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Exploring Integer Solutions of Negative Pell's Equation with Two-digit Keith Number

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Abstract: ***This work explores non-trivial integer solutions to the negative Pell's equation, specifically considering the two-digit Keith number. Furthermore, we identity interesting recurrence relations that emerge from these solutions.***
Keywords: Pell equation, Integer solutions, 2-digit Keith number, Negative Pell equation.

I. INTRODUCTION

Let $x^2 - Dy^2 = -1$ be the negative Pell equation where *D* is a non-negative integer that is not a perfect square. A necessary condition for solvability is that *D* is not divisible 4 or by a prime of form $4k+3.$ For example, $x^2 - 3y^2 = -1$ is never solvable, but $x^2 - 5y^2 = -1$ may be solvable.

This paper explores the negative Pell equation in connection with Keith number specifically the equation $x^2 = 75y^2 - 11, y \in N$ is exanimate for its distinct non- zero integer solutions along with the corresponding recurrence relation.

II. BASIC DEFINITION

A Keith number is a special type of number that appears in a Fibonacci-like sequence generated from its own digits. It is also called a repfigit number. Some of the Keith numbers are 14, 19, 28, 47, 61, 75, 197, 742, …

III. METHOD OF ANALYSIS

Consider the negative Pell equation $x^2 = 75y^2 - 11$ (1)

Let (x_0, y_0) be the initial solution of (1) is given by

$$x_0 = 8, y_0 = 1$$

To discover the other solutions of (1), consider the Pell Equation

$$x^2 = 75y^2 + 1$$

The initial solution is $\tilde{x}_0 = 26, \tilde{y}_0 = 3$ is given by

$$\tilde{x}_n = \frac{1}{2} f_n$$

$$\tilde{y}_n = \frac{1}{2\sqrt{75}} g_n$$

where,

$$f_n = \left(26 + 3\sqrt{75}\right)^{n+1} + \left(26 - 3\sqrt{75}\right)^{n+1}$$

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$$g_n = \left(26+3\sqrt{75}\right)^{n+1} - \left(26-3\sqrt{75}\right)^{n+1}$$

Applying the Brahma Gupta Lemma between (x_0, y_0) and $(\tilde{x}_0, \tilde{y}_0)$, the sequence of non-zero distinct integer solutions are obtained as

$$x_{n+1} = \frac{1}{2}\left[8f_n + \sqrt{75}g_n\right]$$

$$y_{n+1} = \frac{1}{2\sqrt{75}}\left[\sqrt{75}f_n + 8g_n\right]$$

The recurrence relation from the above solutions are obtained as

$$x_{n+3} - 52x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 52y_{n+2} + y_{n+1} = 0$$

IV. CONCLUSION

This paper explores integer solutions to the negative Pell equation involving Keith number. The diverse nature of Diophantine equation, future research can extend this approach by investigating other types of Keith number within different Diophantine equations.

REFERENCES

[1] AI - Zaid. H, Brindza .B and Printer .A, "On Positive integer solutions of the Equation $xy + yz + zn = n''$, Canad.Math.Bull, Vol.39,pp.199,1996.

[2] Bhanumurthy T.S, Ancient Indian Mathematics , New Age International Published Limited, New Delhi,1995.

[3] Borevich Z.I and Shafarevich I.R, Number Theory, Academic press, New York,1968.

[4] Boyer C.B, "A History of Mathematics ", John Wiley & Sons Inc., New York, 1966.

[5] Brown .E, "Diophantine Equation of the from $x^2 + D = y^n$ ",J. ReineAngew.Math,Vol.274/275,pp.385- 389,1975.

[6] Carmichael R.D, "Theory of Number and Diophantine Analysis", Dover Publications Inc., New York, 1959.

[7] Davenport Harold, "The Higher Arithmetic: An Introduction to the Theory of Number", 7^{th} Edition, Cambridge University Press, 1999.

[8] Dudley .U, "Elementary Number Theory", W.H. Freeman and Co., New York, 1969.

[9] Gopal M.A, Sangeetha .V and Manju Somanath, "On the integer solution of the Pell equation $x^2 = 13y^2 - 3^t$ ", International Journal of Applied Mathematical Research, Vol.3,No.1,pp.58-64,2014.

[10] S.Vidhya and G.Janaki, "An integral solution of negative Pell's equation involving two digit sphenic numbers", International Journal of Computer Science and Engineering, vol 6, Issue 7, Pp: 444-445, July 2018.

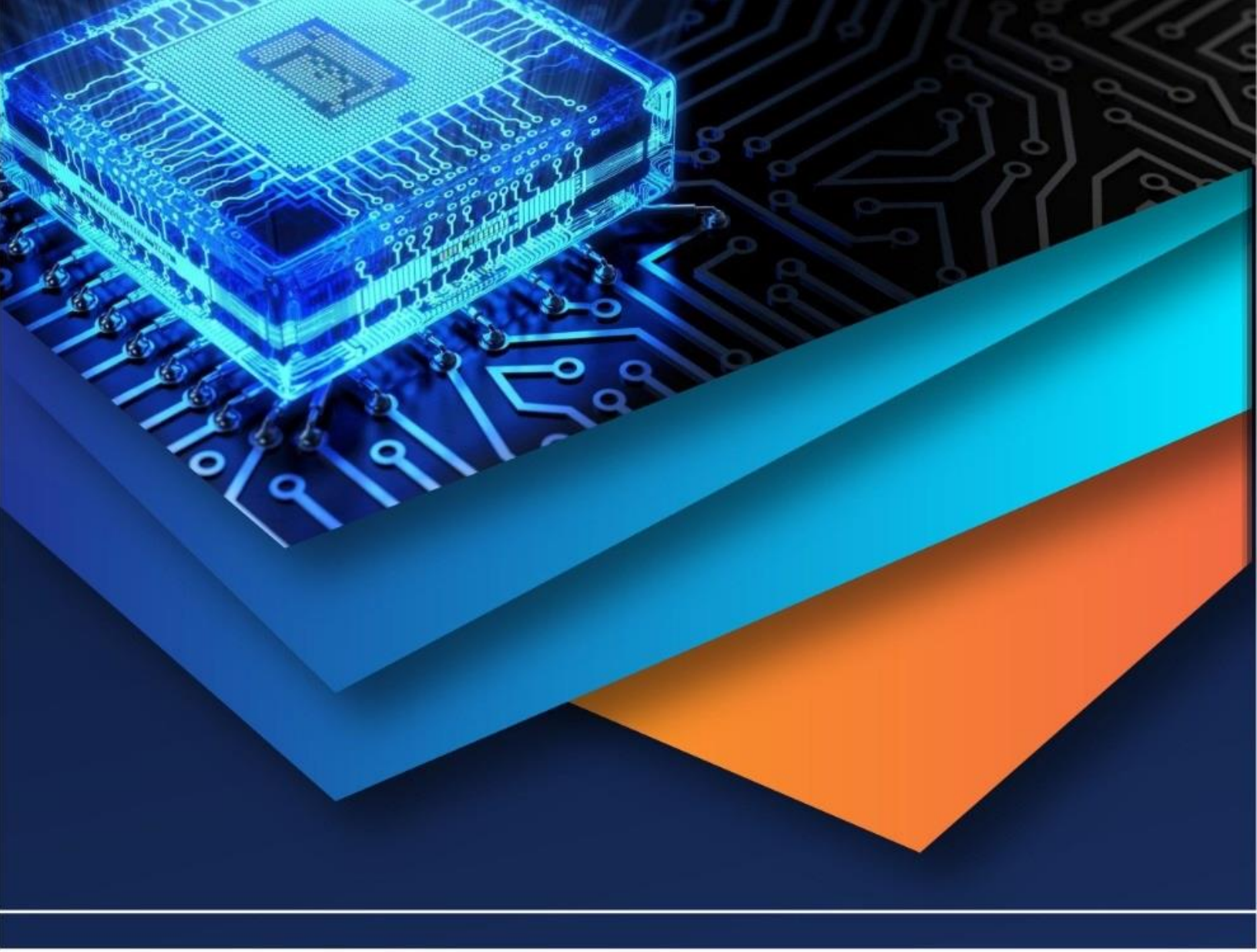
[11] G Janaki, S Vidhya "Observation on $y^2 = 6x^2 + 1$ ", International Journal of mathematics, Vol.2,Issue.3,pp.04-05,2017.

[12] G.Janaki and S.Vidhya, "On the integer solutions of Pell equation $x^2 - 79y^2 = 9^k$ ", International Journal of Scientific Research in Science, Engineering and Technology, Vol.2, Issue.2 , pp.1195-1197, 2016.

[13] G.Janaki and S.Vidhya, "On the integer solutions of the Pell equation $x^2 = 20y^2 - 4^t$ ", International Journal of Multidisciplinary Research and Development, Vol.3, Issue.5, pp.39-42, 2016.

[14] G.Janaki and S.Vidhya, "On the negative Pell equation $y^2 = 21x^2 - 3$ ", International Journal of Applied Research, Vol.2, Issue.11, pp.462-466, 2016.

[15] S.Vidhya and G.Janaki, "Observation on $y^2 = 11x^2 + 1$ ". International Journal for Science and Advance Research in Technology, Vol.5, Isssue.12, pp.232-233, 2019.



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