iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 10 **Issue:** III **Month of publication:** March 2022

DOI: https://doi.org/10.22214/ijraset.2022.40903

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 10 Issue III Mar 2022- Available at www.ijraset.com

Estimation Based on Optimally Selected Single Order Statistic in Pareto: Rayleigh Distribution

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***Abstract:** A special case of the well known T – X family of distributions proposed by Ayman Alzaatreh et al. (2012) called Pareto – Rayleigh (P – R) distribution is considered. Its scale parameter is estimated using a single order statistic in small samples. Optimal criterion for the choice of single order statistic in a given small sample is worked out. Comparison is made with the corresponding optimal choice of single order statistic with respect to the criterion of asymptotic variance. The results are extended to estimate parametric functions like reliability and hazard rate.*
***Keywords:** T-X family, Pareto-Rayleigh distribution, order statistics, optimal estimation, asymptotic variance.*

I. INTRODUCTION

Mixtures and compounds of existing distributions, generalization to existing distributions, convex combinations of cumulative distribution functions, are some methods of obtaining new models. However recently Ayman Alzaatreh *et al.* (2012) proposed generating families of continuous probability models named as T – X family of distributions. Such models are observed to be having a good number of applications in reliability studies, survival analysis and related areas. The principle in generating a T – X family is as follows. Let T and X be two continuous random variables with respective cumulative distribution functions $F(\cdot)$,$G(\cdot)$ and $f(\cdot)$, $g(\cdot)$are the respective density functions.

Let $F(x)$ be the cumulative distribution function (CDF) of any random variable X and $r(t)$ be the probability density function (PDF)of a random variable, T, defined on $[0,\infty)$. The CDF of the Transformed-Transformer family (or T-X family) of distributions defined by Ayman Alzaatreh, *et al.* (2012) is given by

$$G(x) = \int_0^{-\log[1-F(x)]} r(t)dt \quad (1.1)$$

When X is continuous random variable, the probability density function of the T-X family is

$$g(x) = \frac{f(x)}{1-F(x)} r(-\log(1-F(x))) \quad (1.2)$$

Taking Pareto Type IV model for T and Rayleigh model for X, the T – X family named as Pareto – Rayleigh model is considered by Subba Rao *et al.* (2015). We know that the probability density function (pdf), cumulative distribution function (cdf) of a Pareto Type IV distribution are given by

$$f(t,\alpha,\theta) = \frac{\alpha}{\theta}\left[1+\frac{t}{\theta}\right]^{-(\alpha+1)} \quad if\ t \geq 0$$

$$= \alpha[1+t]^{-(\alpha+1)} \quad if\ \theta = 1 \quad (1.3)$$

$$F(t,\alpha,1,\theta) = \left[1+\frac{t}{\theta}\right]^{-\alpha} \quad (1.4)$$

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In these two equations, α is called shape parameter and θ is called scale parameter. Its hazard function is given by

$$h(t,\alpha,\theta) = \frac{\frac{\alpha}{\theta}\left[1+\frac{t}{\theta}\right]^{-1}}{\left[\left[1+\frac{t}{\theta}\right]^{\alpha}-1\right]}. \qquad (1.5)$$

The standard version of these three functions are obtained by taking $\theta = 1$ in their respective expressions. Let us denote T as a random variable defining the above distribution. The pdf and cdf of a Rayleigh distribution are given by

$$f(x,\theta) = \frac{x}{\theta^2}e^{-\frac{x^2}{2\theta^2}};\ x \geq 0, \theta > 0, \qquad (1.6)$$

$$F(x)1 - e^{-x^2/2\theta^2};\ x > 0, \theta > 0. \qquad (1.7)$$

Let X be the random variable corresponding to this Rayleigh distribution. Then the T – X family distribution generated by Pareto, Rayleigh distributions is obtained by using (1.1) with the corresponding expressions of Rayleigh and Pareto distributions. Thus we get the Pareto – Rayleigh (P – R) distribution with the following cdf and pdf functions:

$$G(x) = 1 - \left[1 + \frac{x^2}{2\theta^2}\right]^{-\alpha}; x > 0, \alpha > 1, \theta > 0 \qquad (1.8)$$

$$g(x) = \frac{\alpha}{\theta^2}x\left[1 + \frac{x^2}{2\theta^2}\right]^{-\alpha-1};\ x > 0, \alpha > 1, \theta > 0 \qquad (1.9)$$

where α is shape parameter and θ is scale parameter.
It can be seen that P – R distribution is continuous with a positive valued random variable having scale and shape parameters. The shape of the frequency curve of P – R distribution changes as α changes. For some values of α the graphs of pdf of P – R distribution are given in the following figures which indicate that it is a positively skewed distributions.

Figure 1.1

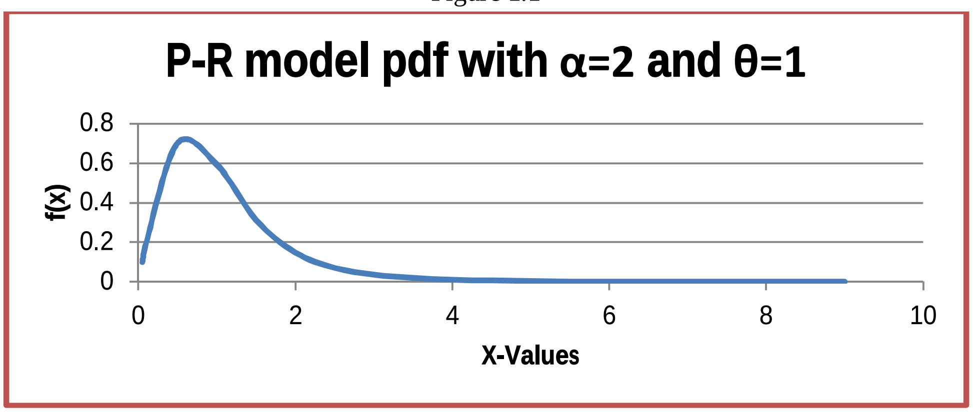

Figure 1.2

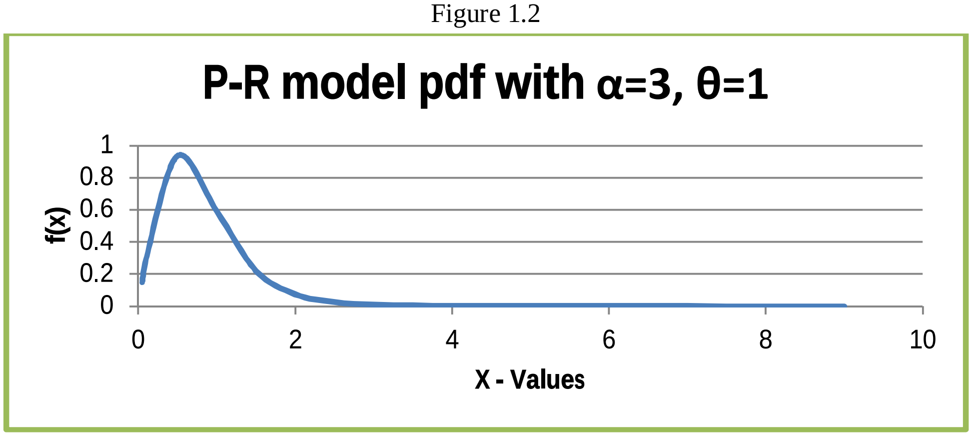

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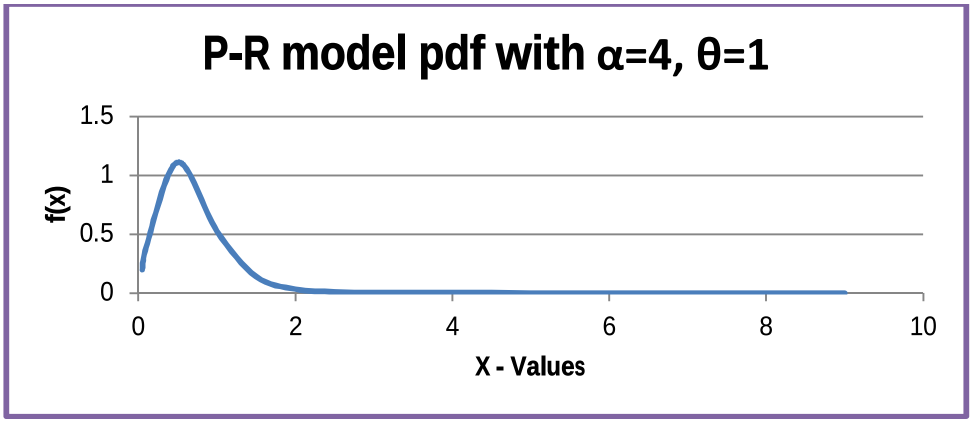
Figure 1.3



The Pareto – Rayleigh (P – R) distribution is considered to develop point estimation of its parameters by Subba Rao *et al.* (2015). It consists of theory using order statistics also.

From Subba Rao *et al.* (2015) we know that reliability and hazard function of a P – R distribution are given by

$$R(x) = \left[1 + \frac{x^2}{2\theta^2}\right]^{-\alpha}; x > 0, \alpha > 1, \theta > 0 \quad (1.10)$$

$$h(x) = \frac{\alpha}{\theta^2} x \left[1 + \frac{x^2}{2\theta^2}\right]^{-1}; \; x > 0, \alpha > 1, \theta > 0 \quad (1.11)$$

It has the following distributional characteristics:

$$Mean = \alpha(2\theta^2)^{1/2}\beta\left(\frac{3}{2}, \alpha - \frac{1}{2}\right)$$

$$Median = M_d = \sqrt{2\theta^2[(0.5)^{-1/\alpha} - 1]}$$

$$Mode = \alpha\theta^2\left(\frac{x^2}{2\theta^2} + 1\right)^{-(\alpha+1)} - (\alpha + 1)\alpha x^2\left(\frac{x^2}{2\theta^2} + 1\right)^{-(\alpha+2)} = 0$$

Mode is the iterative solution of above equation for x.

$$Variance = 2\alpha\theta^2\left[\beta(\alpha - 1,2) - \alpha\left(\beta\left[\alpha - \frac{1}{2}, \frac{3}{2}\right]\right)^2\right]$$

$$Standard\ Deviation = \theta\sqrt{2\alpha\left[\beta(\alpha - 1,2) - \alpha\left(\beta\left[\alpha - \frac{1}{2}, \frac{3}{2}\right]\right)^2\right]}$$

$$Coefficient\ of\ Variation = \frac{\theta\sqrt{2\alpha\left[\beta(\alpha-1,2) - \alpha\left(\beta\left[\alpha - \frac{1}{2}, \frac{3}{2}\right]\right)^2\right]}}{\alpha(2\theta^2)^{1/2}\beta\left(\frac{3}{2}, \alpha - \frac{1}{2}\right)} \times 100$$

For a known α the lower bound for the variance of estimator of θ based on random sample of size n popularly known as Cramer – Rao lower bound (CRLB) is given by

$$CRLB(\theta) = \frac{-n}{E\left[\frac{\partial^2 f(x,\alpha,\theta)}{\partial\theta^2}\right]} = \left[\frac{2n(3\alpha+2)}{\theta^2(\alpha+2)} - \frac{2n}{\theta^2}\right]^{-1} \quad (1.12)$$

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Subba Rao *et al.* (2015) studied maximum likelihood method estimation, modified maximum likelihood method estimation of the parameter θ for a given shape parameter. He also suggested method of estimation of θ based on single order statistic corresponding to minimum asymptotic variance. In this paper we attempt to propose estimation of θ based on single order statistic with respect to optimum small sample variance along with the asymptotic relative efficiency. The procedure is extended to estimate of reliability function as well as hazard function adopting the invariance property for estimation using order statistics. The paper is organized with Section – 2 dealing with estimation of parameters and Section – 3 dealing with estimation of parametric functions.

II. PARAMETRIC ESTIMATION

We know that the cdf of P – R model is given by

$$G(x) = 1 - \left[1 + \frac{x^2}{2\theta^2}\right]^{-\alpha};x > 0, \alpha > 1, \theta > 0 \quad (2.1)$$

It can be written as

$$\theta = \frac{x}{\sqrt{2[[1-G(x)]^{-\frac{1}{\alpha}}-1]}} \quad (2.2)$$

Correspondence of probability theory says that $x, G(x)$ have one to one correspondence in the case of cumulative distribution functions. We also know that $G(x)$ lies between 0 and 1. Specifying hypothetical value say 'p' for G(x) between 0 and 1, we can get a unique value of x say x_p called the p – th quantile of the distribution under consideration. With this notation the above equation (2.2) can be rewritten as

$$\theta = \frac{x_p}{\sqrt{2[[1-p]^{-\frac{1}{\alpha}}-1]}} \quad (2.3)$$

In a given random sample of size n, for a given p between 0 and 1 the order statistic with $[np] + 1$ as the suffix is called p^{th} sample quantile generally denoted by x_p . Using the fact that sample quantiles are consistent estimators of the corresponding population quantiles, the above equation (2.3) can be considered as the one giving an estimate of θ based on a single sample order statistic.

$$i.e., \hat{\theta} = \frac{x_p}{\sqrt{2[[1-p]^{-1/\alpha}-1]}} \quad (2.4)$$

But in a sample of size n among the available n separate single order statistics we have to choose that single order statistic which yields a minimum variance of the estimator of θ. At the same time exact variances of order statistics in statistical distributions are not of analytic expressions even in standard distributions like exponential, normal leading to the publication of tables of moments of order statistics by researchers like Sarhan and Greenberg (1962), Gupta (1962), Balakrishnan and Cohen (1991) and various other researchers. However asymptotic theory of order statistics facilitates to give analytic expressions for asymptotic variances of order statistics. Accordingly asymptotic variance of $\hat{\theta}$ given in (2.4) is

$$Asymptotic\ Variance\ of\ \hat{\theta} = \frac{ASVAR\ (x_p)}{2[(1-p)^{-(1/\alpha)}-1]} \quad (2.5)$$

From David (1981), [Chapter 6],

$$ASVAR(x_p) = \frac{p(1-p)}{4\alpha^2[(1-p)^{-(1/\alpha)}-1]^2(1-p)^{2\left(\frac{\alpha+1}{\alpha}\right)}} \quad (2.6)$$

For an optimum choice of a sample quantile we have to minimize the asymptotic variance given by (2.6) with respect to p.

That is,

$$\frac{d}{dp}[ASVAR(x_p)] = 0$$

$$\Rightarrow \frac{d}{dp}\left[\frac{p(1-p)}{4\alpha^2\left[(1-p)^{-(1/\alpha)}-1\right]^2(1-p)^{2\left(\frac{\alpha+1}{\alpha}\right)}}\right] = 0$$

$$\Rightarrow \alpha\left[(1-p)^{-\frac{1}{\alpha}}-1\right](1-p)^{-\frac{1}{\alpha}}(1-2p) - 2p\left[(\alpha+1)\left(1-p^{\frac{1}{\alpha}}\right)-\alpha\right] = 0 \qquad (2.7)$$

$$and\ \frac{d^2}{dp^2}\left[ASVAR(x_p)\right]$$

$$\Rightarrow \frac{d^2}{dp^2}\left[\frac{p(1-p)}{4\alpha^2\left[(1-p)^{-(1/\alpha)}-1\right]^2(1-p)^{2\left(\frac{\alpha+1}{\alpha}\right)}}\right] > 0$$

The equation (2.7) has to be solved iteratively for p to get its optimum value corresponding to minimum of the asymptotic variance of $\hat{\theta}$.

A non trivial solution in p for the equation (2.6) is obtained by numerical iterative methods only. Such a solution and the corresponding x_p would give estimator of θ based on optimally selected single ordered statistic. Subba Rao *et al.* (2015) obtained a solution of the above equation for α = 2, 3, 4 as p = 0.618034, 0.669467, 0.698069.

The practical implication of these p – values is as follows. If we have a random sample of size n, if we need to find an estimate of θ using one and only of these n order statistics for a known α say $\alpha = 2$, we have to choose that order statistic in the ordered sample at the serial position $[n(0.618034)] + 1$. For example if $n = 9$ when α=2 the asymptotically optimally suggested order statistic to estimate θ is $[9(0.618034)] + 1 = [5.562306] + 1 = 5 + 1 = 6$. Therefore 6th order statistic in the given sample of size 9 when $\alpha = 2$ gives when used in the formula (2.4) of $\hat{\theta}$ an estimator with asymptotic least variance. However in small sample theory the same 6th order statistic may or may not yield to in the estimator $\hat{\theta}$ with a minimum small sample variance. At the same time because of non analytic nature of small sample variances we resort to simulation method in verifying small sample optimality of variances in estimating θ as described in the following lines.

For a specified α, and a sample size n, we generate a random sample from a standard P – R distribution and arranged the ascending order of the sample in order to get n order statistics. Each of these order statistics is substituted in succession in the formula to get $\hat{\theta}$. This procedure is repeated 1000 times called the simulation runs. In all the 1000 cases we get 1000 estimates of θ obtained by using 1st order statistic in a sample of size n, another 1000 estimates of θ using 2nd order statistic of a sample of size n and so on, another 1000 estimates of θ using the last order statistic of sample of size n. Across the 1000 simulation runs the mean and variance of estimate of θ obtained by using each order statistic are calculated. This procedure is carried out for α=2,3,4 and n=5(5)20. These empirical means and variances of the estimates of θ are given in Table 2.1. The last row of each table indicates the suffix of the order statistics in a sample of that size at which the empirical variance of the estimate of $\hat{\theta}$ is minimum and the minimum value of such variance.

Table – 2.1:: Empirical Variance of $\hat{\theta}$ for **α=2,3,4.**

Sample Size	**$\alpha = 2$**		**$\alpha = 3$**		**$\alpha = 4$**	
	Means(θ)	**Var(θ)**	**Means(θ)**	**Var(θ)**	**Means(θ)**	**Var(θ)**
05	0.956876	0.264416	0.975696	0.268071	0.919623	0.253137
	1.032166	0.18757	1.01755	0.158322	0.982391	0.143757
	1.083435	0.174407	1.047322	0.129741	1.021122	0.114303
	1.116226	0.190518	1.082889	0.138203	1.047132	0.112221
	1.267035	0.441119	1.200499	0.241203	1.139944	0.170291
	EMV	0.174407	**EMV**	0.129741	**EMV**	0.112221

	OS	3	**OS**	3	**OS**	4
	ARE	57.337148%	**ARE**	64.230274%	**ARE**	66.832411%
10	0.926985	0.246507	0.907584	0.219538	0.916131	0.247207
	0.9923	0.144637	0.97144	0.136148	0.961923	0.127517
	1.002171	0.102087	0.9979	0.105002	0.978267	0.088708
	1.011856	0.085368	1.016388	0.085307	0.989887	0.07373
	1.026705	0.082336	1.03122	0.074304	0.996185	0.065501
	1.03515	0.076373	1.038142	0.064993	1.013103	0.056944
	1.043701	0.077316	1.043052	0.064803	1.033572	0.05423
	1.065551	0.083113	1.052304	0.067896	1.046638	0.060434
	1.104791	0.122272	1.082497	0.08612	1.062058	0.06591
	1.242569	0.319723	1.175234	0.182052	1.137614	0.130012
	EMV	0.076373	**EMV**	0.064803	**EMV**	0.05423
	OS	6	**OS**	7	**OS**	7
	ARE	65.468163%	**ARE**	64.297949%	**ARE**	69.149917%
15	0.919586	0.244355	0.901466	0.226511	0.90113	0.221524
	0.969787	0.135782	0.958931	0.129196	0.947359	0.129419
	0.981301	0.091151	0.98336	0.092246	0.975239	0.086917
	0.983731	0.068665	0.98829	0.072321	0.988038	0.067143
	0.985081	0.057644	1.000628	0.060597	0.997734	0.060204
	0.990988	0.052703	1.011968	0.053017	1.006715	0.050279
	0.997281	0.048015	1.015909	0.047898	1.012667	0.04527
	1.007173	0.046682	1.017918	0.045595	1.016738	0.041541
	1.01216	0.046885	1.024954	0.04198	1.020391	0.038753
	1.025017	0.049468	1.028175	0.042304	1.023686	0.037734
	1.03293	0.051593	1.031965	0.042894	1.025113	0.036879
	1.048879	0.056353	1.050409	0.049058	1.039579	0.039806
	1.075878	0.071499	1.066131	0.056324	1.053059	0.043166
	1.11935	0.118422	1.093538	0.076808	1.071988	0.053972
	1.286672	0.476491	1.194784	0.200985	1.140694	0.110803
	EMV	0.046682	**EMV**	0.04198	**EMV**	0.036879
	OS	8	**OS**	9	**OS**	11
	ARE	71.404396%	**ARE**	66.169605%	**ARE**	67.789257%

*__EMV__=Empirical Minimum Variance, **O.S**= Order Statistic,
ARE=Asymptotic Relative Efficiency.

Table – 2.1 :: Empirical Variance of $\hat{\theta}$ for **α=2,3,4.**(Continuation)

Sample Size	$\alpha = 2$		$\alpha = 3$		$\alpha = 4$	
	Means(θ)	**Var(θ)**	**Means(θ)**	**Var(θ)**	**Means(θ)**	**Var(θ)**
20	0.909083	0.22582	0.877578	0.221822	0.899513	0.228047
	0.954926	0.126308	0.933976	0.119771	0.953922	0.129468
	0.97857	0.090056	0.963724	0.08815	0.972717	0.093929
	0.988146	0.070345	0.979956	0.070368	0.987996	0.067542
	0.9955	0.05729	0.982763	0.060142	0.993519	0.056763
	0.999814	0.049775	0.991623	0.051769	0.991557	0.047046
	1.004314	0.044718	0.994914	0.045356	0.996747	0.040508
	1.01268	0.042016	0.997048	0.040053	1.000225	0.037083
	1.01542	0.039391	0.999969	0.037378	1.003829	0.032996
	1.013439	0.036151	1.005628	0.034485	1.005283	0.031536
	1.014145	0.034564	1.00798	0.031781	1.007919	0.030666
	1.017078	0.03499	1.010137	0.031684	1.010136	0.02953
	1.02176	0.035024	1.013027	0.030495	1.013498	0.028409
	1.026139	0.036097	1.015912	0.030161	1.017416	0.028975
	1.028834	0.038172	1.026007	0.031925	1.01679	0.028807
	1.03186	0.041159	1.03591	0.035308	1.022395	0.02915
	1.044467	0.049604	1.044758	0.038792	1.028822	0.031596
	1.072274	0.06158	1.058292	0.048059	1.037529	0.03651
	1.119123	0.095624	1.081945	0.061074	1.057089	0.047085
	1.242236	0.293165	1.179521	0.161449	1.121368	0.100071
	EMV	0.034564	**EMV**	0.030161	**EMV**	0.028409
	OS	11	**OS**	14	**OS**	13
	ARE	72.329591%	**ARE**	69.072643%	**ARE**	66.000211%

***EMV**=Empirical Minimum Variance, **OS**= Order Statistic,
ARE=Asymptotic Relative Efficiency.

From the above tables it can be seen that the ratio of CRLB to the empirical optimum variance which is also called the asymptotic relative efficiency is at the minimum about 57.337148% and this relative efficiency is increasing as the sample size increases reaching a maximum of 72.329591%, at the sample size $n = 20$. These observations show that asymptotic optimality of our proposed estimator operates from the sample size $n = 20$ onwards.

III. ESTIMATION OF PARAMETRIC FUNCTIONS

As mentioned earlier, P – R distribution can be used as a probability model in reliability studies also. We know that Reliability function and hazard rate function are two important concepts of reliability studies where the former talks of the quality aspect of any manufactured product which has life and eventual failure and the latter speaks of failure behaviour of any product whose life time is modeled by P – R distribution. In this section we attempt to study the estimation of reliability function and hazard function of P – R model using an optimally selected single order statistic in a random sample of size n from P –R distribution. The efficiency of the resulting estimator for a given 'α', at a specified population reliability.

The reliability function and hazard function of a P – R distribution with parameters α, θ are given by

$$R(x) = \left(1 + \frac{x^2}{2\theta^2}\right)^{-\alpha}; x > 0, \theta > 0, \alpha > 0 \qquad (3.1)$$

$$h(x) = \frac{2\alpha x}{x^2 + 2\theta^2} \qquad (3.2)$$

In a random sample of size n if x_p stands for sample $p-$ th quantile, where $p = \frac{i}{n+1}$, for given x, α estimators of R(x) and h(x) using x_p may be taken as those obtained by replacing θ with the estimate of θ obtained by using x_p given in the formula:

$$\hat{\theta} = \frac{x_p}{\sqrt{2[[1-p]^{-1/\alpha}-1]}} \quad (3.3)$$

Accordingly $\hat{R}(x) = \left(1 + \frac{x^2}{2\hat{\theta}^2}\right)^{-\alpha}$ (3.4)

$$\hat{h}(x) = \frac{2\alpha x}{x^2+2\hat{\theta}^2} \quad (3.5)$$

Here we assumed invariance property for $\hat{\theta}$ though it is theoretically true for method of maximum likelihood estimation but not for other methods of estimation and $\hat{\theta}$ in the present situation is not MLE of θ. Such adoption of invariance principle for non ML methods is advised in Sinha (1986). Though the exact variances of $\hat{R}(x), \hat{h}(x)$ are not known, their Asymptotic variances can be obtained from the following expressions.

$$ASVAR\ \hat{R}(x) = \left(\frac{\partial R}{\partial \theta}\right)^2 . ASVAR\ (\hat{\theta}). \quad (3.6)$$

$$ASVAR\ \hat{h}(x) = \left(\frac{\partial h}{\partial \theta}\right)^2 . ASVAR\ (\hat{\theta}) \quad (3.7)$$

We have already got the values of ASVAR ($\hat{\theta}$) for α=2,3,4 at n=5,10,15,20.

It can be seen that

$$\frac{\partial R}{\partial \theta} = \frac{\alpha x^2}{\theta^3}\left(1 + \frac{x^2}{2\theta^2}\right)^{-(\alpha+1)}$$

$$\left(\frac{\partial R}{\partial \theta}\right)^2 = \frac{\alpha^2 x^4}{\theta^6}\left(1 + \frac{x^2}{2\theta^2}\right)^{-(\alpha+1)}$$

For a standard density $\left(\frac{\partial R}{\partial \theta}\right)^2 = \alpha^2 x^4 \left(1 + \frac{x^2}{2}\right)^{-(\alpha+1)}$ (3.8)

Similarly for the asymptotic variance of $\hat{h}(x)$ we need $\left(\frac{\partial h}{\partial \theta}\right)^2$ which is given by

$$\left(\frac{\partial h}{\partial \theta}\right)^2 = \frac{\theta^2(2\theta^2+x^2)^4}{\alpha^4 x^4}$$

For a standard density $\left(\frac{\partial h}{\partial \theta}\right)^2 = \frac{(2+x^2)^4}{\alpha^4 x^4}$ (2.4.9)

Using these in (3.6) and (3.7) we get Asymptotic Variance of $\hat{R}(x)$ and $\hat{h}(x)$ at any specified value of x. The Asymptotic Variance of estimates of $R(x)$ and $h(x)$ for our chosen values of α and x are given in the Tables – 3.1 and 3.2.

However in small samples we get the variance of $\hat{\theta}$ by simulation as described in Section – 2 in order to get small sample variance of $\hat{\theta}$ with optimally selected order statistic, which in turn if substituted in 3.1 and 3.2 we get estimates of $R(x)$ and $h(x)$ over the simulation runs.

We have chosen α=2,3,4, x is corresponding to reliabilities of 0.95,0.75,0.50,0.25,0.05. The empirical variance of $\hat{R}(x), \hat{h}(x)$ for the above selected values of α, x, sample sizes are given in Table – 3.3 to 3.5.

Table – 3.1 :: Asymptotic Variance of Estimate of $R(x)$

x \ α	**2**	**3**	**4**
$\boldsymbol{R^{-1}}$**(0.95)**	0.003465	0.004318	0.005126
$\boldsymbol{R^{-1}}$**(0.75)**	0.086195	0.095444	0.100665
$\boldsymbol{R^{-1}}$**(0.50)**	0.336366	0.304109	0.261887
$\boldsymbol{R^{-1}}$**(0.25)**	0.693136	0.44312	0.26983
$\boldsymbol{R^{-1}}$**(0.05)**	0.747406	0.213685	0.058191

Table – 3.2 :: Asymptotic Variance of Estimate of $h(x)$

x \ α	2	3	4
$R^{-1}(0.95)$	284.5036	47.90319	13.84426
$R^{-1}(0.75)$	12.87218	2.167348	0.626374
$R^{-1}(0.50)$	4.03989	0.680215	0.196585
$R^{-1}(0.25)$	2.772542	0.466826	0.134915
$R^{-1}(0.05)$	5.74943	0.968058	0.279773

Table – 3.3:: Empirical Variance of $\hat{R}(x)$, $\hat{h}(x)$ for **α=2**.

	Empirical Variance of $\hat{R}(x)$ at α=2					Empirical Variance of $\hat{h}(x)$ at α=2				
Sample Size	0.95	0.75	0.50	0.25	0.05	0.95	0.75	0.50	0.25	0.05
05	0.030334	0.068695	0.064409	0.039302	0.007891	4.903403	2.364631	1.114199	0.413478	0.055567
	0.007421	0.036548	0.045641	0.031644	0.006376	0.818832	0.945792	0.621722	0.281092	0.042609
	0.00323	0.025114	0.037427	0.029128	0.00678	0.302345	0.56247	0.448782	0.229925	0.039468
	0.002249	0.022242	0.036902	0.031041	0.00789	0.198738	0.462634	0.411742	0.228087	0.042467
	0.002018	0.022919	0.043041	0.043303	0.017468	0.173633	0.454711	0.442984	0.275424	0.065519
EMV	0.002018	0.022242	0.036902	0.029128	0.006376	0.173633	0.454711	0.411742	0.228087	0.039468
OS	5	4	4	3	2	5	5	4	4	3
10	0.033946	0.069677	0.063504	0.037136	0.006835	5.241596	2.473909	1.12785	0.407402	0.052048
	0.007407	0.033836	0.040741	0.026145	0.004283	0.845911	0.892855	0.570524	0.248011	0.03413
	0.003648	0.022442	0.030122	0.019702	0.002976	0.424976	0.532953	0.389178	0.180325	0.024935
	0.001822	0.017202	0.025361	0.017097	0.00253	0.160422	0.367279	0.306111	0.150514	0.021299
	0.001172	0.013918	0.023071	0.016722	0.00263	0.099167	0.275613	0.258033	0.137262	0.020768
	0.001026	0.012251	0.02075	0.015479	0.002525	0.086972	0.241619	0.228803	0.123981	0.019272
	0.000882	0.011264	0.020239	0.015764	0.002643	0.074198	0.216466	0.215068	0.121548	0.019608
	0.0008	0.010759	0.020265	0.016605	0.003031	0.066977	0.202811	0.20908	0.122719	0.020901
	0.000886	0.012051	0.023717	0.021271	0.005053	0.074237	0.225369	0.237848	0.146415	0.028138
	0.001086	0.01598	0.034173	0.03625	0.014308	0.08995	0.291086	0.325035	0.21927	0.054087
EMV	0.0008	0.010759	0.020239	0.015479	0.002525	0.066977	0.202811	0.20908	0.121548	0.019272
OS	8	8	7	6	6	8	8	8	7	6
15	0.039606	0.069879	0.061503	0.035855	0.006753	6.897354	2.640615	1.134075	0.398825	0.050642
	0.007137	0.033952	0.039259	0.024513	0.003893	0.74079	0.902702	0.564221	0.238884	0.031974
	0.002955	0.021029	0.028293	0.018095	0.002513	0.282466	0.486493	0.363961	0.168263	0.022602
	0.00145	0.014789	0.022177	0.014434	0.001838	0.125721	0.30826	0.264815	0.130218	0.017547
	0.000949	0.011603	0.018738	0.012447	0.001546	0.079804	0.22833	0.213388	0.109479	0.014939
	0.000849	0.010503	0.017234	0.011531	0.001398	0.071319	0.20555	0.194289	0.100697	0.01375
	0.000705	0.009151	0.015596	0.01066	0.001284	0.05892	0.175867	0.171707	0.091216	0.012633
	0.000662	0.008712	0.015073	0.01042	0.001255	0.055251	0.166487	0.164391	0.088272	0.012323
	0.000645	0.008517	0.014867	0.010429	0.001293	0.053905	0.162472	0.161208	0.087283	0.012369
	0.000609	0.008263	0.014817	0.010783	0.001473	0.050679	0.155988	0.157935	0.087494	0.012938
	0.000567	0.007876	0.014652	0.011148	0.001631	0.047166	0.147066	0.152607	0.087165	0.013474
	0.000514	0.007783	0.015329	0.012203	0.001854	0.042283	0.141182	0.154199	0.091811	0.014782
	0.000504	0.007837	0.016253	0.01417	0.002744	0.041387	0.140462	0.158214	0.099177	0.01786
	0.00063	0.00976	0.020876	0.019877	0.005059	0.051865	0.174788	0.199087	0.130072	0.02651
	0.000895	0.014043	0.032239	0.037003	0.016949	0.073721	0.249367	0.29369	0.210418	0.057318
EMV	0.000504	0.007783	0.014652	0.01042	0.001255	0.041387	0.140462	0.152607	0.087165	0.012323
OS	13	12	11	8	8	13	13	11	11	8

*__EMV__=Empirical Minimum Variance, **OS**= Order Statistic

Table – 3.3 :: Empirical Variance of $\hat{R}(x)$, $\hat{h}(x)$ for **α=2** (Continuation).

	Empirical Variance of $\hat{R}(x)$ at **α=2**					Empirical Variance of $\hat{h}(x)$ at **α=2**				
Sample Size	**0.95**	**0.75**	**0.50**	**0.25**	**0.05**	**0.95**	**0.75**	**0.50**	**0.25**	**0.05**
20	0.031901	0.067343	0.060747	0.034685	0.006025	4.888385	2.368995	1.084316	0.387928	0.048262
	0.007255	0.034646	0.039498	0.023446	0.003266	0.808956	0.914063	0.573586	0.238368	0.030148
	0.002559	0.021395	0.029034	0.018191	0.002369	0.230668	0.476703	0.370607	0.171554	0.02254
	0.001582	0.015643	0.02302	0.014814	0.001837	0.13762	0.329941	0.278301	0.135207	0.017991
	0.001027	0.012132	0.019068	0.012397	0.001482	0.08668	0.241878	0.220947	0.111235	0.01485
	0.000774	0.009824	0.016335	0.01096	0.001316	0.064827	0.190582	0.182661	0.095364	0.013028
	0.000638	0.008463	0.014517	0.009935	0.001202	0.053147	0.16159	0.1592	0.084846	0.01177
	0.000542	0.007483	0.013321	0.009377	0.001165	0.045044	0.140716	0.142751	0.07809	0.011096
	0.000462	0.006628	0.012264	0.008875	0.001114	0.038188	0.1227	0.128327	0.072121	0.010466
	0.000413	0.006077	0.011366	0.008211	0.00101	0.034034	0.111576	0.11819	0.066741	0.009641
	0.000378	0.005688	0.01081	0.007884	0.000971	0.031135	0.103645	0.111332	0.063536	0.009242
	0.000378	0.005682	0.010836	0.007955	0.000996	0.031053	0.103458	0.11135	0.063767	0.009344
	0.000369	0.005576	0.010699	0.00793	0.001016	0.030359	0.101374	0.109519	0.063072	0.00934
	0.000376	0.005614	0.010779	0.008092	0.001078	0.03096	0.102379	0.110241	0.063732	0.009586
	0.000376	0.005681	0.011063	0.008477	0.00118	0.030958	0.103143	0.112138	0.065656	0.010102
	0.000394	0.005974	0.011699	0.009094	0.001298	0.032397	0.108324	0.118138	0.069652	0.010881
	0.000414	0.006342	0.012759	0.010479	0.001732	0.034039	0.114434	0.126614	0.076833	0.012831
	0.000434	0.006827	0.014309	0.012504	0.002342	0.035596	0.121794	0.138413	0.087296	0.015643
	0.000532	0.008579	0.018812	0.017904	0.00407	0.043527	0.151452	0.176883	0.117053	0.023324
	0.000795	0.012665	0.029088	0.032424	0.013152	0.065333	0.223833	0.26513	0.188338	0.048492
EMV	0.000369	0.005576	0.010699	0.007884	0.000971	0.030359	0.101374	0.109519	0.063072	0.009242
OS	13	13	13	11	11	13	13	13	13	11

***EMV**=Empirical Minimum Variance, **OS**= Order Statistic

Table – 3.4 :: Empirical Variance of $\hat{R}(x)$, $\hat{h}(x)$ for **α=3**.

	Empirical Variance of $\hat{R}(x)$ at α=3					Empirical Variance of $\hat{h}(x)$ at α=3				
Sample Size	0.95	0.75	0.50	0.25	0.05	0.95	0.75	0.50	0.25	0.05
05	0.031721	0.073597	0.071641	0.046029	0.011467	10.53851	6.557672	3.72214	1.792181	0.442306
	0.007206	0.037075	0.045699	0.031654	0.007453	1.475766	2.105043	1.688078	0.990648	0.276795
	0.003567	0.023693	0.035575	0.027632	0.006761	0.86186	1.133443	1.059583	0.71216	0.225186
	0.001523	0.018376	0.032669	0.028043	0.007964	0.203718	0.670789	0.800429	0.615506	0.218456
	0.001351	0.018129	0.036178	0.036702	0.014667	0.176919	0.62302	0.7946	0.664443	0.276041
EMV	0.001351	0.018129	0.032669	0.027632	0.006761	0.176919	0.62302	0.7946	0.615506	0.218456
OS	5	5	4	3	3	5	5	5	4	4
10	0.032626	0.06999	0.064362	0.038494	0.008518	12.90446	6.569975	3.569805	1.650753	0.381888
	0.009186	0.03758	0.044252	0.028607	0.005603	2.212216	2.362427	1.743889	0.979741	0.256959
	0.003178	0.025555	0.034808	0.023681	0.004539	0.466778	1.16509	1.120854	0.715394	0.202469
	0.002242	0.018841	0.028307	0.020295	0.003762	0.341079	0.819895	0.828323	0.561513	0.167402
	0.00121	0.013631	0.023429	0.0182	0.003553	0.166109	0.512468	0.594093	0.443576	0.143735
	0.000831	0.011351	0.020782	0.016382	0.003092	0.107771	0.388933	0.492012	0.385081	0.127618
	0.000778	0.010732	0.019855	0.015909	0.003224	0.100838	0.365687	0.465428	0.366926	0.123426
	0.000692	0.01021	0.019999	0.016836	0.00352	0.088596	0.334725	0.443959	0.364119	0.128142
	0.000715	0.010711	0.021694	0.019504	0.004973	0.091447	0.347618	0.467213	0.392658	0.146577
	0.000851	0.013066	0.028442	0.029726	0.011567	0.108783	0.416611	0.574181	0.509525	0.219157
EMV	0.000692	0.01021	0.019855	0.015909	0.003092	0.088596	0.334725	0.443959	0.364119	0.123426
OS	8	8	7	7	6	8	8	8	8	7
15	0.036991	0.074973	0.066808	0.038871	0.008411	13.18851	7.274654	3.864762	1.74819	0.393691
	0.008896	0.040061	0.044024	0.02688	0.004958	1.739443	2.466232	1.840142	0.999772	0.249091
	0.003821	0.024717	0.03233	0.021142	0.003672	0.696003	1.234368	1.099111	0.676068	0.183737
	0.001596	0.016536	0.025547	0.017741	0.002996	0.221352	0.655789	0.717392	0.499028	0.146759
	0.001112	0.013012	0.021555	0.01546	0.002508	0.149454	0.483665	0.563506	0.411026	0.124995
	0.000713	0.009949	0.018132	0.013767	0.002333	0.092056	0.337397	0.430446	0.335558	0.108032
	0.000575	0.008449	0.016143	0.012651	0.002147	0.073643	0.277974	0.366446	0.294785	0.097695
	0.000556	0.008192	0.015578	0.012096	0.002015	0.071209	0.269407	0.35507	0.284709	0.093654
	0.000486	0.007317	0.014273	0.01127	0.001857	0.062032	0.237345	0.317696	0.259055	0.086465
	0.000514	0.007506	0.014416	0.011304	0.001861	0.065963	0.247372	0.325869	0.262845	0.087049
	0.000503	0.007449	0.014391	0.011405	0.001934	0.064425	0.243964	0.323362	0.262034	0.08758
	0.000519	0.007791	0.015388	0.012707	0.002395	0.066287	0.252952	0.338796	0.279011	0.096567
	0.000532	0.007987	0.01619	0.0141	0.003005	0.068095	0.258658	0.348371	0.292235	0.105776
	0.000592	0.009051	0.018996	0.017722	0.004552	0.075672	0.289917	0.396063	0.340982	0.131274
	0.000715	0.011555	0.026978	0.030403	0.013288	0.09097	0.356606	0.510926	0.476481	0.220698
EMV	0.000486	0.007317	0.014273	0.01127	0.001857	0.062032	0.237345	0.317696	0.259055	0.086465
OS	9	9	9	9	9	9	9	9	9	9

***EMV**=Empirical Minimum Variance, **OS**= Order Statistic

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 10 Issue III Mar 2022- Available at www.ijraset.com

Table – 3.4 :: Empirical Variance of $\hat{R}(x)$, $\hat{h}(x)$ for **α=3**(Continuation).

	Empirical Variance of $\hat{R}(x)$ at α=3					Empirical Variance of $\hat{h}(x)$ at α=3				
Sample Size	0.95	0.75	0.50	0.25	0.05	0.95	0.75	0.50	0.25	0.05
20	0.036743	0.074457	0.065897	0.038128	0.008138	14.56103	7.288251	3.841107	1.728145	0.387028
	0.00745	0.03817	0.042704	0.025575	0.004377	1.285565	2.205735	1.728466	0.956514	0.237122
	0.003096	0.023886	0.031627	0.020398	0.003459	0.460406	1.111867	1.048515	0.655252	0.177563
	0.001628	0.016901	0.025509	0.017335	0.002813	0.223608	0.673298	0.731558	0.501763	0.144745
	0.001278	0.013914	0.021947	0.015166	0.002397	0.174963	0.537698	0.602522	0.424981	0.12485
	0.000953	0.011536	0.019112	0.013464	0.002036	0.126536	0.422902	0.498448	0.363794	0.10914
	0.00075	0.009565	0.01662	0.012042	0.001805	0.098687	0.339824	0.413896	0.311641	0.095989
	0.000635	0.008464	0.014953	0.010792	0.001556	0.082715	0.293945	0.365913	0.278691	0.085818
	0.000538	0.007521	0.013723	0.01012	0.001491	0.069462	0.254489	0.325391	0.253333	0.079608
	0.000469	0.006748	0.012596	0.009425	0.001386	0.060261	0.224625	0.292216	0.231008	0.073563
	0.000429	0.006169	0.011579	0.008718	0.001284	0.055246	0.205328	0.26732	0.21203	0.067828
	0.000392	0.005829	0.011232	0.008665	0.001332	0.050165	0.190416	0.252868	0.204257	0.066782
	0.00036	0.005435	0.010629	0.008332	0.001314	0.046033	0.176097	0.23601	0.192575	0.063831
	0.000336	0.005227	0.010455	0.008293	0.001306	0.042616	0.166336	0.2272	0.188259	0.06315
	0.00034	0.005336	0.010819	0.008739	0.00142	0.043151	0.168978	0.232256	0.194265	0.066167
	0.000315	0.005199	0.011097	0.009501	0.00172	0.039676	0.159905	0.227173	0.196991	0.070645
	0.000334	0.005536	0.011919	0.010369	0.001935	0.04202	0.169743	0.242083	0.211297	0.076814
	0.000356	0.006038	0.013426	0.012313	0.002628	0.044667	0.182629	0.264838	0.236688	0.090192
	0.000408	0.006902	0.015514	0.014731	0.003547	0.051234	0.208869	0.303159	0.27335	0.107475
	0.000604	0.010377	0.024761	0.027532	0.010882	0.07585	0.310546	0.458869	0.434298	0.198669
EMV	0.000315	0.005199	0.010455	0.008293	0.001284	0.039676	0.159905	0.227173	0.188259	0.06315
OS	16	16	14	14	11	16	16	16	14	14

***EMV**=Empirical Minimum Variance, **OS**= Order Statistic

Table –3.5 :: Empirical Variance of $\hat{R}(x)$, $\hat{h}(x)$ for **α=4**.

Sample Size	Empirical Variance of $\hat{R}(x)$ at α=4					Empirical Variance of $\hat{h}(x)$ at α=4				
	0.95	0.75	0.50	0.25	0.05	0.95	0.75	0.50	0.25	0.05
05	0.037401	0.0783	0.071509	0.044562	0.012294	23.74682	14.82642	8.781498	4.577093	1.420293
	0.007142	0.039302	0.047826	0.032418	0.007532	1.762248	3.658013	3.400044	2.325696	0.877696
	0.002869	0.025288	0.036819	0.027144	0.00659	0.568782	1.732346	1.993091	1.572867	0.667764
	0.002037	0.020658	0.033287	0.026526	0.0071	0.393623	1.280157	1.583083	1.336348	0.611947
	0.001548	0.018212	0.034741	0.033622	0.012706	0.29291	1.011174	1.359199	1.271159	0.683149
EMV	0.001548	0.018212	0.033287	0.026526	0.00659	0.29291	1.011174	1.359199	1.271159	0.611947
OS	5	5	4	4	3	5	5	5	5	4
10	0.043253	0.080272	0.071784	0.044455	0.011531	34.63744	16.88115	9.371572	4.734524	1.43869
	0.010821	0.04118	0.045783	0.028828	0.005867	4.195313	4.887621	3.842524	2.406918	0.838335
	0.003098	0.023719	0.033046	0.022414	0.004367	0.722384	1.750863	1.902186	1.452568	0.58318
	0.001848	0.01809	0.027258	0.019221	0.003781	0.358741	1.152399	1.394764	1.135385	0.479446
	0.001175	0.014155	0.023644	0.017607	0.003534	0.213597	0.787561	1.053563	0.924449	0.416918
	0.000796	0.011022	0.020349	0.01601	0.003147	0.140662	0.556696	0.801751	0.752243	0.361193
	0.000645	0.009548	0.018696	0.015461	0.003135	0.112321	0.460281	0.687685	0.669879	0.336285
	0.000634	0.00971	0.019612	0.016843	0.003729	0.109696	0.457368	0.696087	0.69164	0.357687
	0.000679	0.010154	0.02038	0.017873	0.004278	0.118093	0.485838	0.73062	0.721403	0.375941
	0.000694	0.011196	0.02502	0.025965	0.009786	0.119416	0.506858	0.797816	0.840449	0.497182
EMV	0.000634	0.009548	0.018696	0.015461	0.003135	0.109696	0.457368	0.687685	0.669879	0.336285
OS	8	7	7	7	7	8	8	7	7	7
15	0.03498	0.075987	0.068172	0.040725	0.009933	23.38045	14.09601	8.443065	4.396926	1.336702
	0.0094	0.04156	0.046003	0.028971	0.006098	2.535474	4.49953	3.784779	2.405457	0.840943
	0.00266	0.023087	0.032337	0.021898	0.004315	0.533459	1.596549	1.821547	1.411184	0.568981
	0.001463	0.016049	0.02553	0.018158	0.003365	0.276935	0.945724	1.211255	1.027859	0.445712
	0.00104	0.013078	0.022186	0.016555	0.003193	0.186681	0.707768	0.966575	0.859172	0.390195
	0.000714	0.009961	0.018377	0.014368	0.002715	0.125627	0.50105	0.723795	0.679233	0.324968
	0.000588	0.00866	0.016576	0.013166	0.002444	0.102475	0.419957	0.624119	0.600553	0.293352
	0.000486	0.007502	0.014933	0.012193	0.0023	0.083882	0.351945	0.536904	0.529993	0.265932
	0.000421	0.006733	0.013848	0.011533	0.002159	0.072267	0.308151	0.479526	0.483298	0.247625
	0.000416	0.006566	0.01345	0.011241	0.002107	0.071527	0.302915	0.46845	0.470398	0.240972
	0.000384	0.006162	0.012858	0.010955	0.002116	0.065848	0.280909	0.438601	0.445473	0.231806
	0.000359	0.006013	0.013108	0.011703	0.002437	0.061231	0.266234	0.42572	0.444453	0.24034
	0.000351	0.005944	0.01321	0.012244	0.002844	0.059729	0.260945	0.420277	0.443789	0.246167
	0.000398	0.006735	0.015216	0.014715	0.003816	0.067809	0.295521	0.476466	0.507387	0.289363
	0.000544	0.009288	0.022105	0.024318	0.009154	0.092843	0.4041	0.657169	0.72023	0.449055
EMV	0.000351	0.005944	0.012858	0.010955	0.002107	0.059729	0.260945	0.420277	0.443789	0.231806
OS	13	13	11	11	10	13	13	13	13	11

*__EMV__=Empirical Minimum Variance, **OS**= Order Statistic

Table – 3.5 :: Empirical Variance of $\hat{R}(x)$ for **α=4**(Continuation).

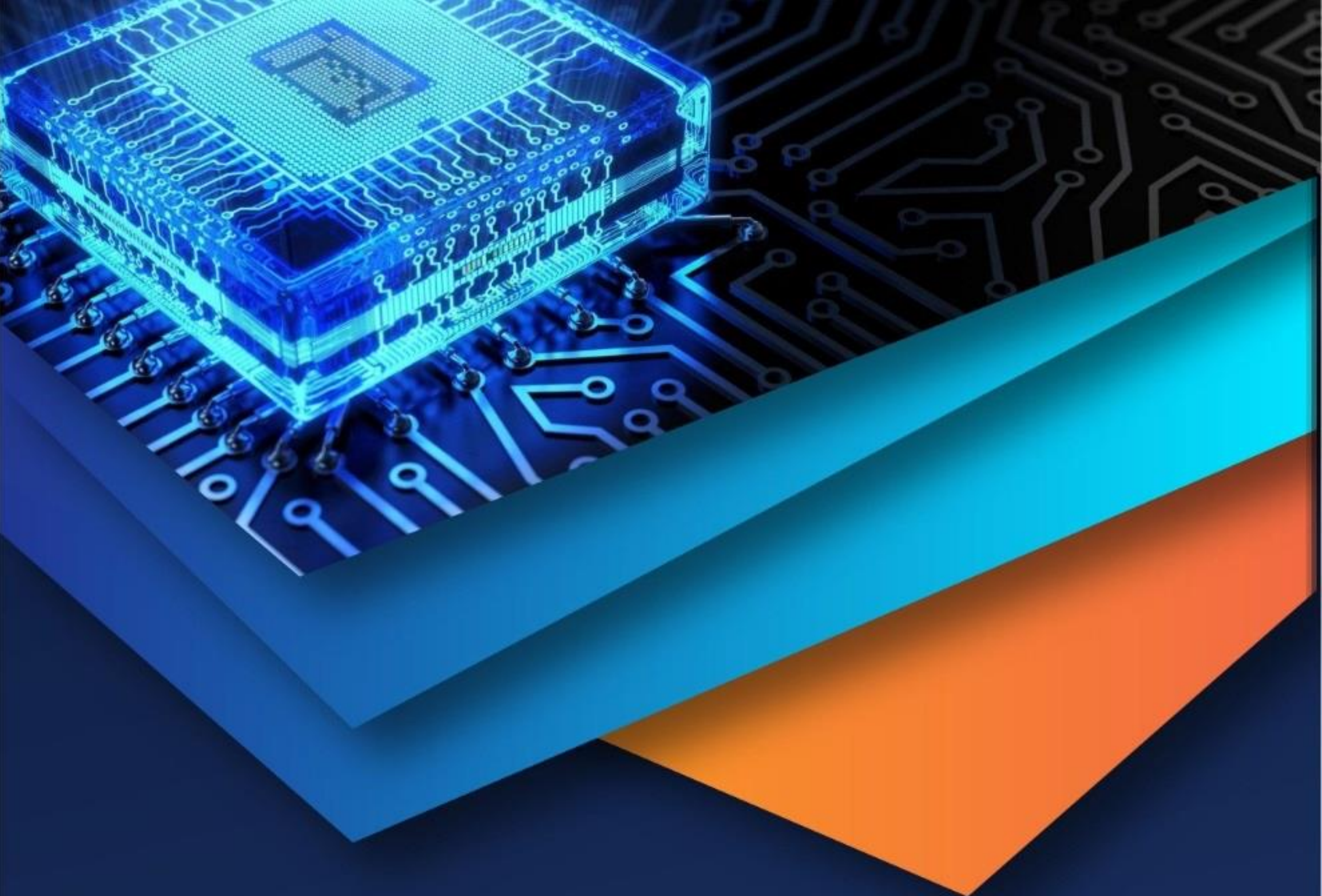
Empirical Variance of $\hat{R}(x)$ at α=4						Empirical Variance of $\hat{h}(x)$ at α=4				
Sample Size	0.95	0.75	0.50	0.25	0.05	0.95	0.75	0.50	0.25	0.05
20	0.036274	0.078892	0.069097	0.041098	0.010256	19.88555	14.48376	8.772451	4.535107	1.363775
	0.007832	0.040116	0.045175	0.028678	0.006299	1.948844	3.955511	3.541889	2.315249	0.822654
	0.003613	0.026006	0.034788	0.023121	0.004619	0.839912	2.004049	2.112752	1.570812	0.614878
	0.001539	0.01591	0.025094	0.017958	0.003477	0.298811	0.97129	1.211977	1.016715	0.439945
	0.001001	0.012534	0.021354	0.015754	0.002923	0.180492	0.678675	0.92642	0.825111	0.373386
	0.000743	0.010024	0.017777	0.013317	0.002431	0.131595	0.516714	0.732105	0.671012	0.310319
	0.000592	0.008413	0.015454	0.011743	0.002074	0.103631	0.418333	0.609368	0.572039	0.269686
	0.000512	0.007529	0.014272	0.010992	0.001873	0.089136	0.365339	0.542482	0.519463	0.249223
	0.000429	0.006491	0.012637	0.009911	0.001678	0.074433	0.30891	0.465782	0.453476	0.221425
	0.000389	0.006078	0.012093	0.009586	0.001599	0.066985	0.282916	0.434107	0.429023	0.21219
	0.000358	0.005716	0.011615	0.009349	0.00158	0.061407	0.261976	0.407014	0.407635	0.204577
	0.000342	0.005506	0.011268	0.009072	0.001503	0.058519	0.250728	0.391547	0.39403	0.198274
	0.000316	0.005135	0.010659	0.00874	0.001481	0.054079	0.232392	0.364818	0.370081	0.188699
	0.000305	0.005049	0.010669	0.008893	0.001549	0.051907	0.225286	0.357712	0.367136	0.189883
	0.000302	0.005028	0.010654	0.008867	0.00153	0.051425	0.223672	0.356004	0.366102	0.189408
	0.000299	0.00499	0.010624	0.008946	0.001581	0.050977	0.221764	0.353294	0.364236	0.189783
	0.000309	0.00512	0.010982	0.009515	0.001823	0.052634	0.228157	0.362809	0.375189	0.198678
	0.000317	0.005365	0.011838	0.010688	0.002269	0.053981	0.235779	0.379351	0.398976	0.217792
	0.000364	0.006233	0.014067	0.013281	0.003173	0.06184	0.2714	0.440133	0.469137	0.26415
	0.000525	0.008904	0.020644	0.021694	0.007704	0.089363	0.389387	0.63011	0.680681	0.410168
EMV	0.000299	0.00499	0.010624	0.00874	0.001481	0.050977	0.221764	0.353294	0.364236	0.188699
OS	16	16	16	13	13	16	16	16	16	13

***EMV**=Empirical Minimum Variance, **OS**= Order Statistic

These tables indicate that the invariance property of $\hat{\theta}$ carried forward to estimation of parameteric function is operating to be true at smaller population reliability values like 0.05, 0.25 for small values of shape shape parameter. However this is observed to be extended for all the reliability values when the values of α become larger and larger.

REFERENCES

[1] Alzaatreh, A., Famoye, F., and Lee, C. (2012). Gamma-Pareto Distribution and Its Applications. Journal of Modern Applied Statistical Methods, Vol. 11, No. 1, 78-94.
[2] Balakrishnan, N. and Cohen, A.C. (1991). Order Statistics and Inference, Academic Press. INC, London.
[3] Gupta, S.S.(1962). Gamma distribution, in Contributions to Order Statistics, John Wiley & Sons, New York.
[4] Sarhan, A. E. and Greenberg, B. G. (1962). Contributions to Order Statistics, Wiley, New York.
[5] Subba Rao, R., Kantam, R.R.L., and Prasad, G. (2015). Modified Maximum likelihood Estimation in Pareto-Rayleigh distribution. Golden Research Thoughts, 140-152.



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