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# Deformable Rough Wide Tapered Land Slider Bearing

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***Abstract: This paper presents an analytical study of the impact of couple stress on the static and dynamic characteristics of a wide tapered land slider bearing under the influence of an applied magnetic field. A modified, non-dimensional Reynolds equation is derived for the bearing system. Closed-form expressions are obtained for the steady-state fluid film pressure, steady load-carrying capacity, and dynamic characteristics, including the dynamic stiffness and dynamic damping coefficients. Numerical computations demonstrate that a magneto hydrodynamic tapered land slider bearing, lubricated with a couple-stress fluid, offers higher steady load-carrying capacity, dynamic stiffness, and dynamic damping coefficients compared to the corresponding Newtonian fluid case. Additionally, the results indicate that the presence of an applied magnetic field, characterized by the Hartmann number, enhances both the static and dynamic performance of the tapered land slider bearing when compared to the case without a magnetic field.***
***Keywords: Reynolds' Expression (RE), Deformable Surface Roughness (DSR), Tapered Land Bearing (TLB); Magneto hydrodynamic (MHD); Couple Stress Fluids (CSF); Dynamic Stiffness (DS); Dynamic Damping (DD).***
***Subject Classification: 74A55 Theories of friction (Tribology), 85A30: Hydrodynamic and hydro magnetic problems.***

## I. LITERATURE REVIEW AND INTRODUCTION

The design of bearings is crucial for ensuring the efficient performance of machines. To achieve high efficiency and reliability, not only the correct size, functional type, and material of the bearing are important, but also the use of the proper lubricant. In many classical hydrodynamic lubrication theories, the lubricant is assumed to be a Newtonian fluid. However, in practical applications, this assumption often fails to account for real-world behavior. A desirable lubricant, which behaves as a non-Newtonian fluid, can be obtained by adding small amounts of long-chain polymers to a Newtonian fluid. In recent years, the study of lubrication with non-Newtonian fluids has garnered significant attention from researchers. The Stokes couple stress theory is the simplest extension of classical lubrication theory that accounts for polar effects, such as the presence of couple stresses and body couples. This theory has been used in various studies to investigate the effects of couple stresses in different bearing configurations. Examples include studies on composite inclined step bearings by Sinha and Singh [2], slider bearings by Ramanaiah and Sarkar [3], circular stepped plates by Naduvinamani and Siddangouda [4], porous stepped composite bearings by Apparao et al. [5], porous parallel stepped plates by Biradar [6], and rough parallel plates by Naduvinamani et al. [7]. These studies have shown that using a couple stress fluids for lubrication improves load carrying capacity and reduces the coefficient of friction compared to the corresponding Newtonian case. Another significant area of study is magneto-hydrodynamics (MHD), which deals with the dynamics of conducting fluids in the presence of an applied magnetic field. The application of a magnetic field is particularly useful for lubricants at high temperatures. Several researchers have examined the effect of an applied magnetic field on bearing performance. For instance, Hughes [8] studied an inclined slider bearing, Malik and Singh [9] investigated finite journal bearings, and Dudzinsky et al. [10] explored journal bearings. Their results showed that an increase in the strength of the applied magnetic field enhances the load-carrying capacity of the bearing compared to cases without a magnetic field. Given the positive effects of both couple stress and MHD individually, researchers have begun investigating their combined influence on bearing performance. The combined effects of MHD and couple stress have been studied in various configurations, including slider bearings by Das [11], squeeze film characteristics between a sphere and a plane surface by Naduvinamani and Rajashekar [12], parallel rectangular plates by Lin et al. [13], and different types of finite plates by Fathima et al. [14]. These studies found that the combination of couple stress and MHD further improves bearing characteristics compared to both the Newtonian and non-magnetic cases.

In bearing design, both static and dynamic characteristics are crucial. Static characteristics provide a baseline, while dynamic characteristics are essential for assessing stability. Taylor and Dowson [15] explored how existing lubrication theories can be applied to bearing design under external magnetic fields. The governing equations for the flow of conducting couple stress fluids in the presence of an applied magnetic field, as described by Das [11], form the foundation for these studies. Lin et al. [16] investigated the dynamic characteristics of an inclined plane slider bearing in the presence of a magnetic field and concluded that the applied magnetic field improves dynamic characteristics. Lin et al. [17] and Lin [18] studied the dynamic characteristics of wide tapered land bearings, first in the absence of a magnetic field and then in the presence of one. Their results indicate that the applied magnetic field improves bearing performance in terms of both static and dynamic characteristics. The objective of this research article is to scrutinize the effects of couple stress on the static and dynamic characteristics of wide deformable rough tapered land bearings in the presence of an external magnetic field. The results obtained will be compared with those from Lin [18] (Newtonian magnetic case) and Lin et al. [17] (Newtonian non-magnetic case). Preliminary findings show that the results are in close agreement with the previous studies by Lin et al. [17] and Lin [18]. The study aims to analyze and examine the static and dynamic characteristics of a MHD wide tapered land rough slider bearing through mathematical analysis.

## II. GEOMETRICAL AND MATHEMATICAL ANALYSIS

The geometry of a wide tapered land slider bearing of length L is shown in figure (A), while the deformable roughness configuration is presented in figures (B and C).

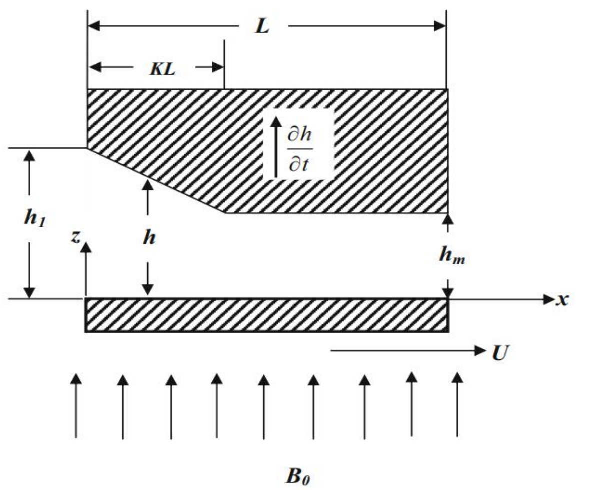


Figure (A) Geometry of tapered land slider bearing

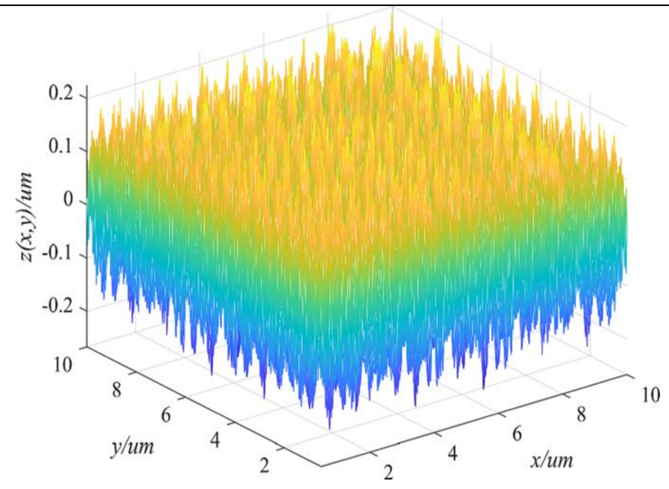


Figure (B): Surface roughness pattern in 3 – D Shuai Mo et. al. 2023

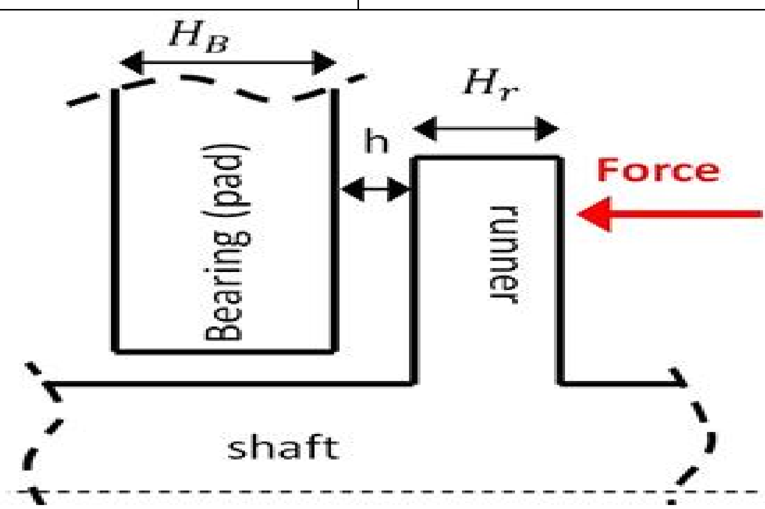


Figure (C): Tapered land thrust bearing side view, Sa Dousti et. al. 2016

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In this problem, a couple stress fluids are considered within the film region, where a uniform transverse magnetic field $B_0$ is applied perpendicular to the bearing. The lower surface of the bearing moves with a sliding velocity U in the x-direction, while the upper surface experiences a squeezing effect, with ∂h/∂t representing the rate of change in the film thickness. The following assumptions are made in formulating the problem: the fluid film is thin, fluid inertia is negligible relative to the viscous forces, and body forces and body coupling effects are insignificant, except for the Lorentz force. Additionally, the induced magnetic field is much smaller than the externally applied magnetic field. Under these conditions, the governing equations for the flow of conducting couple stress fluid in the presence of the applied magnetic field, as described by Das [11], are considered.

$$\mu\frac{\partial^2 u}{\partial z^2} - \eta\frac{\partial^4 u}{\partial z^4} - \sigma B_0^2 u = \frac{\partial p}{\partial x} + \sigma E_y B_0 \tag{1}$$

$$\frac{\partial p}{\partial z} = 0 \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

$$\int_{z=0}^{z=h}\{E_y + B_0 u\}dz = 0 \tag{4}$$

where μ is the lubricant standard viscosity, η is the material constant accountable for couple stresses, σ is the conductivity of the lubricant, p is the film pressure and u, w are the velocity components in the x and z directions respectively. The boundary conditions for velocity components are as follows.

Upper surface: z = h: u = 0, $\frac{\partial^2 u}{\partial z^2} = 0$, w = $h_t$ (5)

Lower surface: z = 0: u = U, $\frac{\partial^2 u}{\partial z^2} = 0$, w = 0 (6)

Use of boundary conditions (2a), (2b) and Equation (1d) the solution of Expression (1a) is obtained as:

$$u = \begin{cases} -\frac{U}{2}[f_{11} - f_{12}] - \frac{h_{ms}^2 h}{2l\mu M^2}\frac{\partial p}{\partial x}[f_{13} - f_{14}] \, for \, \frac{4M^2 l^2}{h_{ms}^2} < 1 \\ -\frac{U}{2}[f_{21} - f_{22}] - \frac{h_{ms}^2 h}{2l\mu M^2}\frac{\partial p}{\partial x}[f_{23} - f_{24}] \, for \, \frac{4M^2 l^2}{h_{ms}^2} = 1 \\ \frac{U}{2}[f_{31} \quad f_{32}] \quad \frac{h_{ms}^2 h}{2l\mu M^2}\frac{\partial p}{\partial x}[f_{33} \quad f_{34}] \, for \, \frac{4M^2 l^2}{h_{ms}^2} > 1 \end{cases} \tag{7}$$

where $M = B_0 h_{ms}\left(\frac{\sigma}{\mu}\right)^{1/2}$ is the Hartmann number and $l = \left(\frac{\eta}{\mu}\right)^{1/2}$ is the couple stress parameter. The related in (7) are given in Appendix (Z). With the use of boundary conditions (1a and 1b) and the expression (3) for u, the integration of continuity equation (1c) over the film thickness gives the modified Reynolds' equation in the form

$$\frac{\partial}{\partial x}\left[g(h, l, M)\frac{\partial p}{\partial x}\right] = 6\mu U\frac{\partial h}{\partial x} + 12\,\mu\frac{\partial h}{\partial t} \tag{8}$$

Here,

$$DRS(G(h), l, M) = \begin{cases} \frac{5h_{ms}^2 G(h)^{2/3}}{lM^2}\left(\frac{\alpha^2 - \beta^2}{\frac{\alpha^2}{\beta}\tanh\left(\frac{\beta h}{2l}\right) \quad \frac{\beta^2}{\alpha}\tanh\left(\frac{\alpha h}{2l}\right)} - \frac{2l}{h} \, for \, \frac{4M^2 l^2}{h_{ms}^2} < 1\right. \\ \frac{6h_{ms}^2 G(h)^{2/3}}{lM^2}\left(\frac{2\left[\cosh\left(\frac{h}{\sqrt{2}l}\right)+1\right]}{3\sqrt{2}\sinh\left(\frac{h}{\sqrt{2}l}\right) - \frac{h}{l}} - \frac{2l}{h} \, for \, \frac{4M^2 l^2}{h_{ms}^2} - 1\right. \\ \frac{6h_{ms}^2 G(h)^{2/3}}{lM^2}\left(\frac{M(\cos\beta_1 h + \cos\alpha_1 h)}{h_{ms}(\alpha_2\sin\beta_1 h + \beta_2\sin\alpha_1 h)} \quad \frac{2l}{h} \, for \, \frac{4M^2 l^2}{h_{ms}^2} > 1\right. \end{cases} \tag{9}$$

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$$\alpha = \left\{\frac{1 + \sqrt{1 \;\; \frac{4l^2M^2}{h_{ms}^2}}}{2}\right\}^{1/2} \quad \text{(a)} \qquad \beta = \left\{\frac{1 \;\; \sqrt{1 \;\; \frac{4l^2M^2}{h_{ms}^2}}}{2}\right\}^{1/2} \quad \text{(b)}$$

$$\alpha_1 = \left\{\frac{M}{lh_{ms}} cos\left(\frac{\varphi}{2}\right)\right\}^{1/2} \quad \text{(c)} \qquad \beta_1 = \left\{\frac{M}{lh_{ms}} sin\left(\frac{\varphi}{2}\right)\right\}^{1/2} \quad \text{(d)}$$

$$\varphi = tan^{-1}\left\{\frac{4l^2M^2}{h_{ms}^2} - 1\right\}^{1/2} \quad \text{(e)} \qquad \alpha_2 = [\beta_1 - \alpha_1 \cot\varphi] \quad \text{(f)}$$

$$\beta_2 = [\alpha_1 - \beta_1 \cot\varphi] \quad \text{(g)}$$

Here,

$$g(h) = \left(h + \frac{p}{p_a}\frac{\delta}{h_0}\right)^3 + 3\left(h + \frac{p}{p_a}\frac{\delta}{h_0}\right)^2 \frac{\alpha}{h_0} + 3\left(h + \frac{p}{p_a}\frac{\delta}{h_0}\right)\left[\left(\frac{\sigma}{h_0}\right)^2 + \left(\frac{\alpha}{h_0}\right)^2\right] + \left(\frac{\sigma}{h_0}\right)^2\left(\frac{\alpha}{h_0}\right) + \left(\frac{\alpha}{h_0}\right)^3 + \left(\frac{\varepsilon}{h_0}\right) + 12\frac{\phi}{h_0^{\,3}} \quad (R_1)$$

Non dimensional deformable roughness function:

$$G(h^*) = (1 + p^*\delta^*)^3 + 3(1 + p^*\delta^*)^2\alpha^* + 3(1 + p^*\delta^*)\{\sigma^{*2} + \alpha^{*2}\} + 3\{\sigma^{*2}\alpha^{*2}\} + \alpha^{*3} + \varepsilon^* + 12\psi \quad (R_2)$$

Where

$$\delta^* = \frac{\delta}{h_0} \qquad \sigma^* = \frac{\sigma}{h_0} \qquad \alpha^* = \frac{\alpha}{h_0}$$

$$\varepsilon^* = \frac{\varepsilon}{h_0^{\,3}} \qquad p^* = \frac{p}{p_a} \qquad \psi = \frac{12\phi}{H_0^{\,3}}$$

The modified Reynolds' type equation (8) is applicable to one-dimensional slider bearings lubricated with couple stress fluid in the presence of transverse magnetic field with the squeezing effect $\partial h/\partial t$. The film thickness in the flow region of bearing is given by

$$h(x, t) = h_m(t) + h_1(x) \quad (10)$$

where $h_m(t)$ is the minimum film thickness and $h_1(x)$ is the known profile for the slider bearing. For wide tapered land bearing, $h_1(x)$ is given by

$$h_1(x) = \begin{cases} d\left[1 - \left(\frac{x}{KL}\right)\right], & 0 \le x \le KL \\ 0, & KL \le x \le L \end{cases} \quad (11)$$

Further, $d = h_1 - h_m$ is the difference between the inlet and outlet film thickness and K is the ratio of the inclined part length to the length of the bearing, known as the geometric parameter.

Defining the non-dimensional quantities:

$x^* = x/L$ $\quad l^* = 2l/h_{ms}$ $\quad p^* = ph_{ms} / \mu UL$ $\quad h_m^{\,*} = h_m(t) / h_{ms}$

$h^* = h / h_{ms}$ $\quad h_1^{\,*} = h_1 / h_{ms}$ $\quad t^* = U_t / L$ $\quad l^* = 2l / h_{ms}$

using these in Eq. (8), the non-dimensional modified Reynolds equation is obtained in the form

$$\frac{\partial}{\partial x^*}\left\{G^*(h^*, l^*, M)\frac{\partial p^*}{\partial x^*}\right\} = 6\frac{\partial h^*}{\partial x^*} + 12\frac{\partial h^*}{\partial t^*} \quad (12)$$

where

$$DRS(G(h),l,M)=\begin{cases}\dfrac{6h_{ms}^{2}G(h)^{2/3}}{lM^{2}}\left(\dfrac{\alpha^{2}-\beta^{2}}{\frac{\alpha^{2}}{\beta}\tanh\left(\frac{\beta h}{2l}\right)-\frac{\beta^{2}}{\alpha}\tanh\left(\frac{\alpha h}{2l}\right)}-\dfrac{2l}{h}\right. & for\ \dfrac{4M^{2}l^{2}}{h_{ms}^{2}}<1\\ \dfrac{6h_{ms}^{2}G(h)^{2/3}}{lM^{2}}\left(\dfrac{2\left[\cosh\left(\frac{h}{\sqrt{2}l}\right)+1\right]}{3\sqrt{2}\sinh\left(\frac{h}{\sqrt{2}l}\right)-\frac{h}{l}}-\dfrac{2l}{h}\right. & for\ \dfrac{4M^{2}l^{2}}{h_{ms}^{2}}-1\\ \dfrac{6h_{ms}^{2}G(h)^{2/3}}{lM^{2}}\left(\dfrac{M(\cos\beta_{1}h+\cos\alpha_{1}h)}{h_{ms}(\alpha_{2}\sin\beta_{1}h+\beta_{2}\sin\alpha_{1}h)}-\dfrac{2l}{h}\right. & for\ \dfrac{4M^{2}l^{2}}{h_{ms}^{2}}>1\end{cases}\tag{13}$$

$h^*$ is the non-dimensional film thickness and is given by

$$h^*(x^*,t^*)=h_m^*(t^*)+h_1^*(x^*)\tag{14}$$

where

$$h_1^*\left(x^*\right)=\begin{bmatrix}\delta\left\{1-\dfrac{x^*}{K}\right\};0\le x^*\le K\\ 0;K\le x^*\le 1\end{bmatrix}$$

$\delta = d / h_{ms}$ is the profile parameter.

The pressure boundary conditions are provided by

$$p^*=0 \text{ at } x^*=0 \text{ and } x^*=1\tag{15}$$

Integrating the non-dimensional Reynolds equation (12) with respect to $x^*$ twice and applying the pressure boundary conditions given in Eq. (15), the non-dimensional pressure is obtained as

$$p^*=\begin{pmatrix}6\xi_A\left[x^*,h_m^*\right]+12V^*\xi_B\left[x^*,h_m^*\right]+C_1\left[V^*,h_m^*\right]\xi_C\left[x^*,h_m^*\right];0\le x^*\le K\\ 12V^*\xi_D\left[x^*,h_m^*\right]+C_1\left[V^*,h_m^*\right]\xi_E\left[x^*,h_m^*\right];K\le x^*\le 1\end{pmatrix}\tag{16}$$

Where

$$C_1\left(V^*,h_m^*\right)=\frac{12V^*\xi_{DK}\left(h_m^*\right)-12V^*\xi_{BK}\left(h_m^*\right)-6V^*\xi_{AK}\left(h_m^*\right)}{\xi_{CK}\left(h_m^*\right)-\xi_{EK}\left(h_m^*\right)}$$

The associated relations are given in Appendix A through Equations. (A2a)–(A2j). On integration of non-dimensional film pressure over the film region, the film force in non-dimensional form is obtained as

$$F^*=6\chi_{AK}\left(h_m^*\right)+12V^*\left\{\chi_{BK}\left(h_m^*\right)+\chi_{DK}\left(h_m^*\right)\right\}+C_1\left(V^*,h_m^*\right)\left[\chi_{CK}\left(h_m^*\right)+\chi_{EK}\left(h_m^*\right)\right]\tag{17}$$

## III. STEADY-STATE CHARACTERISTICS

The steady film pressure and the steady load capacity in non-dimensional form are obtained from (16) and (17) by setting the non-dimensional minimum film height to be a constant and the non-dimensional squeezing velocity to be zero.

$$p_S^*=\begin{pmatrix}6\xi_A\left[x^*,h_m^*\right]_s+C_1\left[0,h_m^*\right]\xi_C\left[x^*,h_m^*\right]_s;0\le x^*\le K\\ C_1\left[0,h_m^*\right]\xi_E\left[x^*,h_m^*\right]_s;K\le x^*\le 1\end{pmatrix}\tag{18}$$

$$W_S^*=6\left[\chi_{AK}\left(h_m^*\right)\right]_s+C_1\left[0,h_m^*\right]\left\{\chi_{CK}\left(h_m^*\right)+\chi_{EK}\left(h_m^*\right)\right\}_s\tag{19}$$

Here, subscript denotes the steady state situation.

## IV. DYNAMIC CHARACTERISTICS

The non-dimensional dynamic stiffness coefficient is obtained by calculating the partial derivative of film force with respect to non-dimensional minimum film thickness and thereafter taking the value in steady state:

$$S_d^*=-6\left[\frac{\partial\chi_{AK}}{\partial h_m^*}\right]-C_1\left(0,h_m^*\right)\left[\left\{\frac{\partial\chi_{CK}}{\partial h_m^*}\right\}_s+\left\{\frac{\partial\chi_{EK}}{\partial h_m^*}\right\}_s\right]-\left\{\frac{\partial C_1}{\partial h_m^*}\right\}_s\left[\chi_{CK}\left(h_m^*\right)+\chi_{EK}\left(h_m^*\right)\right]_s\tag{20}$$

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The non-dimensional dynamic damping coefficient is obtained by calculating the partial derivative of film force with respect to non-dimensional squeezing velocity and thereafter taking the value in steady state:

$$C_d^{\ *} = -12\left[\chi_{BK}\left(h_m^{\ *}\right) + \chi_{DK}\left(h_m^{\ *}\right)\right]_s - \left\{\frac{\partial C_1}{\partial V^*}\right\}_s \left[\chi_{CK}\left(h_m^{\ *}\right) + \chi_{EK}\left(h_m^{\ *}\right)\right]_s \qquad (21)$$

## V. RESULTS AND DISCUSSIONS

This paper analytically investigates the effect of couple stress on the performance characteristics of a wide tapered land slider bearing in the presence of an applied magnetic field. The bearing's performance is influenced by the couple stress parameter $l^*$, the Hartmann number M, the profile parameter $\delta$, and the geometric parameter K. By selecting specific values for the parameters, some previous results can be recovered from this study.

(a) when $l^* = 0$ the study conducted by Lin [18] examines an MHD tapered land slider bearing lubricated with a Newtonian fluid

(b) when $l^* = 0$ and $M = 0$, Lin et al. [17] studied a tapered land slider bearing in the non-conducting Newtonian case.

The use of couple stress fluid in the film region results in a higher steady-state pressure compared to the corresponding Newtonian case (where $l^* = 0$). When a magnetic field is applied, it further increases the steady-state pressure, even when the bearing is lubricated with couple stress fluid. This effect becomes more pronounced for larger values of $l^*$. The increase in steady pressure is attributed to the reduced velocity of the fluid in the film region caused by the magnetic field. The non-dimensional pressure $p_s^*$ depends on the non-dimensional coordinate $x^*$ with variations observed for different values of $l^*$ and M when $\delta = 1$ and $K = 0.8$. For $K = 0.8$, $x^* = 0.65$ and $\delta = 1$, a 24% increase in steady film pressure is observed for couple stress fluid ($l^* = 0$) compared to the Newtonian fluid. This increase rises to 68% when a magnetic field with $M = 5$ and $l^* = 0.4$ is applied, compared to the non-magnetic case. Additionally, it is observed that the steady load-carrying capacity increases with the profile and geometric parameters, reaching a maximum and then decreasing thereafter. The application of couple stress and a magnetic field enhances the steady load-carrying capacity compared to the Newtonian and non-magnetic cases. This increase is due to the higher pressure resulting from the reduced fluid velocity under the magnetic field. The steady load-carrying capacity achieves its maximum value at the geometric parameter $K = 0.8$ irrespective of the values of the couple stress parameter and the Hartmann number. This suggests that the tapered land slider bearing provides optimal performance when $K = 0.8$. For $K = 0.8$ and $\delta = 1$ with couple stress fluid ($l^* = 0.4$) a 21% increase in steady load capacity is observed compared to the Newtonian fluid. This increase rises to 76% with a magnetic field ($M = 5$, $l^* = 0.4$) compared to the non-magnetic case. The dynamic stiffness coefficient increases with both the profile and geometric parameters until it reaches its maximum, after which it decreases. The effect of couple stress and the applied magnetic field increases the dynamic stiffness coefficient compared to the Newtonian and non-magnetic cases. The maximum value of the dynamic stiffness coefficient occurs at $K = 0.8$ for all values of M and $l^*$. Moreover, the point at which the maximum value of the dynamic stiffness coefficient occur shifts toward higher values of $\delta$ for larger values of M and $l^*$. With $K = 0.8$ and $\delta = 1$, for couple stress fluid ($l^* = 0.4$) a 36% increase in the dynamic stiffness coefficient is observed compared to the Newtonian fluid. In the presence of a magnetic field with $M = 5$ and $l^* = 0.4$, this increase is about 39% compared to the non-magnetic case. The dynamic damping coefficient decreases as the values of the profile and geometric parameters increase. However, when compared to the Newtonian and non-magnetic cases, the dynamic damping coefficient increases due to the effects of couple stress and the applied magnetic field. For $K = 0.8$ and $\delta = 1$, for couple stress fluid ($l^* = 0.4$), a 23% increase in the dynamic damping coefficient is observed compared to the Newtonian fluid. When a magnetic field with $M = 5$ and $l^* = 0.4$ is applied, the increase is about 72% compared to the non-magnetic case.

## VI. FRUITFUL CONCLUSIONS

The effect of couple stress on static and dynamic characteristics of tapered land slider bearing in the presence of applied magnetic field is analysed. In accordance with the results and discussion, the following conclusions can be drawn.

1) Compared with the Newtonian case, the couple stress on MHD tapered land slider bearing effects higher values for steady film pressure, steady load carrying capacity, dynamic stiffness coefficient and dynamic damping coefficient.
2) Applied magnetic field also improves bearing characteristics and bearings are even lubricated with couple stress fluid.
3) Tapered land slider bearing lubricated with couple stress fluid in the presence of applied magnetic field shows higher performance for higher values of Hartmann number and couple stress parameter.

## VII. FUTURE RESEARCH

1) In conclusion, the study of porous roughness annular discs is an important are of research with several applications in fluid dynamics, materials science, and engineering.
2) Several promising directions stand out as having high potential for future research in this area, including advanced materials and fabrication modeling, and simulation, applications in energy and environmental engineering, surface modification techniques, data-driven approaches, etc.
3) We can learn much more about porous roughness annular discs and unlock their potential to solve challenging problems across a range of industries by incorporating these research directions.
4) Researchers can help make future engineering solutions more effective and sustainable by improving their design, performance, and applications.

## VIII. DATA AVAILABILITY

No data has been used for this research, and if any data has been included, it has been properly mentioned and cited with due respect and regards in the appropriate sections.

### *A. Declaration of Competing Interest*

Authors declare that they have no known financial conflicts of interest or personal relationships that could have influenced the work reported in this paper.

## IX. ACKNOWLEDGMENTS

Authors would like to express their sincere gratitude to the editor and the review team for their insightful comments and valuable suggestions, which greatly contributed to the improvement of this article.

### *A. Appendix: A (Normal Functions)*

N B NADUVINAMANI ET. AL. 2018, Effect of couple stresses on static and dynamic characteristics of MHD wide tapered land slider bearing , Sådhanå, 43:162 Indian Academy of Sciences https://doi.org/10.1007/s12046-018-0940-9 1

$$f_{11} = \frac{\beta^2}{\left(\alpha^2 - \beta^2\right)}\left\{\frac{\sinh\left(\frac{\alpha h}{l}\right) - \sinh\left(\frac{\alpha z}{l}\right) + \sinh\left(\frac{\alpha(h-z)}{l}\right)}{\sinh\left(\frac{\alpha h}{l}\right)}\right\} \quad \text{(NF1)}$$

$$f_{12} = \frac{\beta^2}{\left(\alpha^2 - \beta^2\right)}\left\{\frac{\sinh\left(\frac{\beta h}{l}\right) - \sinh\left(\frac{\beta z}{l}\right) + \sinh\left(\frac{\beta(h-z)}{l}\right)}{\sinh\left(\frac{\beta h}{l}\right)}\right\} \quad \text{(NF2)}$$

$$f_{13} = \frac{\beta^2}{\frac{\beta^2}{\alpha}\tanh\left(\frac{\alpha h}{2l}\right) - \frac{\alpha^2}{\beta}\tanh\left(\frac{\beta h}{2l}\right)}\left\{\frac{\sinh\left(\frac{\alpha h}{l}\right) - \sinh\left(\frac{\alpha z}{l}\right) + \sinh\left(\frac{\alpha(h-z)}{l}\right)}{\sinh\left(\frac{\alpha h}{l}\right)}\right\} \quad \text{(NF3)}$$

$$f_{14} = \frac{\alpha^2}{\frac{\beta^2}{\alpha}\tanh\left(\frac{\alpha h}{2l}\right) - \frac{\alpha^2}{\beta}\tanh\left(\frac{\beta h}{2l}\right)}\left\{\frac{\sinh\left(\frac{\beta h}{l}\right) - \sinh\left(\frac{\beta z}{l}\right) + \sinh\left(\frac{\beta(h-z)}{l}\right)}{\sinh\left(\frac{\beta h}{l}\right)}\right\} \quad \text{(NF4)}$$

$$f_{21} = \frac{\sinh\left(\frac{z-h}{\sqrt{2}l}\right) + \sinh\left(\frac{z}{\sqrt{2}l}\right) - \sinh\left(\frac{h}{\sqrt{2}l}\right)}{\sinh\left(\frac{h}{\sqrt{2}l}\right)} \quad \text{(NF5)}$$

$$f_{22} = \frac{z\cosh\left(\frac{z-h}{\sqrt{2}l}\right) + y\cosh\left(\frac{z}{\sqrt{2}l}\right) - h\coth\left(\frac{h}{2\sqrt{2}l}\right)\sinh\left(\frac{z}{\sqrt{2}l}\right)}{2\sqrt{2}l\sinh\left(\frac{h}{\sqrt{2}l}\right)} \quad \text{(NF6)}$$

$$f_{23} = \frac{z\sinh\left(\frac{z-h}{\sqrt{2}l}\right) + z\sinh\left(\frac{z}{\sqrt{2}l}\right) - h\sinh\left(\frac{h}{\sqrt{2}l}\right)}{\left\{6l\sinh\left(\frac{h}{\sqrt{2}l}\right) - \sqrt{2}h\right\}} \quad \text{(NF7)}$$

$$f_{24} = \frac{2\cosh\left(\frac{z-h}{\sqrt{2}l}\right) + 2\cosh\left(\frac{z}{\sqrt{2}l}\right) - -2\cosh\left(\frac{h}{\sqrt{2}l}\right)}{\left\{3\sqrt{2}\sinh\left(\frac{h}{\sqrt{2}l}\right) - \frac{h}{l}\right\}} - 2 \quad \text{(NF8)}$$

$$f_{31} = \frac{\cosh\alpha_1 z\cos\beta_1(z-h)) - \cosh\beta_1 z\cos\alpha_1(z-h))}{\cosh\alpha_1 h - \cosh\beta_1 h} \quad \text{(NF9)}$$

$$f_{32} = \frac{\cot\phi\{\sinh\alpha_1 z\sin\beta_1(z-h) - \sin\beta_1 z\sinh\alpha_1(z-h)\} + (\cosh\alpha_1 h - \cos\beta_1 h)}{\cosh\alpha_1 h - \cosh\beta_1 h} \quad \text{(NF10)}$$

$$f_{33} = \frac{M}{h_{ms}}\left[\frac{\cot\phi\{\sinh\alpha_1 z\cosh\alpha_1(z-h) + \sin\beta_1 z\sinh\alpha_1(z-h)\} + (\cos\beta_1 h - \cosh\alpha_1 h)}{(\beta_1 - \alpha_1\cot\phi)\sin\beta_1 h + (\alpha_1 - \beta_1\cot\phi)\sinh\alpha_1 h}\right] \quad \text{(NF11)}$$

$$f_{34} = \frac{M}{h_{ms}}\left[\frac{\{\cos\beta_1 z\cosh\alpha_1(z-h) + \cosh\alpha_1 z\cos\beta_1(z-h)\}}{(\beta_1 - \alpha_1\cot\phi)\sin\beta_1 h + (\alpha_1 - \beta_1\cot\phi)\sinh\alpha_1 h}\right] \quad \text{(NF12)}$$

### B. *Appendix: B (Integral Functions)*

$$\xi_A(x^*, h_m^*) = \int_{x^*=0}^{x^*} \frac{h_1^*(x^*, t)}{G^*(h^*, l^*, M)} dx^* \qquad \chi_{AK}(h_m^*) = \int_{x^*=0}^{K}\int_{x^*=0}^{x^*} \frac{h_1^*(x^*, t)}{G^*(h^*, l^*, M)} dx^* dx^*$$

$$\xi_B(x^*, h_m^*) = \int_{x^*=0}^{x^*} \frac{x^*}{} dx^* \qquad \chi_{BK}(h_m^*) = \int_{x^*=0}^{K}\int_{x^*=0}^{x^*} \frac{x^*}{G^*(h^*, l^*, M)} dx^* dx^*$$

$$\xi_C(x^*, h_m^*) = \int_{x^*=0}^{x^*} \frac{1}{G^*(h^*, l^*, M)} dx^* \qquad \chi_{CK}(h_m^*) = \int_{x^*=0}^{K}\int_{x^*=0}^{x^*} \frac{1}{G^*(h^*, l^*, M)} dx^* dx^*$$

$$\xi_D(x^*, h_m^*) = \int_{x^*=1}^{x^*} \frac{x^*}{G^*(h^*, l^*, M)} dx^* \qquad \chi_{DK}(h_m^*) = \int_{x^*=0}^{K}\int_{x^*=1}^{x^*} \frac{x^*}{G^*(h^*, l^*, M)} dx^* dx^*$$

$$\xi_E\left(x^*,h_m^*\right)=\int_{x^*=1}^{x^*}\frac{1}{G^*(h^*,l^*,M)}dx^* \qquad \chi_{EK}\left(h_m^*\right)=\int_{x^*=0}^{K}\int_{x^*=1}^{x^*}\frac{1}{G^*(h^*,l^*,M)}dx^*dx^*$$

$$\xi_{Ak}\left(h_m^*\right)=\int_{x^*=0}^{K}\frac{h_1^*\left(x^*,t\right)}{G^*(h^*,l^*,M)}dx^* \qquad \frac{\partial\chi_{Ak}}{\partial h_m^*}=-\int_{x^*=0}^{K}\int_{x^*=0}^{x^*}\frac{h_1^*(x^*)}{\left[G^*(h^*,l^*,M)\right]^2}\frac{\partial G^*}{\partial h_m^*}dx^*dx^*$$

$$\xi_{Bk}\left(h_m^*\right)=\int_{x^*=0}^{K}\frac{x^*}{G^*(h^*,l^*,M)}dx^* \qquad \frac{\partial\chi_{Bk}}{\partial h_m^*}=-\int_{x^*=0}^{K}\int_{x^*=0}^{x^*}\frac{x^*}{\left[G^*(h^*,l^*,M)\right]^2}\frac{\partial G^*}{\partial h_m^*}dx^*dx^*$$

$$\xi_{Ck}\left(h_m^*\right)=\int_{x^*=0}^{K}\frac{1}{G^*(h^*,l^*,M)}dx^* \qquad \frac{\partial\chi_{Ck}}{\partial h_m^*}=-\int_{x^*=0}^{K}\int_{x^*=0}^{x^*}\frac{1}{\left[G^*(h^*,l^*,M)\right]^2}\frac{\partial G^*}{\partial h_m^*}dx^*dx^*$$

$$\xi_{Dk}\left(h_m^*\right)=\int_{x^*=1}^{K}\frac{x^*}{G^*(h^*,l^*,M)}dx^* \qquad \frac{\partial\chi_{Ek}}{\partial h_m^*}=-\int_{x^*=0}^{K}\int_{x^*=1}^{x^*}\frac{1}{\left[G^*(h^*,l^*,M)\right]^2}\frac{\partial G^*}{\partial h_m^*}dx^*dx^*$$

$$\xi_{Ek}\left(h_m^*\right)=\int_{x^*=1}^{K}\frac{1}{G^*(h^*,l^*,M)}dx^* \qquad \left(\frac{\partial C_1}{\partial V^*}\right)_s=12\left[\frac{\xi_{DK}-\xi_{BK}}{\xi_{CK}-\xi_{EK}}\right]$$

$$\left(\frac{\partial C_1}{\partial h_m^*}\right)_s=-6\left[\frac{\left\{\xi_{CK}-\xi_{EK}\right\}\frac{\partial\xi_{EK}}{\partial h_m^*}-\xi_{AK}\left(\frac{\partial\xi_{CK}}{\partial h_m^*}-\frac{\partial\xi_{EK}}{\partial h_m^*}\right)}{\left\{\xi_{CK}-\xi_{EK}\right\}^2}\right]$$

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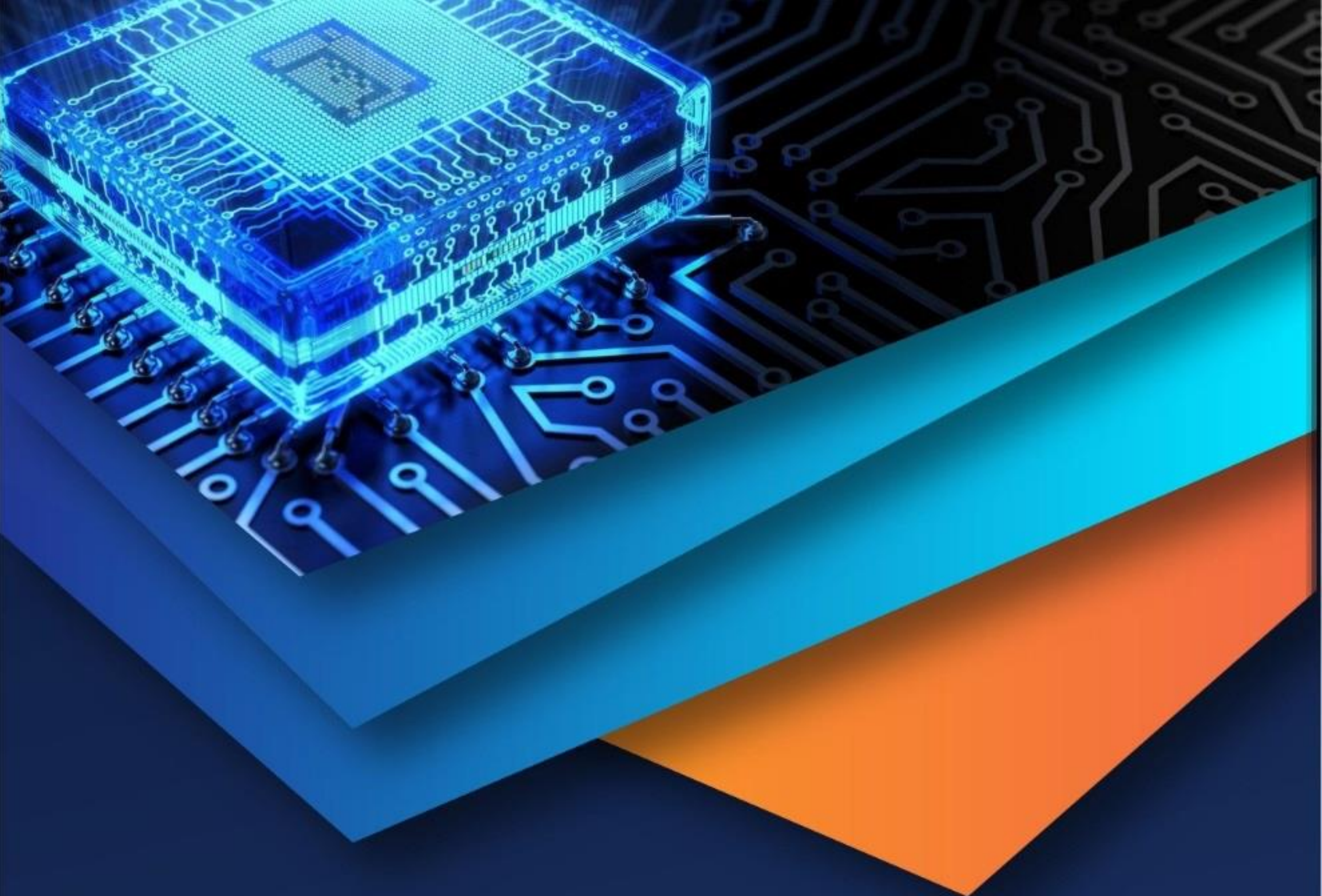
**Nomenclature**

| | |
|---|---|
| $B_0$ | Applied magnetic field (Wb/m$^2$) |
| $C_d$* | Non-dimensional dynamic damping coefficient |
| d | Difference between the inlet and outlet film thickness (m) |
| F | Frictional force (N) |
| F* | Non-dimensional frictional force $F^* = \frac{Fh^2{}_{ma}}{\mu U L^2 B_0}$ |
| h(x,t) | Film thickness (m) |
| h* | Non-dimensional film thickness h*(x*, t*) = h(x, t)/hms |
| hm(t) | Minimum squeezing film thickness (m) |
| hm*(t*) | Non-dimensional mini squeezing film thickness hm*(t*) = hm(t)/hms |
| hms | Steady-state reference minimum film thickness at outlet (m) |
| h1 | Inlet film thickness (m) |
| L | Length of the bearing (m) |
| l | Couple stress parameter (m) |
| l* | Non-dimensional couple stress parameter l* = 2l/hms |
| M | Hartmann number, $M = B_0 h_{ms}\left(\frac{\sigma}{\mu}\right)^{1/2}$ |
| p | Film pressure (Pa) |
| ps | Steady film pressure (Pa) |
| p* | Non-dimensional film pressure $p^* = \frac{ph^2{}_{ms}}{\mu U L}$ |
| ps* | Dimensionless steady-state film pressure |
| t, t* | Time, $t^* = \frac{Ut}{L(s)}$ |
| U | Sliding velocity of lower part (m/s) |

| | |
|---|---|
| $V^*$ | Non-dimensional squeezing velocity $V^* = dhm^*/dt^*$ |
| u, w | Velocity components in x and z directions |
| Ws | Steady load carrying capacity (N) |
| Ws* | Non-dimensional steady load carrying capacity, |
| x, z | Cartesian coordinates |
| x* | Non-dimensional coordinate $x^* = x/L$ |
| $\delta$ | Profile parameter $\delta = d/hms$ |
| $\eta$ | Material constant responsible for couple stress parameter (Ns) |
| F | Thrust force |
| Hr | Thickness of rotating disk (runner) |
| HB | Thickness of bearing (pad) |

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