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Exploring Integer Solutions of Negative Pell's Equation with Two-digit Keith Number

S. Vidhya[1], N. Priyabharathy[2]
[1]*Associate Professor, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 620018.*
[2]*PG Student, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 620018.*

Abstract: This work explores non-trivial integer solutions to the negative Pell's equation, specifically considering the two-digit Keith number. Furthermore, we identity interesting recurrence relations that emerge from these solutions.
Keywords: Pell equation, Integer solutions, 2-digit Keith number, Negative Pell equation.

I. INTRODUCTION

Let $x^2 - Dy^2 = -1$ be the negative Pell equation where D is a non-negative integer that is not a perfect square. A necessary condition for solvability is that D is not divisible 4 or by a prime of form $4k+3$. For example, $x^2 - 3y^2 = -1$ is never solvable, but $x^2 - 5y^2 = -1$ may be solvable.

This paper explores the negative Pell equation in connection with Keith number specifically the equation $x^2 = 75y^2 - 11, y \in N$ is exanimate for its distinct non- zero integer solutions along with the corresponding recurrence relation.

II. BASIC DEFINITION

A Keith number is a special type of number that appears in a Fibonacci-like sequence generated from its own digits. It is also called a repfigit number. Some of the Keith numbers are 14, 19, 28, 47, 61, 75, 197, 742, …

III. METHOD OF ANALYSIS

Consider the negative Pell equation $x^2 = 75y^2 - 11$ (1)

Let (x_0, y_0) be the initial solution of (1) is given by

$$x_0 = 8, y_0 = 1$$

To discover the other solutions of (1), consider the Pell Equation

$$x^2 = 75y^2 + 1$$

The initial solution is $\tilde{x}_0 = 26, \tilde{y}_0 = 3$ is given by

$$\tilde{x}_n = \frac{1}{2} f_n$$

$$\tilde{y}_n = \frac{1}{2\sqrt{75}} g_n$$

where,

$$f_n = \left(26 + 3\sqrt{75}\right)^{n+1} + \left(26 - 3\sqrt{75}\right)^{n+1}$$

$$g_n = \left(26+3\sqrt{75}\right)^{n+1} - \left(26-3\sqrt{75}\right)^{n+1}$$

Applying the Brahma Gupta Lemma between $\left(x_0, y_0\right)$ and $\left(\tilde{x}_0, \tilde{y}_0\right)$, the sequence of non-zero distinct integer solutions are obtained as

$$x_{n+1} = \frac{1}{2}\left[8 f_n + \sqrt{75} g_n\right]$$

$$y_{n+1} = \frac{1}{2\sqrt{75}}\left[\sqrt{75} f_n + 8 g_n\right]$$

The recurrence relation from the above solutions are obtained as

$$x_{n+3} - 52 x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 52 y_{n+2} + y_{n+1} = 0$$

IV. CONCLUSION

This paper explores integer solutions to the negative Pell equation involving Keith number. The diverse nature of Diophantine equation, future research can extend this approach by investigating other types of Keith number within different Diophantine equations.

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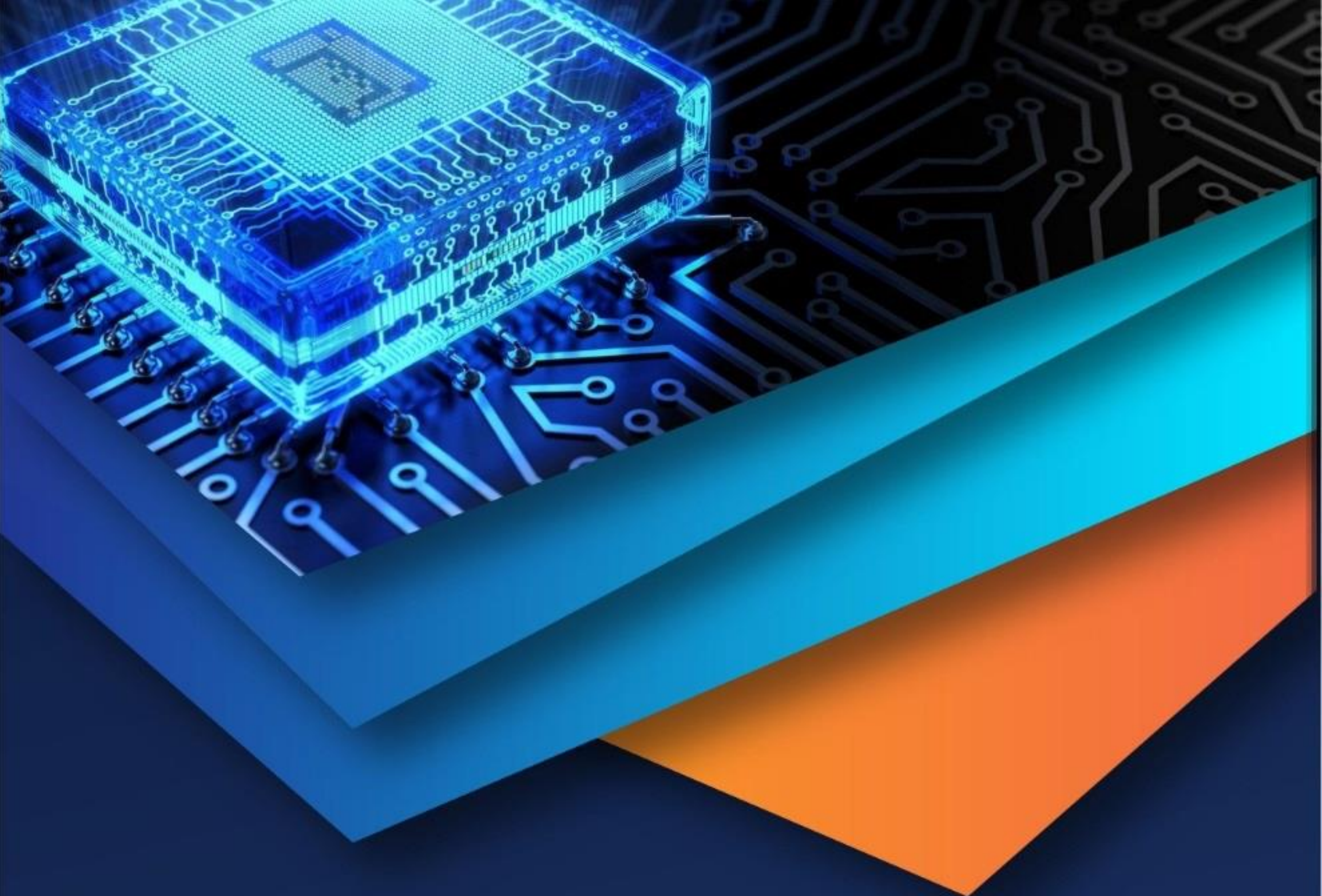

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