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Exposure of Positive Inte ger Solutions to an Equation $l^{\alpha} + m^{\beta} = \omega^{\gamma}$ Comprising Kaprekar Numbers

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Abstract: In this manuscript, the technique of evaluating all possible positive integer solutions to the Kaprekar number and an even perfect number of base 2 based on an equation $l^{\alpha} + m^{\beta} = \omega^{\gamma}$ is enlightened.
Keywords: Diophantine equation, Even perfect number, Kaprekar number.

I. INTRODUCTION

A number sequence is an ordered list of numbers that follow a specific rule or pattern. There are abundant research studies in the literature that concern number sequences, see [1 − 4]. In [6], Saranya. C and Janaki. G presented an approach to finding an infinite number of integer solutions to the generalized Fermat equations using Jarasandha numbers. In [10], Pandichelvi.V and Vanaja.R exhibited that there is no solution for the equation $p^x + (p+2)^y = z^2$ together with the condition of $x + y$. In [11], the authors analysed an equation $(3u^2 + 5)p + (6u^2 + 11)q = w^2$ wtth $p + q = 0,1,2,3$. In [12], an exponential Diophantine equation $q^x + p^y = z^2$ where p is a Safe prime, q is a Sophie Germain prime and x, y, z are non-negative integers such that $x + y = 0,1,2,3$. is scrutinized. For a more comprehensive study of numbers, one may refer [5,7 − 9].

In this manuscript, an integer solution to the equation $l^{\alpha} + m^{\beta} = \omega^{\gamma}$ concerning a Kaprekar number and an even perfect number of base 2, a special type of Kaprekar number is exposed.

II. PRELIMINARIES

A. Even Perfect Number

Every number of the form $2^{p-1}(2^p - 1)$ where $2^p - 1$ is a Mersenne prime number for all prime values of p is called an even perfect number. Some even perfect numbers are 6, 28, 496, 8128, 33550336,etc

B. Kaprekar Number

A Kaprekar number is a number whose square is split up into two pieces such that their sum equal to the original number and in which none of the sections has a value of zero.

Take a look at an illustration, 7272 is a Kaprekar number because the square of 7272 is 52881984; divide it into two equal parts 5288 and 1984; and then add the two parts together 5288 + 1984 to get back to 7272.

Here are a few examples.

$9^2 = 81$, $8 + 1 = 9$;
$45^2 = 2025$, $20 + 25 = 45$;
$55^2 = 3025$ $30 + 25 = 55$.

C. Special Kind of Kaprekar Number

An even perfect number is a Kaprekar number in base 2.

Example: 6 is an even perfect number, which is a Kaprekar number in base 2.

It is known that $6_2 = 110$

Then,

$110^2 = 100100 = 10 + 0100 = 110$

$110^2 = 110$

Hence, 6 is a Kaprekar number in base 2.

III. TECHNIQUE OF ANALYSIS

In this section, the technique for establishing numerous collections of positive integer solutions to the equation $l^\alpha + m^\beta = \omega^\gamma$ where k is a Kaprekar number is analysed in the following theorems.

A. *Theorem 3.1*

If k is a Kaprekar number, then the set of positive integer solutions of $l^\alpha + m^\beta = \omega^\gamma$ are

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(k+1)k^2t}\left(a^{(k+1)k^2t}+1\right)^{k+1} \\ a^{(k-2)k^2t}\left(a^{(k+1)k^2t}+1\right)^{k-1} \\ a^{(k+1)(k-2)t}\left(a^{(k+1)k^2t}+1\right) \end{pmatrix}$$

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(k+2)k^2t}\left(a^{(k-1)k^2t}+1\right)^{k+1} \\ a^{(k-1)k^2t}\left(a^{(k-1)k^2t}+1\right)^{k-1} \\ a^{(k^2-1)t}\left(a^{(k-1)k^2t}+1\right) \end{pmatrix}$$

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(k+1)(k^2+1)t}\left(a^{(k^2-1)t}+1\right)^{k+1} \\ a^{(k-1)k^2t}\left(a^{(k^2-1)t}+1\right)^{k-1} \\ a^{(k^2-1)t}\left(a^{(k^2-1)t}+1\right) \end{pmatrix}$$

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(k-1)^2kt}\left(a^{(k-1)^2kt}+1\right)^{k-1} \\ a^{(k-1)k^2t}\left(a^{(k-1)^2kt}+1\right)^{k+1} \\ a^{(k-1)^2t}\left(a^{(k-1)^2kt}+1\right) \end{pmatrix}$$

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{k^3t}\left(a^{(k+1)k^2t}+1\right)^{k-1} \\ a^{(k+1)k^2t}\left(a^{(k+1)k^2t}+1\right)^{k+1} \\ a^{(k^2-1)t}\left(a^{(k+1)k^2t}+1\right) \end{pmatrix} \quad \text{and}$$

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(k-1)(k^2+1)t}\left(a^{(k^2-1)t}+1\right)^{k-1} \\ a^{(k+1)k^2t}\left(a^{(k^2-1)t}+1\right)^{k+1} \\ a^{(k^2-1)t}\left(a^{(k^2-1)t}+1\right) \end{pmatrix}$$

Proof

The equation to be analysed is $l^\alpha + m^\beta = \omega^\gamma$ (3.1)

where l, m, ω belongs to Z^+ .

Let us consider the following cases.

1) *Case 1*

Let $\alpha = k - 1$, $\beta = k + 1$ and $\gamma = k^2$ where k is a Kaprekar number. Then the above equation becomes,

$$l^{k-1} + m^{k+1} = \omega^{k^2} \tag{3.2}$$

Subcase 1(i):

Assume that $l = a^x(a^x + 1)^{k+1}$, $= a^y(a^x + 1)^{k-1}$, $\omega = a^z(a^x + 1)$

where $x, y, z \in Z^+$ and $n > 0$

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Then, the original equation (3.2) can be transformed into the equation

$$a^{(k-1)x}(a^x+1)^{k^2-1}+a^{(k+1)y}(a^x+1)^{k^2-1}=a^{k^2 z}(a^x+1)^{k^2}$$

Equating the exponent on either side of the equation provides that

$$(k-1)x=x+k^2 \quad (3.3)$$

$$(k+1)y=k^2 z \quad (3.4)$$

It is deeply experiential that (3.3) and (3.4) are fulfilled by the succeeding chances of x, y and z

$$x=(k+1)k^2 t\,,\ y=(k-2)k^2 t \ \text{ and } \ z=(k+1)(k-2)t$$

Subsequently, the values of the variables l, m and ω are invented by

$$l=a^{(k+1)k^2 t}\left(a^{(k+1)k^2 t}+1\right)^{k+1}$$

$$m=a^{(k-2)k^2 t}\left(a^{(k+1)k^2 t}+1\right)^{k-1}$$

$$\omega=a^{(k+1)(k-2)t}\left(a^{(k+1)k^2 t}+1\right)$$

The following Table 3.1 displays specific numerical solutions for (3.1)

Table 3.1

k	a	t	l	m	ω
9	2	1	$2^{810}(2^{810}+1)^{10}$	$2^{567}(2^{810}+1)^{8}$	$2^{70}(2^{810}+1)$
45	3	2	$3^{186300}(3^{186300}+1)^{46}$	$3^{174150}(3^{186300}+1)^{44}$	$3^{3956}(3^{186300}+1)$
55	2	2	$2^{338800}(2^{338800}+1)^{56}$	$2^{320650}(2^{338800}+1)^{54}$	$2^{5936}(2^{338800}+1)$
99	2	3	$2^{2940300}(2^{2940300}+1)^{100}$	$2^{2852091}(2^{2940300}+1)^{98}$	$2^{29100}(2^{2940300}+1)$

The values of each side of (3.2) are illustrated in the following table

Table 3.1(i)

LHS of (3.2)	RHS of (3.2)
$2^{5670}(2^{810}+1)^{81}$	$2^{5670}(2^{810}+1)^{81}$
$3^{8010900}(3^{186300}+1)^{2025}$	$3^{8010900}(3^{186300}+1)^{2025}$
$2^{17956400}(2^{338800}+1)^{3025}$	$2^{17956400}(2^{338800}+1)^{3025}$
$2^{285209100}(2^{2940300}+1)^{9801}$	$2^{285209100}(2^{2940300}+1)^{9801}$

Subcase 1(ii):

Replacing the chances of l, m and ω by $l=a^x(a^y+1)^{k+1}$, $m=a^y(a^y+1)^{k-1}$ and $\omega=a^z(a^y+1)$ in (3.2) and employing the analogous approach described in subcase 1(i), the feasible values of x, y and z pointed out by

$$x=(k+2)k^2 t\,,\ y=(k-1)k^2 t \ \text{ and } \ z=(k^2-1)t$$

The corresponding triples (l, m, ω) satisfying the given equation is stated by

$$l=a^{(k+2)k^2 t}\left(a^{(k-1)k^2 t}+1\right)^{k+1}$$

$$m=a^{(k-1)k^2 t}\left(a^{(k-1)k^2 t}+1\right)^{k-1}$$

$$\omega=a^{(k^2-1)t}\left(a^{(k-1)k^2 t}+1\right)$$

Numerical specimens fulfilling (3.2) for few Kaprekar are shown in the following table 3.2

Table 3.2

k	a	t	l	m	ω
9	3	2	$3^{1782}(3^{1296}+1)^{10}$	$3^{1296}(3^{1296}+1)^{8}$	$3^{160}(3^{1296}+1)$
45	2	3	$2^{285525}(2^{267300}+1)^{46}$	$2^{267300}(2^{267300}+1)^{44}$	$2^{6072}(2^{267300}+1)$
55	2	2	$2^{344850}(2^{326700}+1)^{56}$	$2^{326700}(2^{326700}+1)^{54}$	$2^{6048}(2^{326700}+1)$
99	3	1	$3^{989901}(3^{960498}+1)^{100}$	$3^{960498}(3^{960498}+1)^{98}$	$3^{9800}(3^{960498}+1)$

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The following table displays the values of each side of (3.2)

Table 3.2(i)

LHS of (3.2)	RHS of (3.2)
$3^{12960}(3^{1296}+1)^{81}$	$3^{12960}(3^{1296}+1)^{81}$
$2^{12295800}(2^{267300}+1)^{2025}$	$2^{12295800}(2^{267300}+1)^{2025}$
$2^{18295200}(2^{326700}+1)^{3025}$	$2^{18295200}(2^{326700}+1)^{3025}$
$3^{96049800}(3^{960498}+1)^{9801}$	$3^{96049800}(3^{960498}+1)^{9801}$

Subcase 1(iii):

Interchange the values of l, m, ω in (3.2) as mentioned below

$$l = a^x(a^z+1)^{k+1},\ m = a^y(a^z+1)^{k-1} \text{ and } \omega = a^z(a^z+1)$$

Following the technique as explained in subcase 1(i), the possible values of x, y and z are evaluated by

$$x = (k+1)(k^2+1)t,\ y = (k-1)k^2t \text{ and } z = (k^2-1)t$$

Consequently, the assessment of l, m and ω are indicated by

$$l = a^{(k+1)(k^2+1)t}\left(a^{(k^2-1)t}+1\right)^{k+1}$$

$$m = a^{(k-1)k^2t}\left(a^{(k^2-1)t}+1\right)^{k-1}$$

$$\omega = a^{(k^2-1)t}\left(a^{(k^2-1)t}+1\right)$$

Verification of solutions with arithmetic values for little bit Kaprekar numbers are illustrated in the table 3.3

Table 3.3

k	a	t	l	m	ω
9	2	1	$2^{820}(2^{80}+1)^{10}$	$2^{648}(2^{80}+1)^{8}$	$2^{80}(2^{80}+1)$
45	3	2	$3^{186392}(3^{4048}+1)^{46}$	$3^{178200}(3^{4048}+1)^{44}$	$3^{4048}(3^{4048}+1)$
55	2	2	$2^{338912}(2^{6048}+1)^{56}$	$2^{326700}(2^{6048}+1)^{54}$	$2^{6048}(2^{6048}+1)$
99	2	3	$2^{2940600}(2^{29400}+1)^{100}$	$2^{2881494}(2^{29400}+1)^{98}$	$2^{29400}(2^{29400}+1)$

The values of (3.2) on either side are shown in the table below

Table 3.3(i)

LHS of (3.2)	RHS of (3.2)
$2^{6480}(2^{80}+1)^{81}$	$2^{6480}(2^{80}+1)^{81}$
$3^{8197200}(3^{4048}+1)^{2025}$	$3^{8197200}(3^{4048}+1)^{2025}$
$2^{18295200}(2^{6048}+1)^{3025}$	$2^{18295200}(2^{6048}+1)^{3025}$
$2^{288149400}(2^{29400}+1)^{9801}$	$2^{288149400}(2^{29400}+1)^{9801}$

2) Case 2:

Let $\alpha = k+1$, $\beta = k-1$ and $\gamma = k^2$ where k is a kaprekar number. Then the equation (3.1) becomes,

$$l^{k+1} + m^{k-1} = \omega^{k^2} \qquad (3.5)$$

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Subcase 2(i):

Assume that $l = a^x(a^x + 1)^{k-1}$, $m = a^y(a^x + 1)^{k+1}$ and $\omega = a^z(a^x + 1)$ in the place l, m and ω in (3.5) and appealing the corresponding methodology labelled in subcase 1(i), the reasonable value of $x, y,$ and z are scrutinized by

$x = (k-1)^2kt$, $y = (k-1)k^2t$ and $z = (k-1)^2t$

Hence the value of l, m and ω satisfying (3.5) are given by

$$l = a^{(k-1)^2kt}\left(a^{(k-1)^2kt} + 1\right)^{k-1}$$

$$m = a^{(k-1)k^2t}\left(a^{(k-1)^2kt} + 1\right)^{k+1}$$

$$\omega = a^{(k-1)^2t}\left(a^{(k-1)^2kt} + 1\right)$$

The following Table 3.4 shows some numerical instances for the Kaprekar numbers

Table 3.4

k	a	t	l	m	ω
9	3	2	$3^{1152}(3^{1152} + 1)^8$	$3^{1296}(3^{1152} + 1)^{10}$	$3^{128}(3^{1152} + 1)$
45	2	3	$2^{261360}(2^{261360} + 1)^{44}$	$2^{267300}(2^{261360} + 1)^{46}$	$2^{5808}(2^{261360} + 1)$
55	2	2	$2^{320760}(2^{320760} + 1)^{54}$	$2^{326700}(2^{320760} + 1)^{56}$	$2^{5832}(2^{320760} + 1)$
99	3	1	$3^{950796}(3^{950796} + 1)^{98}$	$3^{960498}(3^{950796} + 1)^{100}$	$3^{9604}(3^{950796} + 1)$

The table below offers illustrations illustrating the values for each side of (3.5)

Table 3.4(i)

LHS of (3.5)	RHS of (3.5)
$3^{10368}(3^{1152} + 1)^{81}$	$3^{10368}(3^{1152} + 1)^{81}$
$2^{11761200}(2^{261360} + 1)^{2025}$	$2^{11761200}(2^{261360} + 1)^{2025}$
$2^{17641800}(2^{320760} + 1)^{3025}$	$2^{17641800}(2^{320760} + 1)^{3025}$
$3^{9694128804}(3^{950796} + 1)^{9801}$	$3^{9694128804}(3^{950796} + 1)^{9801}$

Subcase 2(ii):

Adopting $l = a^x(a^y + 1)^{k-1}$, $m = a^y(a^y + 1)^{k+1}$ and $\omega = a^z(a^y + 1)$ instead of the values of l, m and ω in (3.5) and applying the procedure using in subcase 1(i), the above system of equation is fulfilled

$x = k^3t$, $y = (k+1)k^2t$ and $z = (k^2-1)t$

Therefore, the values of l, m and ω satiating (3.5) are given by,

$$l = a^{k^3t}\left(a^{(k+1)k^2t} + 1\right)^{k-1}$$

$$m = a^{(k+1)k^2t}\left(a^{(k+1)k^2t} + 1\right)^{k+1}$$

$$\omega = a^{(k^2-1)t}\left(a^{(k+1)k^2t} + 1\right)$$

Table 3.5 provides numerical verification for specific values of the Kaprekar number.

Table 3.5

	a	t	l	m	ω
9	3	2	$3^{1458}(3^{1620} + 1)^8$	$3^{1296}(3^{1620} + 1)^{10}$	$3^{160}(3^{1620} + 1)$
45	2	3	$2^{273375}(2^{279450} + 1)^{44}$	$2^{267300}(2^{279450} + 1)^{46}$	$2^{132}(2^{279450} + 1)$
55	2	2	$2^{332750}(2^{338800} + 1)^{54}$	$2^{267300}(2^{338800} + 1)^{56}$	$2^{132}(2^{338800} + 1)$
99	3	1	$3^{970299}(3^{980100} + 1)^{98}$	$3^{980100}(3^{980100} + 1)^{100}$	$3^{9800}(3^{980100} + 1)$

The table below displays the values of the equation (3.5) on both sides

Table 3.5(i)

LHS of (3.5)	RHS of (3.5)
$3^{12960}(3^{1620}+1)^{81}$	$3^{12960}(3^{1620}+1)^{81}$
$2^{12295800}(2^{279450}+1)^{2025}$	$2^{12295800}(2^{279450}+1)^{2025}$
$2^{18295200}(2^{338800}+1)^{3025}$	$2^{18295200}(2^{338800}+1)^{3025}$
$3^{96049800}(3^{980100}+1)^{9801}$	$3^{96049800}(3^{980100}+1)^{9801}$

Subcase 2(iii):

Agreeing $l = a^x(a^z+1)^{k-1}$, $m = a^y(a^z+1)^{k+1}$ and $\omega = a^z(a^z+1)$ in (3.5) and recapping the analogous procedure as illuminated in subcase1(i), the significant values of x, y and z are

$$x = (k-1)(k^2+1)t,\ y = (k+1)k^2t \quad \text{and}\ z = (k^2-1)t$$

Consequently, the assessment of l, m and ω gratifying (3.5) are indicated by

$$l = a^{(k-1)(k^2+1)t}\left(a^{(k^2-1)t}+1\right)^{k-1}$$

$$m = a^{(k+1)k^2t}\left(a^{(k^2-1)t}+1\right)^{k+1}$$

$$\omega = a^{(k^2-1)t}\left(a^{(k^2-1)t}+1\right)$$

Integer solution for few numerical cases of Kaprekar numbers are presented in Table 3.6 below

Table 3.6

k	a	t	l	m	ω
9	2	1	$2^{656}(2^{80}+1)^{8}$	$2^{810}(2^{80}+1)^{10}$	$2^{80}(2^{80}+1)$
45	3	2	$3^{178288}(3^{4048}+1)^{44}$	$3^{186300}(3^{4048}+1)^{46}$	$3^{4048}(3^{4048}+1)$
55	2	2	$2^{326808}(2^{6048}+1)^{54}$	$2^{338800}(2^{6048}+1)^{56}$	$2^{6048}(2^{6048}+1)$
99	3	3	$3^{2881788}(3^{29400}+1)^{98}$	$3^{2940300}(3^{29400}+1)^{100}$	$3^{29400}(3^{29400}+1)$

The values of (3.5) on either side are shown in the following table

Table 3.6(i)

LHS of (3.5)	RHS of (3.5)
$2^{6480}(2^{80}+1)^{81}$	$2^{6480}(2^{80}+1)^{81}$
$3^{8197200}(3^{4048}+1)^{2025}$	$3^{8197200}(3^{4048}+1)^{2025}$
$2^{18295200}(2^{6048}+1)^{3025}$	$2^{18295200}(2^{6048}+1)^{3025}$
$3^{288149400}(3^{29400}+1)^{9801}$	$3^{288149400}(3^{29400}+1)^{9801}$

B. *Theorem* 3.2

If k is an even perfect number in base 2, a special sort of Kaprekar number, then all positive integers solution for $l^{k-1} + m^{k+1} = \omega^{k^2}$ are

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}\left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}+1\right)^{2^{2p-1}-2^{p-1}+1}, \\ a^{(2^{2p-1}-2^{p-1}-2)(2^{2p-1}-2^{p-1})^2 t}\left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}+1\right)^{2^{2p-1}-2^{p-1}-1}, \\ a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1}-2)t}\left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}+1\right) \end{pmatrix}$$

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(2^{2p-1}-2^{p-1}+2)(2^{2p-1}-2^{p-1})^2 t}\left(a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t}+1\right)^{(2^{2p-1}-2^{p-1}+1)}, \\ a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t}\left(a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t}+1\right)^{(2^{2p-1}-2^{p-1}-1)}, \\ a^{\left((2^{2p-1}-2^{p-1})^2-1\right)t}\left(a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t}+1\right) \end{pmatrix}$$

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(2^{2p-1}-2^{p-1}+1)\left((2^{2p-1}-2^{p-1})^2+1\right)t}\left(a^{\left((2^{2p-1}-2^{p-1})^2-1\right)t}+1\right)^{2^{2p-1}-2^{p-1}+1}, \\ a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t}\left(a^{\left((2^{2p-1}-2^{p-1})^2-1\right)t}+1\right)^{2^{2p-1}-2^{p-1}-1}, \\ a^{((2^{2p-1}-2^{p-1})^2-1)t}\left(a^{((2^{2p-1}-2^{p-1})^2-1)t}+1\right) \end{pmatrix}$$

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t}\left(a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t}+1\right)^{2^{2p-1}-2^{p-1}-1} \\ a^{(2^{2p-1}-2^{p-1}-1)k^2 t}\left(a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t}+1\right)^{2^{2p-1}-2^{p-1}+1} \\ a^{(2^{2p-1}-2^{p-1}-1)^2 t}\left(a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t}+1\right) \end{pmatrix}$$

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(2^{2p-1}-2^{p-1})^3 t}\left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}+1\right)^{2^{2p-1}-2^{p-1}-1} \\ a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}\left(a^{(2^{2p-1}-2^{p-1}+1)k^2 t}+1\right)^{2^{2p-1}-2^{p-1}+1} \\ a^{((2^{2p-1}-2^{p-1})^2-1)t}\left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}+1\right) \end{pmatrix}$$

$$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix} = \begin{pmatrix} a^{(2^{2p-1}-2^{p-1}-1)((2^{2p-1}-2^{p-1})^2+1)t}\left(a^{((2^{2p-1}-2^{p-1})^2-1)t}+1\right)^{2^{2p-1}-2^{p-1}-1} \\ a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}\left(a^{((2^{2p-1}-2^{p-1})^2-1)t}+1\right)^{2^{2p-1}-2^{p-1}+1} \\ a^{\left((2^{2p-1}-2^{p-1})^2-1\right)t}\left(a^{((2^{2p-1}-2^{p-1})^2-1)t}+1\right) \end{pmatrix}$$

Proof

The equation to be analysed is $l^{\alpha} + m^{\beta} = \omega^{\gamma}$ (3.6)

where l, m, ω belongs to Z^{+} .

Let us deliberate on the subsequent cases.

1) *Case 1*

Let $\alpha = k - 1$, $\beta = k + 1$ and $\gamma = k^2$ where k is a Kaprekar number which is an even perfect number in base 2. That is $k = 2^{p-1}(2^p - 1)$ in base 2 where p is a prime number.

Then the above equation (3.6) becomes,

$$l^{k-1} + m^{k+1} = \omega^{k^2} \qquad (3.7)$$

Subcase 1(i):

Acknowledge that $= a^x(a^x + 1)^{2^{2p-1}-2^{p-1}+1}$, $m = a^y(a^x + 1)^{2^{2p-1}-2^{p-1}-1}$

$\omega = a^z(a^x + 1)$ in (3.7) and repeating the corresponding technique as illuminated in the Theorem 3.1, the preference of x, y and z are

$$x = (2^{2p-1} - 2^{p-1} + 1)(2^{2p-1} - 2^{p-1})^2 t$$
$$y = (2^{2p-1} - 2^{p-1} - 2)(2^{2p-1} - 2^{p-1})^2 t$$
$$z = (2^{2p-1} - 2^{p-1} + 1)(2^{2p-1} - 2^{p-1} - 2)t$$

and hence

$$l = a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}\left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}+1\right)^{2^{2p-1}-2^{p-1}+1}$$

$$m = a^{(2^{2p-1}-2^{p-1}-2)(2^{2p-1}-2^{p-1})^2 t}\left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}+1\right)^{2^{2p-1}-2^{p-1}-1}$$

$$\omega = a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1}-2)t}\left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}+1\right)$$

For relevant choices of k, a few statistical examples are explicitly described in the next table 3.7

Table 3.7

p	a	t	k	$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix}$	LHS and RHS of $(\mathbf{3.7})$
2	2	1	6	$\begin{pmatrix} 2^{252}(2^{252}+1)^7 \\ 2^{144}(2^{252}+1)^5 \\ 2^{28}(2^{252}+1) \end{pmatrix}$	$2^{1008}(2^{252}+1)^{36}$
3	3	3	28	$\begin{pmatrix} 3^{68208}(3^{68208}+1)^{29} \\ 3^{61152}(3^{68208}+1)^{27} \\ 3^{2262}(3^{68208}+1) \end{pmatrix}$	$3^{96049800}(3^{9800}+1)^{9801}$
5	2	1	496	$\begin{pmatrix} 2^{122269952}(2^{122269952}+1)^{497} \\ 2^{121531904}(2^{122269952}+1)^{495} \\ 2^{245518}(2^{122269952}+1) \end{pmatrix}$	$2^{1992009996000}(2^{1996000}+1)^{998001}$

Subcase 1(ii):

Implementing $= a^x(a^y+1)^{(2^{2p-1}-2^{p-1}+1)}$, $m = a^y(a^y+1)^{(2^{2p-1}-2^{p-1}-1)}$

$\omega = a^z(a^y+1)$ in (3.7) and the possible values of x, y and z are determined by using the similar style presented in Theorem 3.1,

$$x = (2^{2p-1}-2^{p-1}+2)(2^{2p-1}-2^{p-1})^2 t$$
$$y = (2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t$$
$$z = ((2^{2p-1}-2^{p-1})^2-1)$$

The subsequent solutions $l, m,$ and ω satisfying the given equation $l^{k-1}+m^{k+1}=\omega^{k^2}$ are

$$l = a^{(2^{2p-1}-2^{p-1}+2)(2^{2p-1}-2^{p-1})^2 t}\left(a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t}+1\right)^{(2^{2p-1}-2^{p-1}+1)}$$
$$m = a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t}\left(a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t}+1\right)^{(2^{2p-1}-2^{p-1}-1)}$$
$$\omega = a^{\left((2^{2p-1}-2^{p-1})^2-1\right)t}\left(a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t}+1\right)$$

The next table 3.8 explicitly lists a few statistical evidence for applicable Kaprekar numeral selections.

Table 3.8

p	a	t	k	$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix}$	LHS and RHS of (3.7)
2	2	3	6	$\begin{pmatrix} 2^{864}(2^{540}+1)^7 \\ 2^{540}(2^{540}+1)^5 \\ 2^{105}(2^{540}+1) \end{pmatrix}$	$2^{3780}(2^{540}+1)^{36}$
3	3	2	28	$\begin{pmatrix} (3^{47040}(3^{42336}+1)^{29} \\ 3^{42336}(3^{42336}+1)^{27} \\ 3^{1566}(3^{42336}+1) \end{pmatrix}$	$3^{1227744}(3^{42336}+1)^{784}$
5	2	1	496	$\begin{pmatrix} 2^{122515968}(2^{121777920}+1)^{497} \\ 2^{121777920}(2^{121777920}+1)^{495} \\ 2^{246015}(2^{121777920}+1) \end{pmatrix}$	$2^{60523626240}(2^{121777920}+1)^{246016}$

Subcase 1(iii):

Approving $= a^x(a^z+1)^{2^{2p-1}-2^{p-1}+1}$, $m = a^y(a^z+1)^{2^{2p-1}-2^{p-1}-1}$, $\omega = a^z(a^z+1)$ in (3.7) and following an approach described in the Theorem 3.1, the potential values of x, y and z are

$$x = (2^{2p-1}-2^{p-1}+1)((2^{2p-1}-2^{p-1})^2+1)t\,,$$
$$y = (2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t \quad \text{and}$$

$z = ((2^{2p-1} - 2^{p-1})^2 - 1)t$

Accordingly, the calculation of l, m and ω are specified by

$$l = a^{(2^{2p-1}-2^{p-1}+1)((2^{2p-1}-2^{p-1})^2+1)t}\left(a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1\right)^{2^{2p-1}-2^{p-1}+1},$$

$$m = a^{(2^{2p-1}-2^{p-1}-1)(2^{2p-1}-2^{p-1})^2 t}\left(a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1\right)^{2^{2p-1}-2^{p-1}-1} \text{ and}$$

$$\omega = a^{((2^{2p-1}-2^{p-1})^2-1)t}\left(a^{((2^{2p-1}-2^{p-1})^2-1)t} + 1\right)$$

A few precise cases in point for the Kaprekar numbers are listed in table 3.9 below.

Table 3.9

p	a	t	k	$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix}$	LHS and RHS of (3.7)
2	2	3	6	$\begin{pmatrix} 2^{259}(2^{35}+1)^7 \\ 2^{180}(2^{35}+1)^5 \\ 2^{35}(2^{35}+1) \end{pmatrix}$	$2^{1260}(2^{35}+1)^{36}$
3	3	2	28	$\begin{pmatrix} 3^{45530}(3^{1566}+1)^{29} \\ 3^{42336}(3^{1566}+1)^{27} \\ 3^{1566}(3^{1566}+1) \end{pmatrix}$	$3^{1227744}(3^{1566}+1)^{784}$
5	2	1	496	$\begin{pmatrix} 2^{366811347}(2^{738045}+1)^{497} \\ 2^{365333760}(2^{738045}+1)^{495} \\ 2^{738045}(2^{738045}+1) \end{pmatrix}$	$2^{181570878720}(2^{738045}+1)^{246016}$

2) Case 2:

Let $\alpha = k+1$, $\beta = k-1$ and $\gamma = k^2$ where $k = 2^{2p-1} - 2^{p-1}$.

Then the given equation turns out to be

$$l^{k+1} + m^{k-1} = \omega^{k^2} \qquad (3.8)$$

Subcase 2(i):

Assume that

$l = a^x(a^x + 1)^{2^{2p-1}-2^{p-1}-1}$,

$m = a^y(a^x + 1)^{2^{2p-1}-2^{p-1}+1}$,

$\omega = a^z(a^x + 1)$ in equation (3.8) and employing the corresponding process established by the Theorem 3.1, the suitable values of x, y and z are deliberated as

$x = (2^{2p-1} - 2^{p-1} - 1)^2(2^{2p-1} - 2^{p-1})t$,

$y = (2^{2p-1} - 2^{p-1} - 1)(2^{2p-1} - 2^{p-1})^2 t$ and

$z = \left((2^{2p-1} - 2^{p-1}) - 1\right)^2 t$

Hence the value of l, m and ω satisfying (3.8) are given by

$$l = a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t}\left(a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} + 1\right)^{2^{2p-1}-2^{p-1}-1}$$

$$m = a^{(2^{2p-1}-2^{p-1}-1)k^2 t}\left(a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} + 1\right)^{2^{2p-1}-2^{p-1}+1}$$

$$\omega = a^{(2^{2p-1}-2^{p-1}-1)^2 t}\left(a^{(2^{2p-1}-2^{p-1}-1)^2(2^{2p-1}-2^{p-1})t} + 1\right)$$

The following table 3.10 shows some numerical instances for the Kaprekar numbers.

Table 3.10

p	a	t	k	$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix}$	LHS and RHS of $(\mathbf{3.8})$
2	2	3	6	$\begin{pmatrix} 2^{450}(2^{450}+1)^5 \\ 2^{540}(2^{450}+1)^7 \\ 2^{75}(2^{450}+1) \end{pmatrix}$	$2^{2700}(2^{450}+1)^{36}$
3	3	2	28	$\begin{pmatrix} 3^{40824}(3^{40824}+1)^{27} \\ 3^{42336}(3^{40824}+1)^{29} \\ 3^{1458}(3^{40824}+1) \end{pmatrix}$	$3^{1143072}(3^{40824}+1)^{784}$
5	4	2	496	$\begin{pmatrix} 4^{243064800}(4^{243064800}+1)^{495} \\ 4^{243555840}(4^{243064800}+1)^{497} \\ 4^{490050}(4^{243064800}+1) \end{pmatrix}$	$4^{120560140800}(4^{243064800}+1)^{246016}$

Subcase 2(ii):

Adopting $= a^x(a^y+1)^{2^{2p-1}-2^{p-1}-1}$, $m = a^y(a^y+1)^{2^{2p-1}-2^{p-1}+1}$, $\omega = a^z(a^y+1)$ in (3.8) and applying the procedure used in Theorem.3.1, the above system is satisfied when

$x = (2^{2p-1} - 2^{p-1})^3 t$,

$y = (2^{2p-1} - 2^{p-1} + 1)k^2 t$ and

$z = ((2^{2p-1} - 2^{p-1})^2 - 1)t$

Therefore, the value of l, m and ω gratifying (3.8) are given by

$$l = a^{(2^{2p-1}-2^{p-1})^3 t}\left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}+1\right)^{2^{2p-1}-2^{p-1}-1}$$

$$m = a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}\left(a^{(2^{2p-1}-2^{p-1}+1)k^2 t}+1\right)^{2^{2p-1}-2^{p-1}+1}$$

$$\omega = a^{((2^{2p-1}-2^{p-1})^2-1)t}\left(a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}+1\right)$$

The next table 3.11 explicitly displays a few pieces of evidence for suitable Kaprekar numeral choices.

Table 3.11

p	a	t	k	$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix}$	LHS and RHS of $(\mathbf{3.8})$
2	2	3	6	$\begin{pmatrix} (2^{648}(2^{756}+1)^5 \\ 2^{756}(2^{756}+1)^7 \\ 2^{105}(2^{756}+1) \end{pmatrix}$	$2^{3780}(2^{756}+1)^{36}$
3	3	2	28	$\begin{pmatrix} 3^{43904}(3^{45472}+1)^{27} \\ 3^{45472}(3^{45472}+1)^{29} \\ 3^{1566}(3^{45472}+1) \end{pmatrix}$	$3^{1227744}(3^{45472}+1)^{784}$
5	5	1	496	$\begin{pmatrix} 5^{122023936}(5^{122269952}+1)^{495} \\ 5^{122269952}(5^{122269952}+1)^{497} \\ 5^{246015}(5^{122269952}+1) \end{pmatrix}$	$5^{60523626240}(5^{122269952}+1)^{246016}$

Subcase 2(iii):

Agreeing $l = a^x(a^z+1)^{2^{2p-1}-2^{p-1}-1}$, $m = a^y(a^z+1)^{2^{2p-1}-2^{p-1}+1}$ and $\omega = a^z(a^z+1)$ in (3.8) and repeating the equivalent method as shown in the Theorem 3.1, x, y and z are acknowledged as

$x = (2^{2p-1} - 2^{p-1} - 1)((2^{2p-1} - 2^{p-1})^2 + 1)t$,

$y = (2^{2p-1} - 2^{p-1} + 1)(2^{2p-1} - 2^{p-1})^2 t$ and

$z = ((2^{2p-1} - 2^{p-1})^2 - 1)t$

Consequently, the assessment of l, m and ω satisfying (3.8) are indicated by,

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$$l = a^{(2^{2p-1}-2^{p-1}-1)((2^{2p-1}-2^{p-1})^2+1)t}\left(a^{((2^{2p-1}-2^{p-1})^2-1)t}+1\right)^{2^{2p-1}-2^{p-1}-1}$$

$$m = a^{(2^{2p-1}-2^{p-1}+1)(2^{2p-1}-2^{p-1})^2 t}\left(a^{((2^{2p-1}-2^{p-1})^2-1)t}+1\right)^{2^{2p-1}-2^{p-1}+1}$$

$$\omega = a^{((2^{2p-1}-2^{p-1})^2-1)t}\left(a^{((2^{2p-1}-2^{p-1})^2-1)t}+1\right)$$

Numerical verification for certain values of the Kaprekar number offered in the table 3.12

Table 3.12

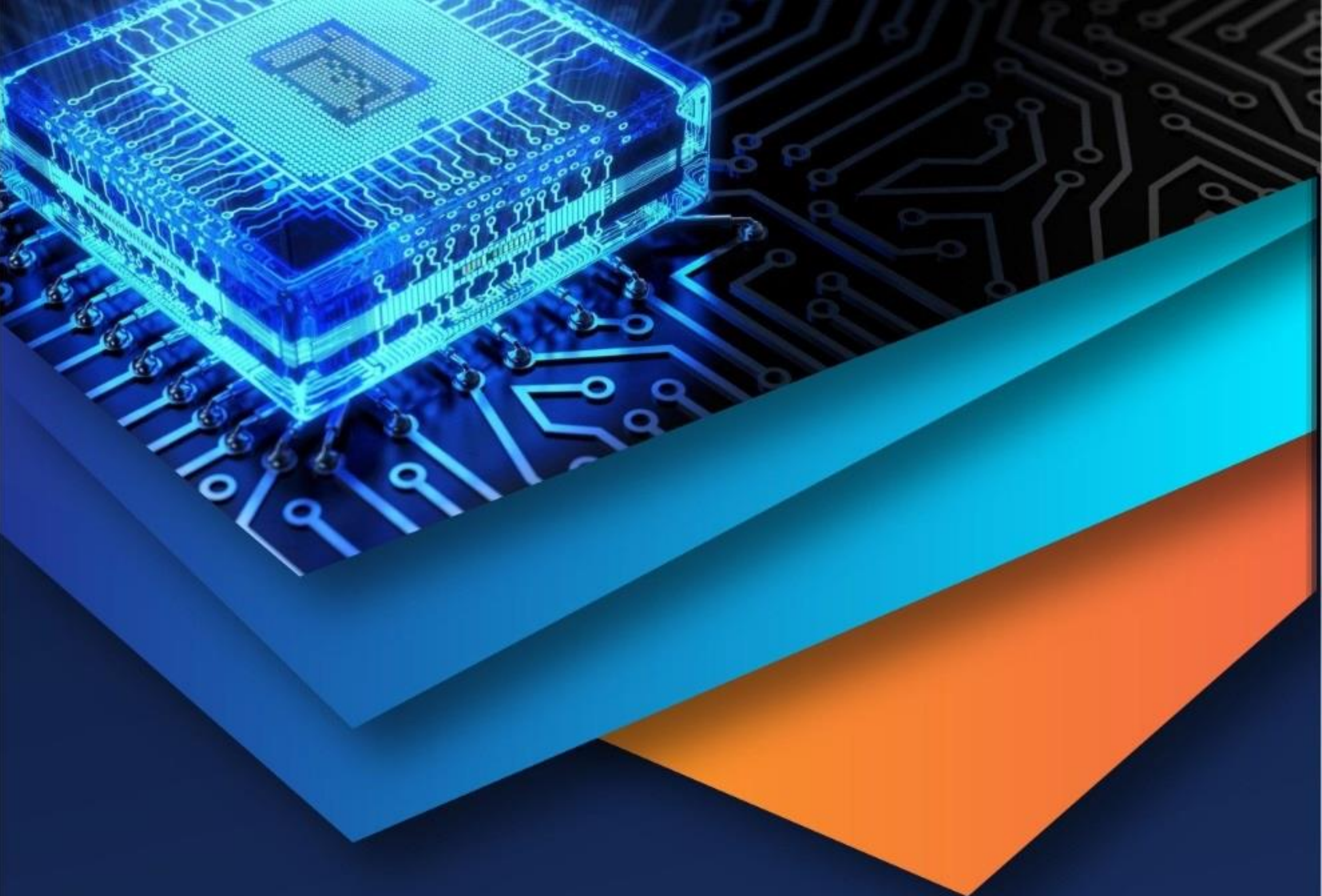
p	a	t	k	$\begin{pmatrix} l \\ m \\ \omega \end{pmatrix}$	LHS and RHS of $(\mathbf{3.8})$
2	4	2	6	$\begin{pmatrix} 4^{370}(4^{70}+1)^5 \\ 4^{504}(4^{70}+1)^7 \\ 4^{70}(4^{70}+1) \end{pmatrix}$	$4^{2520}(4^{70}+1)^{36}$
3	3	1	28	$\begin{pmatrix} 3^{21195}(3^{783}+1)^{27} \\ 3^{22736}(3^{783}+1)^{29} \\ 3^{783}(3^{783}+1) \end{pmatrix}$	$3^{613872}(3^{783}+1)^{784}$
5	2	2	496	$\begin{pmatrix} 2^{243556830}(2^{492030}+1)^{495} \\ 2^{244539904}(2^{492030}+1)^{497} \\ 2^{492030}(2^{492030}+1) \end{pmatrix}$	$2^{121047252480}(2^{492030}+1)^{246016}$

IV. CONCLUSION

In this transmission, a method for obtaining positive integer solution to the equation $l^{\alpha} + m^{\beta} = \omega^{\gamma}$ involving Kaprekar numbers is discovered. To conclude, one could look into another equation with different numbers.

REFERENCES

[1] Kaprekar, D.R. An interesting property of the number 6174. Scr. Math. 1955, 21, 304
[2] Kaprekar, D., 1980. On Kaprekar Numbers. Journal of Recreational Mathematics, 13(2), 1980-81.
[3] Hua.L.K, “Introduction to the Theory of Numbers”, Springer-Verlag, Berlin-New york, 1982.
[4] Ivan Niven, Herbert, Zuckerman.S and Hugh Montgomery.L,“An Introduction to the theory of Numbers”, John Wiley and Sons Inc, New York 2004.
[5] Gopalan.M.A, Vidhyalakshmi.S, Thiruniraiselvi.N and Kanaka.D, “On three special Generalized Fermat equations”, International Journal of Trend in Research and Development.2016; 3(1): 97-99.
[6] Saranya. C and Janaki. G “On generalized fermat equations involving jarasandha numbers”, Parishodh Journal.2020; 9(2): 712-716.
[7] Pandichelvi. V and Vanaja. R, “Novel approach of existence of solutions to the exponential equation $(3m^2+3)^x + (7m^2)^y = z^2$”, Turkish Journal of Computer and Mathematics Education,2021; 12(1): 376-381
[8] Pandichelvi.V and Saranya.S, “Frustrating solutions for two exponential Diophantine equations $\mathbb{p}^a + (\mathbb{p}+3)^b - 1 = c^2$ and $(\mathbb{p}+1)^a - \mathbb{p}^b + 1 = c^2$”, Journal of Xi’an Shiyou University, Natural Science Edition.2021; 17(5): 147-156.
[9] Pandichelvi.V and Saranya.S,”Maddening solutions for an exponential Diophantine equation concerning definite prime numbers $(2P+1)^u + (P-1)^v - 2 = w^2$”, Advances and Applications in Mathematical Sciences.2022; 21(11): 6619-6625.
[10] Pandichelvi.V and Vanaja.R, Inspecting integer solutions for an exponential Diophantine equation $p^x + (p+2)^y = z^2$, Advances and applications in mathematical sciences.2022; 21(8): 4693-4701.
[11] Pandichelvi.V and Umamaheswari.B , “Exasperating non-negative integer solutions for an exponential diophantine equation $(3u^2+5)p + (6u^2+11) = w^2$”, 2022; 26(3): 1471-1475.
[12] Pandichelvi.V and Umamaheswari.B “Perceiving solutions for an exponential diophantine equation linking Safe and Sophie germain primes $q^x + p^y = z^2$”. 2022; 52(2): 165-167.



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