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Development of Non-Stretchable Triples with Numerable Features

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Abstract: ***In this paper, some non-extendable triples involving Rook Polynomials, Hermite Polynomials and Laguerre Polynomials with appropriate properties are accomplished. Besides, it is showed that these triples cannot be prolonged into quadruples with the help of property of congruence.***
Keywords: Diophantine quadruple, Rook Polynomial, Hermite Polynomial and Laguerre Polynomial.

I. INTRODUCTION

Various scholars created several Diophantine quadruples with the property $D(n)$ for any integer n [1-4, 6,8-11]. A few non-extendable triples were analysed in [5,7]. In this paper, some non-extendable triples concerning Rook Polynomials, Hermite Polynomials and Laguerre Polynomials with suitable properties are discovered. Additionally, it is exposed that these triples cannot be protracted into quadruples.

II. ELEMENTARY DEFINITIONS

A. *Rook Polynomial*

The rook polynomial $R_B(x)$ of a board B is the generating function for the numbers of arrangements of non-attacking rooks:

$$R_B(x) = \sum_{k=0}^{\min(m,n)} r_k(B)x^k$$

where $r_k(B)$ is the number of ways to place k non-attacking rooks on the board B.

Rook polynomial of square 2×2 is

$$R_2(x) = 2x^2 + 4x + 1$$

B. *Laguerre Polynomial*

Laguerre Polynomial is

$$L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(e^{-x}x^n)$$

If $n = 2$, then $L_2(x) = \frac{(x^2-4x+2)}{2}$

C. *Probabilistic Hermite Polynomial*

The Probabilistic Hermite polynomial is

$$He_n(x) = (-1)^n e^{\frac{x^2}{2}}\frac{d^n}{dx^n}e^{-\frac{x^2}{2}}$$

Third probabilistic Hermite polynomial is

$$He_2(x) = x^2 - 1$$

III. DEVELOPMENT OF NON-EXTENDABLE TRIPLES

In the parts that follow, three different kinds of 3-tuples, where the elements are Rook polynomials, Hermite polynomials, and Laguerre polynomials, but not extended to quadruple are discovered in the following sections A to C.

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A. Assessment of triples consists Rook Polynomial and Hermite Polynomial

Select the values $f_1(x) = R_2(2x) = 8x^2 + 8x + 1$ and $f_2(x) = He_2(2x + 1) = 4x^2 + 4x$ be such that the difference between $f_1(x)$ and $f_2(x)$ is a square of a polynomial.

The above assumption is symbolically represented as

$$f_1(x) - f_2(x) = (2x + 1)^2 = [\alpha_1(x)]^2 \text{ (say).} \quad (1)$$

Let $f_3(x)$ be another polynomial with the two resulting conditions that

$$f_1(x) - f_3(x) = [\alpha_2(x)]^2, \quad (2)$$

$$f_2(x) - f_3(x) = [\alpha_3(x)]^2. \quad (3)$$

The following equation can be obtained by subtracting (3) from (2).

$$f_1(x) - f_2(x) = [\alpha_2(x)]^2 - [\alpha_3(x)]^2. \quad (4)$$

Let us use the conversions listed below to find the third element $f_3(x)$ in an essential triple.

$$\alpha_2(x) = A + 1 \text{ and} \alpha_3(x) = A. \quad (5)$$

By substituting the preferred values of $f_1(x)$, $f_2(x)$ and (5) in (4), the possibility of A and thus $\alpha_3(x)$ is obtained as

$$\alpha_3(x) = A = 2x^2 + 2x. \quad (6)$$

Preserving $f_2(x)$ and the previously derived value of $\alpha_3(x)$ in (3), it is evaluated that

$$f_3(x) = -4x^4 - 8x^3 + 4x. \quad (7)$$

Hence, $(f_1(x), f_2(x), f_3(x))$is a triple such that the difference between two of them is a square of a polynomial.

Suppose $f_4(x)$ is the next element in the triple $(f_1(x), f_2(x), f_3(x))$ such that

$$f_1(x) - f_4(x) = [\alpha_4(x)]^2 \quad (8)$$

$$f_2(x) - f_4(x) = [\alpha_5(x)]^2 \quad (9)$$

$$f_3(x) - f_4(x) = [\alpha_6(x)]^2 \quad (10)$$

Subtracting (10) from (8), it is perceived that

$$4x^4 + 8x^3 + 8x^2 + 4x + 1 = [\alpha_4(x)]^2 - [\alpha_6(x)]^2 \quad (11)$$

Similarly, the following equation is obtained by subtracting (10) from (9)

$$4x^4 + 8x^3 + 4x^2 = [\alpha_5(x)]^2 - [\alpha_6(x)]^2$$

By applying factorization method, it is elucidated that

$$[\alpha_5(x)]^2 = x^4 + 2x^3 + x^2 + 1$$

$$[\alpha_6(x)]^2 = x^4 + 2x^3 + x^2 - 1$$

Note that

$$[\alpha_5(x)]^2 \equiv 1(mod4) \text{ and } [\alpha_6(x)]^2 \equiv 1(mod4) \text{ for all } x \in Z \quad (12)$$

On the other hand, employing the value of $[\alpha_6(x)]^2$in (11), it is attained by

$$[\alpha_4(x)]^2 = [\alpha_6(x)]^2 + 4x^4 + 8x^3 + 8x^2 + 4x + 1$$
$$\equiv 2(mod4)$$

which is impossible because a square number can only be congruent to zero or one modulo 4.

Hence, the triple $\{f_1(x), f_2(x), f_3(x)\} = \{8x^2 + 8x + 1, 256x^2 - 64x, 4x^2 + 4x, -4x^4 - 8x^3 + 4x\}$ cannot be prolonged into a quadruple.

B. Assessment of triples comprises Laguerre Polynomial and Hermite Polynomial

Let $\{g_1(x), g_2(x)\} = \{L_2(4x + 2), He_2(2x + 1)\} = \{8x^2 - 1, 4x^2 + 4x\}$ be a pair of Laguerre Polynomial and Hermite Polynomial such that the difference of these two polynomials added by 2 is a square of some other polynomial. Following the procedure as explained in Section A, this pair is stretched into the triple $\{g_1(x), g_2(x), g_3(x)\}$ with property $D(2)$.

Here $g_3(x) = -4x^4 + 8x^3 + 4x^2 + 1$.

Let us pre assume that $g_4(x)$ is a non-zero polynomial such that

$$g_1(x) - g_4(x) = [\beta_4(x)]^2, \quad (13)$$

$$g_2(x) - g_4(x) = [\beta_5(x)]^2, \quad (14)$$

$$g_3(x) - g_4(x) = [\beta_6(x)]^2, \quad (15)$$

Then,

$$[\beta_4(x)]^2 - [\beta_6(x)]^2 = 4x^4 - 8x^3 + 4x^2 - 2 \quad (16)$$

$$[\beta_5(x)]^2 - [\beta_6(x)]^2 = 4x^4 - 8x^3 + 4x^2 - 1 \quad (17)$$

By retaining factorization method, it is pointed out by

$$[\beta_5(x)]^2 = 2x^4 - 4x^3 + 2x$$
$$[\beta_6(x)]^2 = 2x^4 - 4x^3 + 2x - 1$$

It is keenly observed that

$[\beta_5(x)]^2 \equiv 0(mod4)$ and $[\beta_6(x)]^2 \equiv 1(mod4)$, for all $x \in Z$ (18)

Also form (16), it is noticed that

$$[\beta_4(x)]^2 = [\beta_6(x)]^2 + 4x^4 - 8x^3 + 4x^2 - 2$$
$$\equiv 3(mod4)$$

which is contradicts the fact that a square number can only be congruent to zero or one modulo 4.

Hence, the triple $\{g_1(x), g_2(x), g_3(x)\}$ cannot be extended to quadruple.

C. Assessment of triples embraces Rook Polynomial and Laguerre Polynomial

Let $\{h_1(x), h_2(x)\} = \{R_2(2x+2), 2L_2(2x+2)\} = \{8x^2 + 24x + 17, 4x^2 - 2\}$ be a pair of Rook Polynomial and Lagurre Polynomial such that the difference of these two polynomials added by 17 is a square of a specific polynomial. Succeeding the procedure as clarified in Section A, this pair is stretched into the triple $\{h_1(x), h_2(x), h_3(x)\}$ with property $D(17)$.

Here $h_3(x) = -4x^4 - 48x^3 - 176x^2 - 216x - 66$.

Undertake that $h_4(x)$ is the polynomial such that

$$h_1(x) - h_4(x) = [\gamma_4(x)]^2 \quad (18)$$
$$h_2(x) - h_4(x) = [\gamma_5(x)]^2 \quad (19)$$
$$h_3(x) - h_4(x) = [\gamma_6(x)]^2 \quad (20)$$

Following the similar technique as described in section3.1, it is scrutinized that

$$[\gamma_5(x)]^2 = x^4 + 12x^3 + 45x^2 + 54x + 17 \equiv 1(mod4)$$
$$[\gamma_6(x)]^2 = x^4 + 12x^3 + 45x^2 + 54x + 15 \equiv 3(mod4) \text{ for all } x \in Z$$

Therefore, $h_3(x) - h_4(x)$ is not a square of a polynomial.

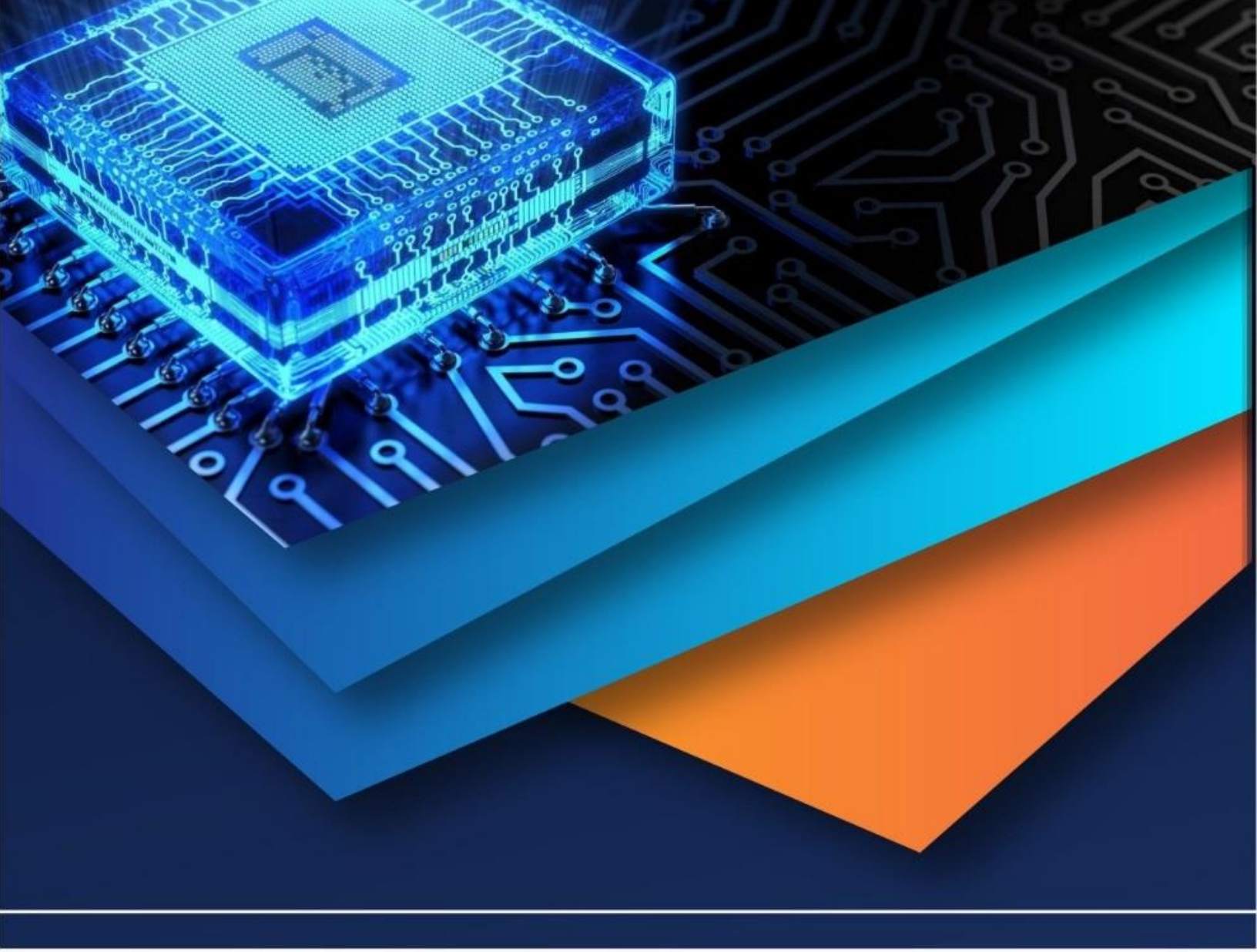
Hence the triple $\{h_1(x), h_2(x), h_3(x)\}$ cannot be stretched to quadruple.

IV. CONCLUSION

In this paper, Diophantine triples comprising Rook polynomials, Hermite Polynomials and Laguerre Polynomials are presented. Also, all these triples cannot be stretched into quadruple is proved by using the basic concepts of congruence. In this manner, one can search triples, quadruples etc with fascinating conditions.

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