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# Exploring FEA Techniques for Bending Rectangular Glass Plates into Parabolic Shapes

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**Abstract:** A structural Finite Element Analysis (FEA) was conducted, incorporating both linear and nonlinear analysis. The investigation considered geometric nonlinearity, material nonlinearity, and boundary nonlinearity, also known as contact analysis. All boundary conditions, loadings, and material properties used in the analysis were based on real-world conditions. Experimental validations were performed to confirm the accuracy of the FEA results, providing strong confidence in the investigation's findings.

**Keywords:** FEA, Parabolic deflection of plate, dynamic loading, Ultimate strength

## I. INTRODUCTION

This investigation focuses on using Finite Element Analysis (FEA) to bend rectangular glass plates into parabolic shapes for solar collectors. With global warming becoming a critical issue due to coal and diesel power plants, the shift toward non-conventional energy sources, particularly solar energy, offers a sustainable alternative. Parabolic solar collectors, commonly used in small-scale power generation, maximize radiation concentration, often utilizing aluminium due to its high reflectivity and yielding properties. However, aluminium's reflectivity isn't perfect, and glass mirrors with 100% reflectivity offer better radiation concentration but are brittle. Achieving a precise parabolic shape is crucial, as deviations impact radiation focus. Imperfect parabolic shapes in glass bending lead to radiation and power loss. Currently, sand is used as a dead weight in the bending process, but this can result in suboptimal shapes. This project studies the behaviour of glass under bending at ambient conditions to develop a method that creates a perfect parabolic profile for more efficient solar collectors.

Plates are flat, non-curved structures with a thickness that is small compared to their other dimensions. They typically experience loads that cause transverse deflections and can be bounded by straight or curved lines. Plates can have free, simply supported, or fixed edges, and they primarily support loads perpendicular to their surfaces. Their load-carrying behaviour is similar to beams or cables and can be modelled as such.

Plates are classified into four types based on structural action:

- 1) *Stiff Plates*: Thin plates with flexural rigidity that carry loads in two dimensions, typically assumed in engineering unless specified otherwise.
- 2) *Membranes*: Thin plates without flexural rigidity, which handle lateral loads through axial shear, resembling a network of stressed cables.
- 3) *Flexible Plates*: A mix of stiff plates and membranes, bearing loads through internal moments and transverse shear.
- 4) *Thick Plates*: Plates with stress conditions resembling three-dimensional structures.

### A. Equation of the Plate

After the development of the Euler–Bernoulli beam theory, several plate theories emerged, with two being particularly prominent in engineering:

- 1) Kirchhoff–Love Theory (**Classical Plate Theory**)
- 2) Mindlin–Reissner Theory (**First-Order Shear Plate Theory**)

In Kirchhoff's approach, a mid-surface plane represents a three-dimensional plate in two dimensions, based on these assumptions:

- Normal lines (perpendicular to the surface) remain straight after deformation.
- Normals retain their original length (unstretched).
- Normals remain perpendicular to the mid-surface post-deformation.

The plate equation is derived under lateral forces using three equilibrium conditions:

$$\sum M_x = 0 \dots\dots\dots 1.1$$

$$\sum M_y = 0 \dots\dots\dots 1.2$$

$$\sum P_z = 0 \dots\dots\dots 1.3$$

In this context,  $M_x$  and  $M_y$  represent the bending moments, while  $P_z$  is the external load. This load  $P_z$  is transmitted through the transverse shear forces  $Q_x$  and  $Q_y$  as well as the bending moments  $M_x$  and  $M_y$ . Unlike beams, plates exhibit significant deviations due to the presence of twisting moments  $M_x$ .

In plate theory, it's essential to consider the internal forces and moments per unit length of the mid-surface. To derive the differential equation of the plate in equilibrium, one must select a coordinate system, sketch the plate element, and illustrate all internal forces, both positive and negative, using Taylor's series.

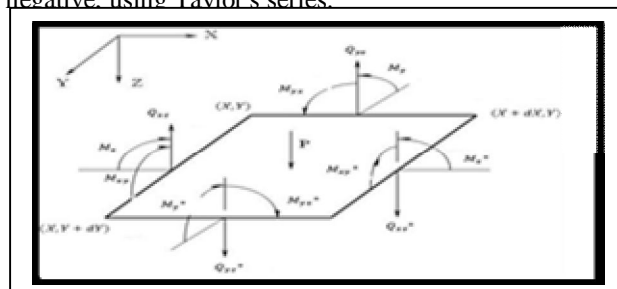
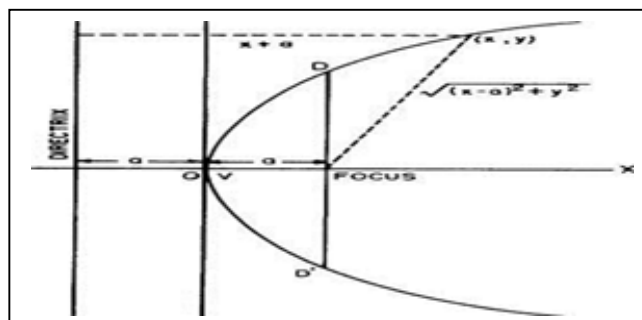


Figure 1: Stress Resultants of the plate

A parabola is defined as the set of points equidistant from a fixed point (the focus) and a fixed line (the directrix). In the parabola illustrated, the vertex V, positioned midway between the focus and the directrix, is typically at the origin. However, the vertex can be located at any point in the plane, similar to how a circle's center can vary.



### B. Defining the support condition

In the analysis, a key focus is the support condition. For a mirrored plate with simply supported boundary conditions, the edges can move freely in a horizontal direction, as illustrated in the figure below.

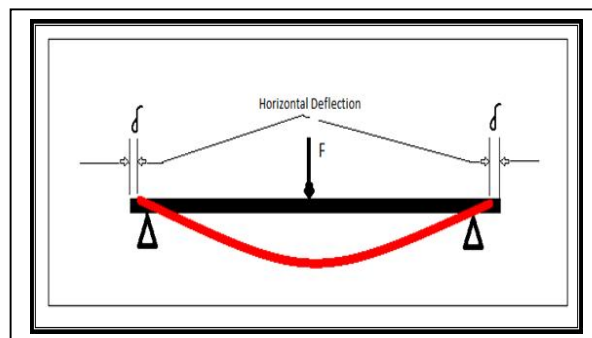


Figure 3 : Actual deflection of beam

Analytically if,  $F=1000$  N, beam span  $L= 0.5$ m, C/S area  $A= 10*10$  mm<sup>2</sup>,  $E= 2.1 \times 10^5$  N/mm<sup>2</sup>. Then the deflection can be calculates as the mathematical formulae  $\delta = FL^3/48EI$ , this gives,  $\delta= 14.88$  mm

This type of constraints is not possible to solve by the FEA analysis. So this is slightly modified as follows

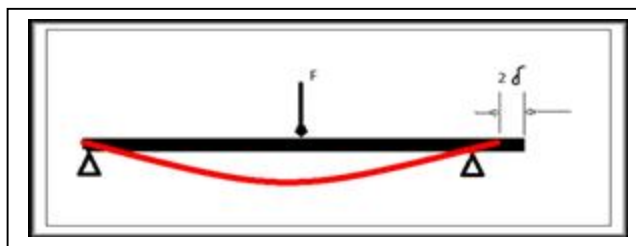


Figure 4: Actual FEA Model

## II. EXPERIMENTAL SETUP

Specimen specification: : 250mm \* 145mm mirror plate, thickness = 1 mm ,  $E = 70 \text{ GPa}$ , poisons ratio = 0.2 , Density =  $2.5 \times 10^{-9} \text{ Tons/mm}^3$ ,

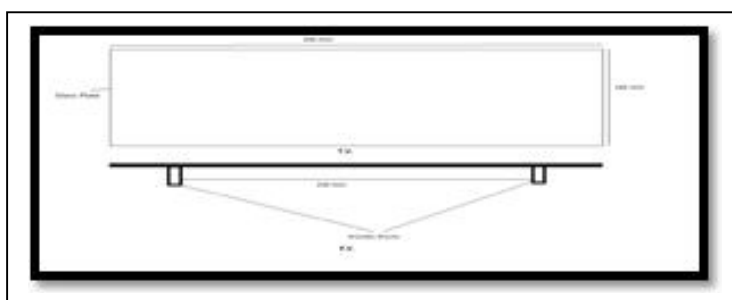


Figure 5: Experimental setup

### A. FEM Setup

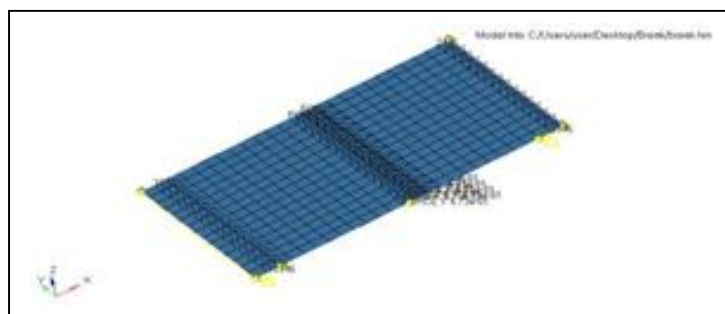
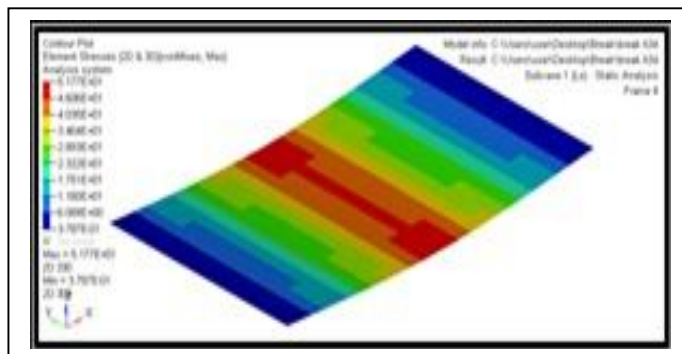


Figure 6: FEA Model

The results obtained by software considering 22.2N as a load: Deflection

### B. Stress





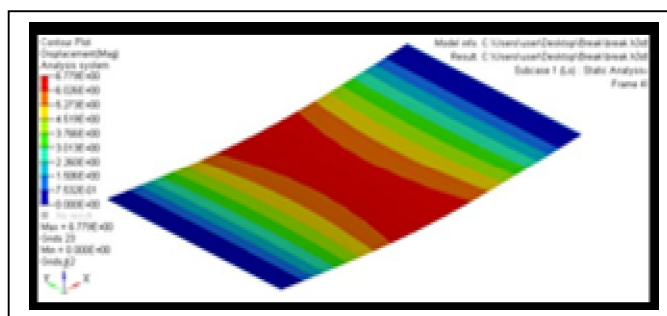


Figure 7: FEA stress results

The stress value got from the software after applying the 22.2 N load is 51.77 N/mm<sup>2</sup>. So we can consider the ultimate strength of the glass is 50 Mpa.

So finally the properties of glass used for analysis are finalizes as:

Modulus of elasticity E = 70 Gpa. Poisson's ratio = 0.2

Density = 2.5e-09 Tons/mm<sup>3</sup> Ultimate strength U = 50 Mpa.

Table 1: Comparison linear and nonlinear Results

Loading Weight (N)	Software deflection (mm) Linear	Software deflection (mm) Geometrical Nonlinear	Experimental deflection (mm)
Self weight	-	-	-
0.1	16.94	16.92	17
0.2	25.87	25.82	25.5
0.3	34.79	34.62	34.5
0.4	43.68	43.42	43.5
0.5	52.62	52.14	52
0.6	61.57	60.82	60.5
0.7	70.49	69.42	69.5
0.8	79.4	77.92	77.5
0.9	88.34	86.22	86.5
1	97.26	94.52	94.5

### III. CONCLUSION

After conducting the experiment to validate geometrical nonlinearity, it was found that significant nonlinearity effects become apparent after a 60 mm deflection, with differences between linear and nonlinear results exceeding 1 mm. Experimental results align closely with software simulations. This highlights the necessity of accounting for geometrical nonlinearity throughout the analysis process. Ongoing research is focused on the nonlinear parameters of glass plates.

### REFERENCES

- [1] C B. Dolicanin, V.B.Nikolic and D.C. Dolicanin (2011), "Application of Finite Difference Method to Study of the Phenomenon in the Theory of Thin Plates".
- [2] Ali Ergun and Nahit Kumbasar (2011), "A new approach of improved finite difference scheme on plate bending analysis".
- [3] Debabrata Das, Prasanta Sahoo, Kashinath Saha; "study of static behaviour of thin isotropic skew plates under uniformly distributed load for various mixed flexural boundary conditions."
- [4] Allan Okodi, Yasin N. Ziraba, Jackson A. "Approximate Large Deflection Analysis of Thin Rectangular Plates under Distributed Lateral Line Load".
- [5] J. N. Reddy and E. J. Barbero "A Plate bending element based on a generalized laminated plate theory".
- [6] Yos Sompornjarosuk and Kraiwood Kiattikomol, "Bending behaviours of simply supported rectangular plates with an internal line sagged and unsagged supports"



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