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Estimation of Range of Appearance in Heptadecagonal Numbers

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Abstract: In this document, the range of appearance $I(r)$ of a positive integer r in heptagonal numbers and some results by comparing $I(r)$ with Legendre symbol, $p-$adic range and prime conjunction function of r are accessible. Since $I(r)$ does not possess a meticulous prototype, all the results are confirmed by Java program for all-natural numbers $r \in N$.
Keywords: Heptadecagonal number, range of appearance, $p-$adic range

I. INTRODUCTION

"In 1796, an arithmeticians Gauss showed that a regular polygon of 17 sides can be created using a scale and compass by screening that a primitive 17th root of unity can be found by solving a succession of quadratic equations over the rationals. A heptadecagram is a star polygon with 17-sides.

The sequence of Heptadecagonal number is provoked by $H(m) = \frac{m(15m-13)}{2}, m \in N$" [1-4]. Many prominent arithmeticians have apportioned with concepts of divisibility of Fibonacci numbers. In [5], the author pondered the equation $Z(n) = (2-\frac{1}{K})n$ involving order of appearance in Fibonacci numbers. In this framework, one may grasp [6-10]. In this document, the range of appearance $I(r)$ of $r \in N$ in the pattern of heptagonal numbers and few results by measure up to $I(r)$ with Legendre symbol, $p-$adic range and prime conjunction functions are offered. Since $I(r)$ does not have a particular model, each and every result are entrenched by Java program for all values of $r \in N$.

1) Definition

"A quadratic residue modulo n is an integer t such that it is congruent to square of a number. That is there can exists an integer Y such that $Y^2 \equiv t(mod\ n)$" [1].

2) Definition

In [2], The Legendre symbol is a number hypothetical function $\left(\frac{r}{p}\right)$ which is defined by

$$\left(\frac{r}{p}\right) = \begin{cases} 0, if\ \frac{p}{r} \\ 1, if\ r\ is\ a\ quadratic\ residue\ modulo\ p \\ -1, if\ r\ is\ a\ quadratic\ nonresidue\ modulo\ p \end{cases}$$

3) Definition

"The greater power of a prime number p which divides a positive integer r is called $p-$adic range of r and it is symbolized by $d_p(r)$".

4) Definition

"The prime conjunction function of a positive integer r is the separate prime factors of r and it is designated by $\wedge(r)$".

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5) Definition

"The range of appearance of a positive integer r in the sequence of Heptadecagonal numbers signified by $I(r)$ is defined as the lowest natural number m such that $r|H(m)$" [5].

The values of $I(r)$ for discrete values of r such that $1 \leq r \leq 20$ are listed in the table below.

r	1	2	3	4	5	6	7	8	9	10	11	12
$I(r)$	1	3	3	3	5	3	6	3	9	15	6	3
r	13	14	15	16	17	18	19	20	21	22	23	24
$I(r)$	13	7	15	3	2	27	11	35	6	11	7	3
r	25	26	27	28	29	30	31	32	33	34	35	36
$I(r)$	25	39	27	27	26	15	5	35	6	19	20	27
r	37	38	39	40	41	42	43	44	45	46	47	48
$I(r)$	28	11	39	35	20	27	41	11	45	7	4	3
r	49	50	51	52	53	54	55	56	57	58	59	60
$I(r)$	27	75	36	91	15	27	50	35	30	55	52	75
r	61	62	63	64	65	66	67	68	69	70	71	72
$I(r)$	9	31	27	35	65	39	50	19	30	20	34	99
r	73	74	75	76	77	78	79	80	81	82	83	84
$I(r)$	69	28	75	11	3	39	43	35	81	20	23	27
r	85	86	87	88	89	90	91	92	93	94	95	96
$I(r)$	70	43	84	83	78	135	13	99	36	4	30	99
r	97	98	99	100								
$I(r)$	72	27	72	75								

The range of appearance of a positive integer r in the already mentioned sequence of Heptadecagonal numbers for all other choices of $r \in N$ are found out by the following Java program.

```
import java.util.Scanner;
import java.util.ArrayList; // import the ArrayList class
import java.util.Collections;// import Collections
class Main {
public static void main(String[] args) {
Scanner myObj = new Scanner(System.in); // Create a Scanner object
System.out.println("Enter the value of m:");
String m = myObj.nextLine(); // Read user input
int x = Integer.parseInt(m);
ArrayList < Integer > result = findM(x);
System.out.println( result);
System.out.println("Enter the value of r:");
String r = myObj.nextLine(); // Read user input
int y = Integer.parseInt(r);
ArrayList < Integer > resultN = findN(y);
System.out.println( resultN); }
//set values to m,r and temp as arraylist
// array list will help to have elements of any size.
static ArrayList < Integer > tmNet = new ArrayList < Integer > ();
static ArrayList < Integer > tnNet = new ArrayList < Integer > ();
static ArrayList < Integer > tmp = new ArrayList < Integer > ();
static int m;
static int r;
```

```
static int tm;
static int result = 0;
static ArrayList < Integer > findM(int val){
m = val;
for(int i = 1; i <= m; i + +){
result = i * ((15 * i) – 13)/2;
tmNet. add(result);
} return tmNet; //return m
} static ArrayList < Integer > findN(int val){
r = val;
for(int j = 1; j <= r; j + +){
for(int i = 1; i <= r; i + +){
tm = i * ((15 * i) – 13)/2;
int val2 = (int) Math. round(tm/j);
            if (val2 ! = 0&& tm%j == 0){ //take value divisible by iterator and the value is > 0
 tmp. add(i);
}}
if(! tmp. isEmpty()){
tnNet. add(Collections. min(tmp));//take min val from list
tmp.removeAll(tmp); //reset array to every iteration
} }
return tnNet; //return r
} }
```

II. ESTIMATION OF RANGE OF APPEARANCE IN HEPTADECAGONAL NUMBERS:

Some results based on the range of appearance of a positive integer r in the above held sequence of Heptadecagonal numbers for numerous selections of a natural number r are explored and are verified by Java program.

A. Result 1

Let n be a positive integer of the form $r = p^{x}$, where p is a prime number.

i. If $p = 2$, then $I(2^{x}) \leq 3.(2^{x-1})$, for $x \geq 1$.
ii. If $p = 3$, then $I(3^{x}) \leq 3^{x}$, for $x \geq 1$.
iii. If $p = 5$, then $I(5^{x}) \leq 5^{x}$, for $x \geq 1$.
iv. If $p > 5$, then $I(p^{x}) \leq \left(p - \left(\frac{2}{p}\right)\right).p^{x-1}$, for $x \geq 1$ where $\left(\frac{r}{p}\right)$ is a legendre symbol

Java program for monitoring this result is specified below.

```
import java. util. Scanner;
import java. util. ArrayList; // import the ArrayList class
import java. util. Collections;// import Collections
import java. lang. Math;
import java. util.*;
class Main {
public static void main(String[] args) {
Scanner myObj = new Scanner(System. in); // Create a Scanner object
System. out. println("Enter the value of r: ");
String r = myObj. nextLine(); // Read user input
int y = Integer. parseInt(r);
ArrayList < Integer > resultN = findN(y);
System. out. println("r: " + getRepValues());
```

```
System.out.println("I(r) " + resultN); }
//set values to m,r and temp as arraylist
// array list will help to have elements of any size.
static ArrayList < Integer > tnNet = new ArrayList < Integer > ();
static ArrayList < Integer > tmp = new ArrayList < Integer > ();
static int m; static int r; static int tm;
static int result = 0;
static ArrayList < Integer > rep = new ArrayList < Integer > ();
static ArrayList < Integer > findN(int val){
r = val;
for(int j = 1; j ≤ r; j + +){
for(int i = 1; i <= r; i + +){
tm = i * ((15 * i) − 13)/2;
int val2 = (int) Math.round(tm/j);
if (val2 ! = 0&& tm%j == 0&& j%2 == 0&& (int)(Math.ceil((Math.log(j) / Math.log(2)))) ==
(int)(Math.floor(((Math.log(j) / Math.log(2)))))){ //take value divisible by iterator and the value is
                    > 0 , is divisible by 2 and is also power of 2.
rep.add(j);
tmp.add(i); }}
if(!tmp.isEmpty()){
tnNet.add(Collections.min(tmp));//take min val from list
tmp.removeAll(tmp); //reset array to every iteration
}} return tnNet; //return r
} static ArrayList < Integer > getRepValues(){
Set < Integer > unique_ = new HashSet <> (rep);
rep.clear();
rep.addAll(unique_);
return rep; }}
```

B. Result 2

Let r be an even integer and $d_2(r), \wedge(r)$ are $2-$adic range of r and the prime conjunction function of r respectively.

i. If $d_2(r) = 1$, then

$$I(r) \leq \begin{cases} 2r, & if \wedge(r) = 2 \; and \; r \nmid 5 \\ \frac{4r}{3}, & if \wedge(r) = 2 \; and \; r \nmid 5 \\ 2\left(\frac{5}{2}\right)^{\wedge(r)-\mathfrak{S}_r-2} r, & if \wedge(r) > 2 \end{cases}$$

ii. If $d_2(r) = 2$, then

$$I(r) \leq \begin{cases} \frac{3r}{2}, & if \wedge(r) = 2 \; and \; r \nmid 5 \\ \frac{7r}{6}, & if \wedge(r) = 2 \; and \; r \nmid 5 \\ 3\left(\frac{1}{2}\right)^{\wedge(r)-\mathfrak{S}_r-2} r, & if \wedge(r) > 2 \end{cases}$$

iii. If $d_2(r) = 3$, then

$$I(r) \leq \begin{cases} \frac{5r}{2}, & if \wedge(r) = 2 \; and \; r \nmid 5 \\ \frac{4r}{3}, & if \wedge(r) = 2 \; and \; r \nmid 5 \\ 2\left(\frac{1}{2}\right)^{\wedge(r)-\mathfrak{S}_r-2} r, & if \wedge(r) > 2 \end{cases}$$

iv. If $d_2(r) \geq 4, \wedge(r) = 2$, then $I(r) \leq \frac{7}{4}\left(\frac{2}{3}\right)^{\wedge(r)-\mathfrak{S}_r-1} r$, where $\mathfrak{S}_r = \begin{cases} 0 & if \; r \nmid 5 \\ 1 & if \; r \mid 5 \end{cases}$

Java program for checking all the above four statements are laid down below.

```
import java.util.Scanner;
import java.util.ArrayList; // import the ArrayList class
import java.util.Collections;// import Collections
import java.lang.Math;
import java.util.*;
class Main {
public static void main(String[] args) {
Scanner myObj = new Scanner(System.in); // Create a Scanner object
System.out.println("Enter the value of p:");
String r = myObj.nextLine(); // Read user input
int y = Integer.parseInt(r);
boolean flag = false;
for (int i = 2; i <= y / 2; ++i) {//// condition for nonprime number
if (y % i == 0) {
flag = true; break; }}
if (!flag){
int param = (int) Math.round(Math.pow(2,4) * y);
System.out.println(y + " is a prime number.");
ArrayList <Integer> resultN = findN(param);
System.out.println("r: " + getRepValues());
System.out.println("I(r) " + resultN);
}else{
System.out.println(y + " is not a prime number. Please rerun the program and enter a prime number!!");
}}//set values to m,n and temp as arraylist
// array list will help to have elements of any size.
static ArrayList<Integer> tnNet = new ArrayList<Integer>();
static ArrayList<Integer> tmp = new ArrayList<Integer>();
static ArrayList<Integer> finalresult = new ArrayList<Integer>();
static int m;
static int r;
static double tm;
static int result = 0;
static ArrayList<Integer> rep = new ArrayList<Integer>();
static ArrayList<Integer> findN(int val){
r = val;
for(int j = 1;j <= r;j++){
for(int i = 1;i <= r;i++){
tm = i * ((15 * i) – 13)/2;
int val2 = (int) Math.round(tm/r);
if (val2 != 0 && tm%r == 0){ //take value divisible by iterator and the value is > 0 and is integer value
rep.add(r); // n is a constant here so we add the calulated
tmp.add(i); //
}}
if(!tmp.isEmpty()){
tnNet.add(Collections.min(tmp));//take min val from list
tmp.removeAll(tmp); //reset array to every iteration
}}if(!tnNet.isEmpty()){
finalresult.add(Collections.min(tnNet));//tnNet; //return r
```

```
}return finalresult;
}static ArrayList < Integer >  getRepValues(){
Set < Integer >  unique_ = new HashSet <> (rep);
rep.clear(); rep.addAll(unique_);
return rep; }}
```

III. CONCLUSION

In this paper, the range of appearance $I(r)$ of a positive integer r in heptagonal numbers is defined and limited number of results consisting $I(r)$, legendre symbol, $p-$ adic range and prime conjunction function of r are evaluated. Entire results are inveterate by Java program for all positive values of r. In this approach, one can define the range of appearance of an integer in other polygonal numbers and may analyse their results by comparing the values of $I(r)$ with Jacobi symbol, quadratic residue and non-residue modulo r.

REFERENCES

[1] Ivan Niven, Hibert S, Zuckerman and Huge L, Montgomery, "An Introduction to the Theory of Numbers", Fifth Edn., John Wiley & Sons Inc., 2004.

[2] Carmichael, R.D., "The Theory of Numbers and Diophantine Analysis", Dover Publication, New York (1959).

[3] Mordell, L.J., "Diophantine Equations", Academic press, London (1969).

[4] John,H., Conway and Richard K.Guy, "The Book of Numbers", Springer Verlag, NewYork,1995.

[5] Eva Trojovs, "On the Diophantine Equation $z(n) = \left(2-\frac{1}{k}\right)n$ involving the order of appearance in the Fibonacci Sequence", Mathematics 2020,1-8.

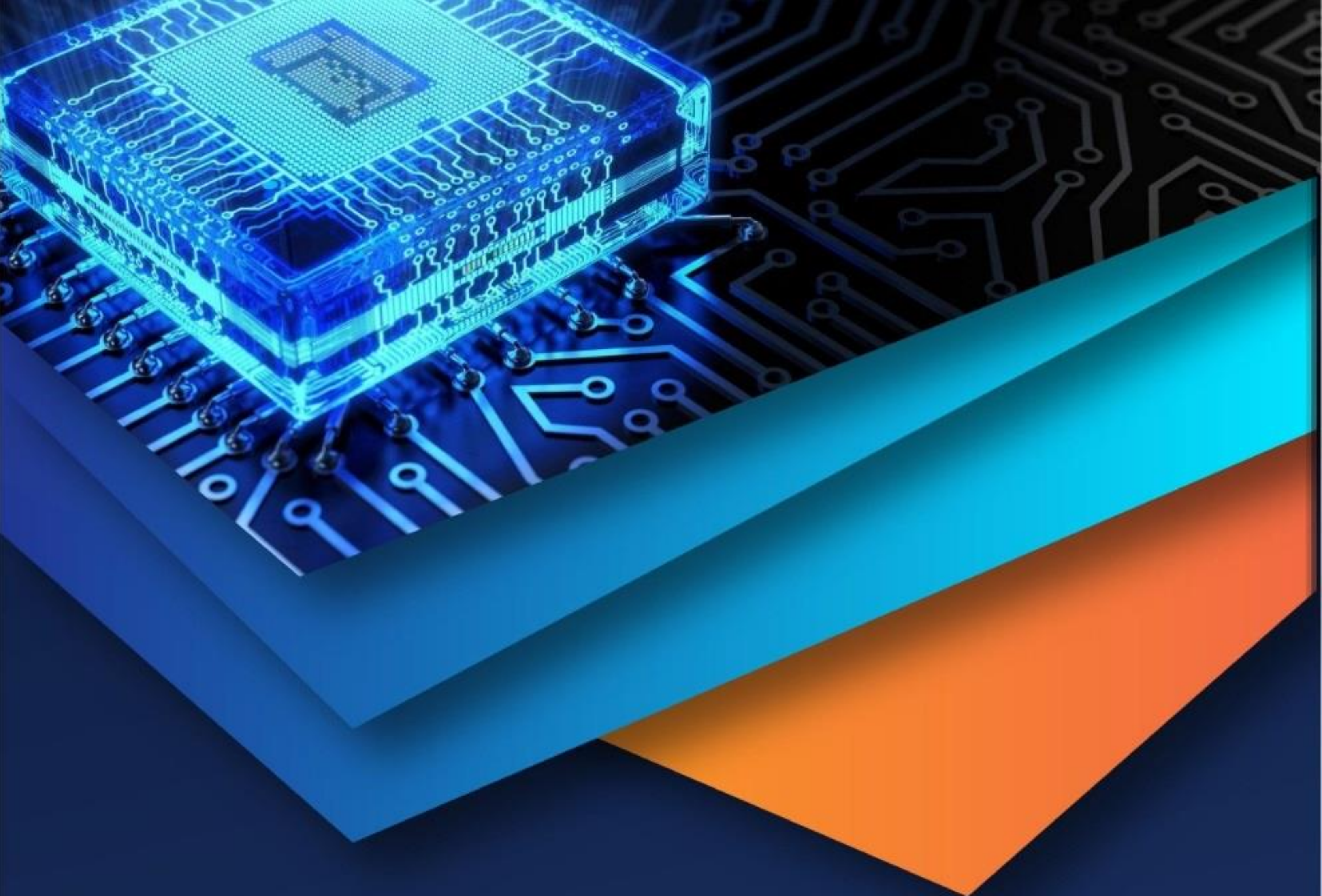
[6] Trojovský, P. "The order of appearance of the sum and difference between two Fibonacci Numbers",Asian-Eur.J.Math.2019,*12*,1950046.

[7] Chung, C.L. "Some Polynomial Sequence Relations". Mathematics 2019, *7*,750.

[8] Knott, R. "Fibonacci Numbers and the Golden Section", The Mathematics Department of the University of Surrey, UK. Available online:Http://www.maths,surrey.ac.uk/hostedsites/R.Knott/Fibonacci/fib.html . (accessed on 2 January 2020).

[9] The Fibonacci Association, Official Website. Available online: https://www.mathstat.dal.ca/fibonacci/(accessed on 2 January 2020).

[10] Kim, S. "The density of the terms in an elliptic divisibility sequence having a fixed G.C.D. with their indices". J. Number Theory2020,*207*, 22–41.



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