2{}^2+p_3{}^2 = \theta$

The energy momentum tensor of cloud string is given by

$$T_{ij} = \rho u_i u_j - \Lambda x_i x_j \tag{2}$$

where, $u^i u_i = -x^i x_i = -1$ and $u^i x_i = 0$

ρ = rest energy density and Λ = string tension density

The field equation for $f(R,T)$ gravity theory is given by,

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \tag{3}$$

To find the particular solution,

Let us consider, $f(T) = \mu T \; and \; f'(T) = \mu$ (4)

Using equation (1), (2), (3) and (4) we get the highly non-linear differential equation as follows,

$$[p_1(s-1) - M]t^{-2} = 8\pi(\rho + \Lambda) + [2\rho + 2\Lambda - 2p - T]\mu \quad (5)$$

$$[p_2(s-1) - M]t^{-2} = 8\pi(\rho + \Lambda) + [2\rho + 2\Lambda - 2p - T]\mu \quad (6)$$

$$[p_3(s-1) - M]t^{-2} = 8\pi(\rho + \Lambda) + [2\rho + 2\Lambda - 2p - T]\mu \quad (7)$$

$$\frac{1}{2}t^{-2}[s^2 - \theta] = -8\pi(\rho + \Lambda) + [-2\rho - 2\Lambda + 2p + T]\mu \quad (8)$$

where, $M = \frac{1}{2}[s^2 - 2s + \theta]$

III.SOLUTION OF THE FIELD EQUATIONS:

On solving equation (5), (6), (7), (8) we get,

$$p_1 = p_2 = p_3 = P \text{ (say)} \quad (9)$$

where, P is arbitrary constant

Hence Kasner form can be written as

$$ds^2 = dt^2 - t^{2P}[dx^2 + dy^2 + dz^2] \quad (10)$$

The spatial Volume V for the model is $V = t^s = t^{3p}$

The mean scale factor of model is obtained as $a = V^{\frac{1}{3}} = t^{\frac{s}{3}}$

The generalized mean Hubble Parameter H is given by,

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{(\dot{t^P})}{t^P} \quad (11)$$

The expansion scalar is given by

$$\theta = 3H = 3\frac{(t^P)^{\cdot}}{t^P} \quad (12)$$

The mean anisotropy Parameter Δ is given by,

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\frac{(H_i - H)^2}{H} = 0 \quad (13)$$

Also the shear Scalar σ is defined as,

$$\sigma^2 = \frac{3}{2}\Delta H^2 = 0 \quad (14)$$

The pressure is given by,

$$p = \frac{-1}{2t^2} + 4\pi(1 + \Lambda) + \frac{3}{2} \quad (15)$$

The energy density is obtained as,

$$\rho = \frac{8\pi(1+\Lambda)+2}{4(4\pi+1)} - \frac{1}{4(4\pi+1)t^2} \quad (16)$$

IV. RESULTS

The physical and kinematical properties are shown in the following graphs

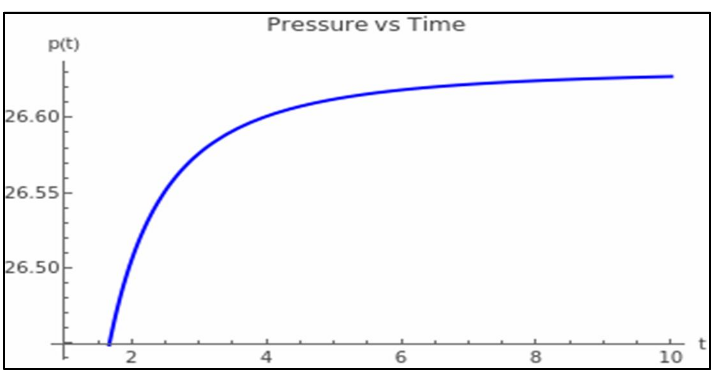

Figure 1
Presssure $p(t)$ vs cosmic time t

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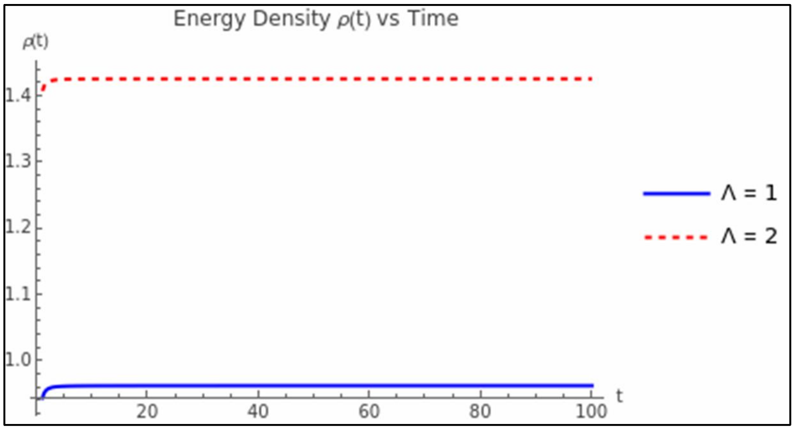

Figure 2

Energy density vs cosmic time t

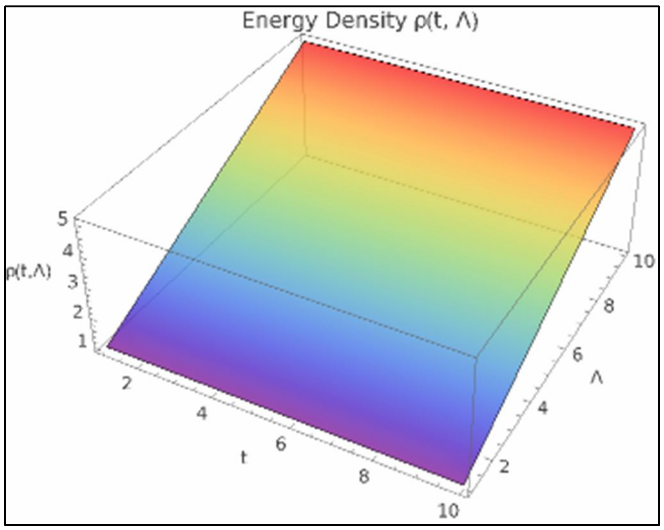

Figure 3

Energy density $\rho(t, \Lambda)$ in 3D

From figure 1, 2, 3 the following results are obtained

1) *Model becomes isotropic:* Bianchi type–I with $\Delta = 0$ and $\sigma = 0$ reduces (kinematically) to an FRW type behaviour (all directional scale factors expand at the same rate).
2) *Directional pressures equal:* Since $H_1 = H_2 = H_3 = H$ the field equations force the directional pressures to coincide. Isotropy implies the effective isotropic pressure p must match that value (or the equations must be arranged so the isotropic $p(t)$ equals the directional value).
3) *Shear terms drop out of dynamics:* Any terms in your field equations proportional to σ *or* Δ vanish simplifying the evolution equations for $a(t), \rho(t)$ *and* $p(t)$. The time dependence remaining in $\rho(t)$ *and* $p(t)$ comes only from the scalar terms and source terms like Λ
4) *Consistency conditions:* If previously-derived expressions give an explicit $p(t)$ (time dependent) while directional pressures were constant consistency requires either a the parameters be chosen so for all t *or* b the assumption that the model is only approximately isotropic at late times (so the equalities hold asymptotically). Otherwise you must revisit the assumptions/solutions used to obtain $p(t)$.
5) *Late-time behaviour:* In your numeric plots you saw both $p(t)$ and $\rho(t)$ approach constants that is consistent with vanishing shear: as shear decays, the universe approaches an isotropic, Λ dominated state.

V. CONCLUSION

1) We have considered Bianchi type-I metric (Kasner form) in the presence of cosmic string in modified theory of gravity.
2) It is interesting to know that our model is isotropic and Shear free. For the present cosmological model the anisotropy parameter $\Delta = 0$ implies $H_1 = H_2 = H_3 = H$; hence the directional Hubble rates coincide. Consequently the shear scalar $\sigma^2 = 0$ which is also vanishes.
3) The vanishing of Δ and σ therefore indicates that the model is kinematically isotropic in effect the Bianchi type–I solutions reduce to an FRW like behaviour.
4) All directional pressures become equal, shear-driven terms drop out of the field equations, and the late-time dynamics are dominated by the constant Λ term (consistent with the numerical plots which show $\rho(t)$ and $p(t)$ approaching constant values).

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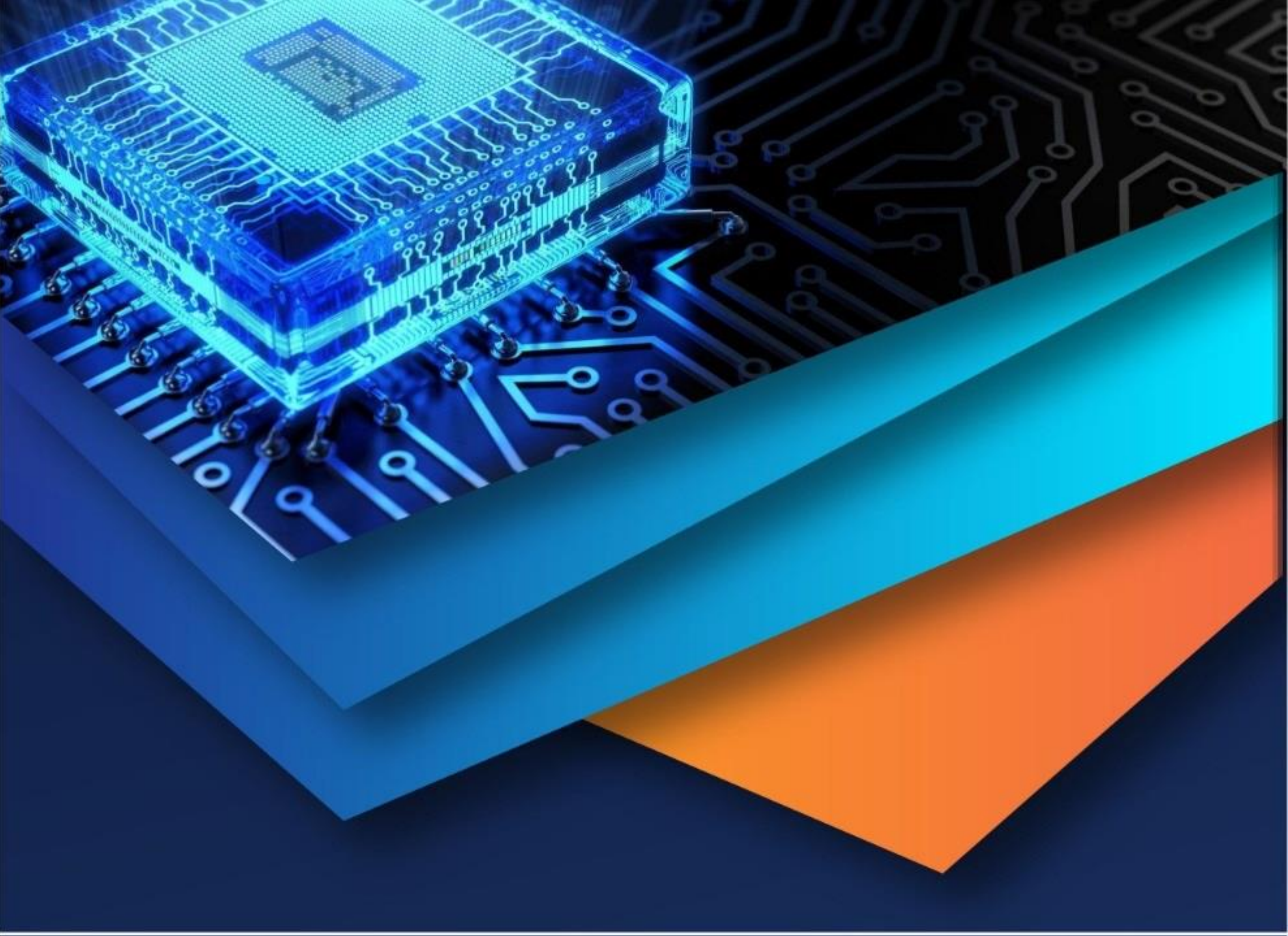

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