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Fermat’z Fuzzy Ideals over Fuzzy Sub Lattice with Respect to (S, T)-Norms

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Abstract: This research delves into the exploration of fermat’z fuzzy sub lattices and fermat’z fuzzy ideals within the context of lattice theory. Through a rigorous analysis of structural theorem concerning these concepts derived fermat’z fuzzy sets. We uncover significant parallels with classical theory. Additionally, we investigate the behavior of fermat’z fuzzy ideals under lattice homomorphism. Our finding shed light on the applicability and utility of fermat’z fuzzy theory in lattice-based structures, offering insights into their properties and relationships.
Keywords: crisp set, fuzzy subset, relation, fermat’z fuzzy set, fermat’z fuzzy sub lattice, prime ideal, fermat’z fuzzy homomorphism.

I. INTRODUCTION

A new chapter in mathematical study opened with the advent of Zadeh’s fuzzy set theory [13], sparking a plethora of studies with far-reaching consequences in a variety of fields. Interestingly, subsequent academic work has taken this ground work further in a number of areas [2], with particular emphasis on group theory and rings. Ajmal and Thomas [1], who have introduced the idea of fuzzy sub lattices by employing fuzzy set theory in lattice theory, have greatly enhanced this treat. Fuzzy group research has increased as a result of Rosenfeld’s new effort [9]. Atanassov made a significant break from traditional thought when he created intuitionistic fuzzy sets [3] in response to these observations. This extension, proposes a frame work for dealing with ambiguity, subgroups [Biswas] and subrings [4]have been created by recent work in abstract algebra, along with other concepts such as intuitionistic fuzzy numbers[Sureghjani. Reta [l0]. In 1998, as part of his work on the treatment of uncertainty, F.smarandache [11] proposed a more general theory than that introduced by Atanassov, called the “Neutrosophic set”. This new theory is characterized by truth membership, an indeterminacy membership and a falsely membership.

Responding to the exigencies posed by pervasive imprecision and uncertainty, Yager [12] introduced pythagorean fuzzy sets [PFSs]. Building upon the foundational work of Zhang and Xu [14], the conceptual frame work aims to translate nebulous and uncertain circumstances into a rigorous mathematical frame work, there by fascinating the derivation of efficacious solutions [8] .In this present study, a comprehensive examination of fermat’z fuzzy sub lattices and fermat’z fuzzy ideals with in the lattice context, elucidating their defining attributes with mathematical rigor. Moreover, our investigation extends to an incisive analysis of fermat’z fuzzy ideals vis-à-vis lattice homomorphism, thereby establishing crucial links between theoretical abstraction and practical applications within the mathematical domain.

II. PRELIMINARIES

In this section, we will list a few concepts. As we go through this essay, we will denote $L = (A, +, .)$ a lattice, where ‘+’ and ‘.’ denote the sup and inf respectively. Here, we will analyze the basic definition of fuzzy set, fermat’z fuzzy set etc.

1) Definition-2.1: Consider a crisp set E. An object with the type $F = \{(x, J(x), K(x) \ /x \in E)\}$ is a fermat’z fuzzy subset F of E. Here $0 \leq J^n(x) + K^n(x) \leq 1$ is satisfied by $J(x), K(x) \in [0,1]$, which represent the membership and non-membership degree of $x \in E$ respectively and
$n \in N = \{1,2,3, \dots\}$.

Below are some of the fermat’z fuzzy sets operations.

2) Definition-2.2: Consider a non-empty crisp set E and let $\Delta_1 = \{(x, J_1(x), K_1(x) \ /x \in E)\}$ and $\Delta_2 = \{(x, J_2(x), K_2(x) \ /x \in E)\}$ be two fermat’z fuzzy subset of E. Then
   (i) $\Delta_1 \subset \Delta_2$ if and only if ${J_1}^n \leq {J_2}^n$ and ${K_1}^n \geq {K_2}^n$.
   (ii) $\Delta_1 = \Delta_2$ if and only if $\Delta_1 \subset \Delta_2$ and $\Delta_1 \supset \Delta_2$.
   (iii) ${\Delta_1}^C = (K_1 J_1)$

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(iv) $\Delta_1 \cap \Delta_2 = (J_1^n \wedge J_2^n, \; K_1^n \vee K_2^n)$
(v) $\Delta_1 \cup \Delta_2 = (J_1^n \vee J_2^n, \; K_1^n \wedge K_2^n)$

3) Definition-2.3: A fermat'z fuzzy set $\delta = \{(x, J_\delta(x), K_\delta(x) \; / x \in E)\}$ of E is called a fermat'z fuzzy set relation of E if each of the following conditions holds for every $x_1, x_2 \in E$.

(i) $J_\delta(x_1 - x_2)^n \geq T\{J_\delta{}^n(x_1), J_\delta{}^n(x_2)\}$ and
$K_\delta(x_1 - x_2)^n \leq S\{K_\delta{}^n(x_1), K_\delta{}^n(x_2)\}$.

(ii) $J_\delta(x_1x_2)^n \geq T\{J_\delta(x_1)^n, J_\delta(x_2)^n\}$ and
$K_\delta(x_1x_2)^n \leq S\{K_\delta(x_1)^n, K_\delta(x_2)^n\}$.

If (ii) is substituted with

$\left(J_\delta(x_1x_2)\right)^n \geq T\{\left(J_\delta(x_1)\right)^n, \; \left(J_\delta(x_2)\right)^n\}$ and

$\left(K_\delta(x_1x_2)\right)^n \leq S\{\left(K_\delta(x_1)\right)^n, \; \left(K_\delta(x_2)\right)^n\}$ the fermat'z fuzzy subset 'δ' is recognized as the fermat'z fuzzy index of E.

4) Definition-2.4: Let Δ_1 and Δ_2 be fermat'z fuzzy subset of E. The product $\Delta_1 \circ \Delta_2$ is defined by $\Delta_1 \circ \Delta_2 = \{(x, J_{\Delta_1 \circ \Delta_2}(x), K_{\Delta_1 \circ \Delta_2}(x) \; / x \in E)\}$where

$$\left(J_\delta(x_1x_2)\right)^n = S\left\{T\left(\left(J_{\Delta_1}(x_1)\right)^n, \; \left(J_{\Delta_2}(x_2)\right)^n\right) \; / \; x_1, x_2 \in E, \; x_1, x_2 = x\right\} \text{ and}$$

$$\left(K_\delta(x_1x_2)\right)^n = T\left\{S\left(\left(K_{\Delta_1}(x_1)\right)^n, \left(K_{\Delta_2}(x_2)\right)^n\right) \; / \; x_1, x_2 \in E, \; x_1, x_2 = x\right\}$$

III. FERMAT'Z FUZZY SUB LATTICES AND IDEALS

In this section, we are discussed fermat'z fuzzy lattice, fermat'z fuzzy ideal and their characterizations.

Definition-3.1: Let 'M' be a lattice and $L = \{(x, J(x), K(x) \; / x \in M)\}$ is a fermat'z fuzzy subset of M. So L is considered as a fermat'z fuzzy sub lattice of M if the following conditions are valid.

(i) $J^n(m_1 + m_2) \geq \; T\{J^n(m_1), J^n(m_2)\}$ and
$K^n(m_1 + m_2) \leq S\{K^n(m_1), K^n(m_2)\}$ for any $m_1, m_2 \in M$.

(ii) $J^n(m_1m_2) \geq \; T\{J^n(m_1), J^n(m_2)\}$ and
$K^n(m_1m_2) \leq S\{K^n(m_1), K^n(m_2)\}$ for any $m_1, m_2 \in M$.

Example-3.2: Consider the lattice L of "divisor of 6". That is $L = \{1, 2, 3, 6\}$.
Let $A = \{(x, J_A(x), K_A(x) \; / x \in L)\}$ be given by $(1, 0.6, 0.3)$, $(2, 0.5, 0.7)$, $(3, 0.5, 0.4)$, $(6, 0.2, 0.4)$. Then A is fermat'z fuzzy sub lattice of L.

Definition-3.3: A fermat'z fuzzy set 'D' of 'A' is called a fermat'z fuzzy ideal of 'A' if $u \leq v$ in A, we have $J^n(u) \geq J^n(v)$ and $K^n(u) \leq K^n(v)$.

Definition-3.4: Consider a fermat'z fuzzy ideal 'D' of 'A'. So D is called a fermat'z fuzzy prime ideal of A, if

$J^n(uv) \leq \; S\{J^n(u), J^n(v)\}$ and
$K^n(uv) \geq T\{K^n(u), K^n(v)\}$.

Definition-3.5: Consider a fermat'z fuzzy set 'D' in 'A'and any $\gamma_1, \gamma_2 \in [0, 1]$, then (γ_1, γ_2) −cut or (γ_1, γ_2) −level set D, denote by $D_{(\gamma_1, \gamma_2)}$ is the crisp set
$D_{(\gamma_1, \gamma_2)} = \{u \in A \; / J^n(u) \geq \; \gamma_1, K^n(u) \leq \gamma_2\}$.

Based on the above definition, the following results have to be obtained.

Proposition-3.6: Consider a fermat'z fuzzy ideal 'D' of 'A'. Then the following propositions are equivalent:

(i) D is a fermat'z fuzzy prime ideal of A.
(ii) $D(t_1, t_2) = (S\{J^n(t_1), J^n(t_2)\}, \;)T\{K^n(t_1), K^n(t_2)\}$ for any $t_1, t_2 \in A$.
(iii) $D(t_1, t_2) = D(t_1)$ or $D(t_2)$ for any $t_1, t_2 \in A$.

We describe fermat'z fuzzy sub lattice, fermat'z fuzzy ideals, fermat'z fuzzy prime ideals in terms of level subsets in the results that follows.

Theorem-3.7: Let 'D' be a fermat'z fuzzy set of 'A'. Then 'D' is a fermat'z fuzzy sub lattice of A if and only if for each $(\gamma_1, \gamma_2) \in Im(D)$, $D(\gamma_1, \gamma_2)$ is a sub lattice of A. (Here $D_{(\gamma_1, \gamma_2)}$ is called a level subset of A).

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Proof:

⇒ Suppose that D is a fermat'z fuzzy sub lattice.

Consider $D_{(\gamma_1,\gamma_2)}$ any non-empty level subset of D and let $t_1, t_2 \in D_{(\gamma_1,\gamma_2)}$.

Then $J^n(t_1) \geq \gamma_1, K^n(t_1) \leq \gamma_2$ and $J^n(t_2) \geq \gamma_1, K^n(t_2) \leq \gamma_2$. Thus

$$J^n(t_1 + t_2) \geq T\{J^n(t_1), J^n(t_2)\} \geq \gamma_1 \text{ and}$$
$$K^n(t_1 + t_2) \leq S\{K^n(t_1), K^n(t_2)\} \leq \gamma_2$$

Also $J^n(t_1t_2) \geq T\{J^n(t_1), J^n(t_2)\} \geq \gamma_1$ and

$$K^n(t_1t_2) \leq S\{K^n(t_1), K^n(t_2)\} \leq \gamma_2$$

Then $t_1 + t_2 \in D_{(\gamma_1,\gamma_2)}$ and $t_1t_2 \in D_{(\gamma_1,\gamma_2)}$.

Hence $D_{(\gamma_1,\gamma_2)}$ is a sub lattice of A.

Conversely, assume that $D_{(\gamma_1,\gamma_2)}$ is a sub lattice of A.

Let $t_1, t_2 \in D_{(\gamma_1,\gamma_2)}$. We can assume that let $t_1 + t_2 = t_1$ and $t_1.t_2 = t_2$ without losing the generality, then

$$J^n(t_1 + t_2) \geq \gamma_1 = T\{J^n(t_1), J^n(t_2)\} \text{ and}$$
$$K^n(t_1 + t_2) \leq \gamma_2 = S\{K^n(t_1), K^n(t_2)\} \text{ and}$$
$$J^n(t_1t_2) \geq \gamma_1 = T\{J^n(t_1), J^n(t_2)\} \text{ and}$$
$$K^n(t_1t_2) \leq \gamma_2 = S\{K^n(t_1), K^n(t_2)\}.$$

D is a fermat'z fuzzy sub lattice as a result. This concludes the evidence.

Theorem-3.8: Let 'D' be a fermat'z fuzzy sub lattice of 'A'. Then 'D' is a fermat'z fuzzy ideal of A if and only if for each $(\gamma_1, \gamma_2) \in \text{Im}(D)$, $D(\gamma_1, \gamma_2)$ is an ideal of A.

Equivalently, for each D is a fermat'z fuzzy set of A, D is a fermat'z fuzzy ideal if and only if each non-empty level subset $D(\gamma_1, \gamma_2)$ is an ideal. In this case, $D(\gamma_1, \gamma_2)$ is called a level ideal of A.

Theorem-3.9: Let 'D' is a fermat'z fuzzy ideal of 'A'. Then D is a fermat'z fuzzy prime ideal of A if and only if for each $(\gamma_1, \gamma_2) \in \text{Im}(D)$, $D(\gamma_1, \gamma_2)$ is a prime ideal of A.

Proof:

⇒ Suppose D is a fermat'z fuzzy prime ideal of A.

Let $(\gamma_1, \gamma_2) \in \text{Im}(D)$ and $t_1, t_2 \in D_{(\gamma_1,\gamma_2)}$.

Then, $J^n(t_1.t_2) \geq \gamma_1$ and $K^n(t_1.t_2) \leq \gamma_2$.

But, by proposition-3.6, we have

$D(t_1.t_2) = D(t_1)$ or $D(t_1.t_2) = D(t_2)$.

Thus, $J^n(t_1) \geq \gamma_1, K^n(t_1) \leq \gamma_2$ or

$J^n(t_2) \geq \gamma_2, K^n(t_2) \leq \gamma_2$.

So, $t_1 \in D(\gamma_1, \gamma_2)$ or $t_2 \in D(\gamma_1, \gamma_2)$.

Hence $D(\gamma_1, \gamma_2)$ is an prime ideal.

⇐ Suppose each level ideal $D(\gamma_1, \gamma_2)$ is prime. Assume that D is not the fermat'z fuzzy prime ideal. Then, by proposition-3.6, there exists $t_1, t_2 \in A$ such that

$D(t_1.t_2) \neq D(t_1)$ and $D(t_1.t_2) \neq D(t_2)$.

Since D is a fermat'z fuzzy ideal of A,

$$J^n(t_1t_2) > J^n(t_1), J^n(t_1t_2) > J^n(t_2) \text{ and}$$
$$K^n(t_1t_2) < K^n(t_1), K^n(t_1t_2) < K^n(t_2).$$

Let $D(t_1, t_2) = (\gamma_1, \gamma_2)$. Then, $t_1, t_2 \in D(\gamma_1, \gamma_2)$, but $t_1 \notin D(\gamma_1, \gamma_2)$ and $t_2 \notin D(\gamma_1, \gamma_2)$.

This runs counter to the idea that $D(\gamma_1, \gamma_2)$ is prime.

Hence, D is a fermat'z fuzzy prime idea of A.

Theorem-3.10: If D_1 and D_2 are two fermat'z fuzzy sub lattice of a lattice A, then $D_1 \cap D_2$ is a fermat'z fuzzy sub lattice of A.

Proof: Consider, $D_1 = \{(x, J_1(x), K_1(x) \,/ x \in A)\}$ and $D_2 = \{(x, J_2(x), K_2(x) \,/ x \in A)\}$ are two fermat'z fuzzy subset of A. Then $D_1 \cap D_2 = \{(x, J_{D_1 \cap D_2}(t), K_{D_1 \cap D_2}(t) \,/ t \in A)\}$, where

$J^n_{D_1 \cap D_2}(t) = T\{J^n_1(t), J^n_2(t)\}$ and $K^n_{D_1 \cap D_2}(t) = S\{K^n_1(t), K^n_2(t)\}$.

So that $J^n_{D_1 \cap D_2}(\wp_1 + \wp_2) = T\{J^n_1(\wp_1 + \wp_2), J^n_2(\wp_1 + \wp_2)\}$

$$\geq T\left\{T\{J^n_1(\wp_1), J^n_1(\wp_2)\}, T\{J^n_2(\wp_1), J^n_2(\wp_2)\}\right\}$$
$$= T\left\{T\{J^n_1(\wp_1), J^n_2(\wp_1)\}, T\{J^n_1(\wp_2), J^n_2(\wp_2)\}\right\}$$
$$= T\left\{J^n_{D_1 \cap D_2}(\wp_1), J^n_{D_1 \cap D_2}(\wp_2)\right\}$$

As D_1 and D_2 are fermat'z fuzzy sub lattice of A, we have

$$J^n_{D_1 \cap D_2}(\wp_1 + \wp_2) \geq T\left\{J^n_{D_1 \cap D_2}(\wp_1), J^n_{D_1 \cap D_2}(\wp_2)\right\}, \text{ for all } \wp_1, \wp_2 \in A.$$

Similarly, we get $J^n_{D_1 \cap D_2}(\wp_1 \wp_2) \geq T\left\{J^n_{D_1 \cap D_2}(\wp_1), J^n_{D_1 \cap D_2}(\wp_2)\right\}$, for all $\wp_1, \wp_2 \in A$.

Also $K^n_{D_1 \cap D_2}(\wp_1 + \wp_2) = S\{K^n_1(\wp_1 + \wp_2), K^n_2(\wp_1 + \wp_2)\}$

$$\leq S\{S\{K^n_1(\wp_1), K^n_1(\wp_2)\}, S\{K^n_2(\wp_1), K^n_2(\wp_2)\}\}$$
$$= S\{S\{K^n_1(\wp_1), K^n_2(\wp_1)\}, S\{K^n_1(\wp_2), K^n_2(\wp_2)\}\}$$
$$= S\{K^n_{D_1 \cap D_2}(\wp_1), K^n_{D_1 \cap D_2}(\wp_2)\}$$

As D_1 and D_2 are fermat'z fuzzy sub lattice of A, we have

$$K^n_{D_1 \cap D_2}(\wp_1 + \wp_2) \leq S\{K^n_{D_1 \cap D_2}(\wp_1), K^n_{D_1 \cap D_2}(\wp_2)\}, \text{ for all } \wp_1, \wp_2 \in A.$$

Similar evidence supports the fermat'z fuzzy ideal.

IV. HOMOMORPHISM AND FERMAT'Z FUZZY SUB LATTICE

In this section, the idea of homomorphism and fermat'z fuzzy sub lattices is analyzed.

Definition-4.1: Consider,$D_1 = \{(x, J_1(x), K_1(x) \ / x \in A)\}$ and
$D_2 = \{(x, J_2(x), K_2(x) \ / x \in B)\}$ are two fermat'z fuzzy subset of A and B respectively. Let φ be a mapping from A to B. Then $\varphi(D)$ is a fermat'z fuzzy subset of on B and defined by

$\varphi(D_1)(x) = \left(\varphi(J_1{}^2), \varphi(K_1{}^2), (x)\right)$ for all $x \in B$, where

$$\varphi(J_1{}^2)(x) = \begin{cases} S\{J_1{}^2(u)/u \in \varphi^{-1}(x)\}, & \text{if } \varphi^{-1}(x) \neq \emptyset \\ 0, & \text{if } \varphi^{-1}(x) = \emptyset \end{cases} \text{ and}$$

$$\varphi(K_1{}^2)(x) = \begin{cases} T\{K_1{}^2(v)/v \in \varphi^{-1}(x)\}, & \text{if } \varphi^{-1}(x) \neq \emptyset \\ 0, & \text{if } \varphi^{-1}(x) = \emptyset \end{cases}.$$

Also $\varphi^{-1}(D_2)$ is a fermat'z fuzzy set of A such that

$\varphi^{-1}(D_2)(u) = \left(\varphi^{-1}(J_2{}^2)(u), \varphi^{-1}(K_2{}^2)(u)\right)$ for all $x \in A$

where $\varphi^{-1}(J_2{}^2)(u) = J_2{}^2(\varphi(u))$ and $\varphi^{-1}(K_2{}^2)(u) = K_2{}^2(\varphi(u))$.

In particular, if $\varphi: A \to A'$ is a lattice homomorphism, D_1 is a fermat'z fuzzy sub lattice of A and D_2 is a fermat'z fuzzy sub lattice of A' , then $\varphi(D_1)$ is called homomorphic image of D_1 under φ and $\varphi^{-1}(D_2)$ is called homomorphic pre image of D_2, where A and A' denote lattices respectively.

The following theorems to be proved relative to the homomorphic.

Theorem-4.2: If $f: A \to A'$ is a lattice epimorphism and D_1 is a fermat'z fuzzy ideal of A, then $f(D_1)$ is fermat'z fuzzy ideal of A'.

Proof:

Consider,$D_1 = \{(x, J_1(x), K_1(x) \ / x \in A)\}$ a fermat'z fuzzy ideal of A. So
$f(D) = \left\{\left(\wp_1, f(J_1)(\wp_2), f(K_1)(\wp_2)\right)/\wp_2 \in A'\right\}$.

Let $\wp_1, p \in A'$ then

$f(J_1{}^n)(\wp_1 + p) = S\{J_1{}^n(\wp_2)/\wp_2 \in A'\}$

$$\geq S\{J_1{}^n(K + l)/K \in f^{-1}(t) \text{ and } l \in f^{-1}(p)\}$$
$$\geq S\{T\{J_1{}^n(K), J_1{}^n(l)\}/K \in f^{-1}(t) \text{ and } l \in f^{-1}(p)\}$$
$$= T\{S\{J_1{}^n(K)/K \in f^{-1}(t)\}, S\{J_1{}^n(l)/l \in f^{-1}(p)\}\}$$
$$= T\{f(J_1{}^n)(t), f(J_1{}^n)(p)\}.$$

since D_1 is a fermat'z fuzzy ideal of A. Also

$$f(J_1{}^n)(t,p) = S\{J_1{}^n(\wp_2)/\wp_2 \in f^{-1}(t,p)\}$$
$$\geq S\{J_1{}^n(K,l)/K \in f^{-1}(t) \text{ and } l \in f^{-1}(p)\}$$
$$\geq S\{S\{J_1{}^n(K), J_1{}^n(l)\}/K \in f^{-1}(t) \text{ and } l \in f^{-1}(p)\}$$
$$= S\{f(J_1{}^n)(t), f(J_1{}^n)(p)\}.$$

since D_1 is a fermat'z fuzzy ideal of A. Also

$$f(K_1{}^n)(t+p) = S\{K_1{}^n(\wp_2)/\wp_2 \in f^{-1}(t+p)\}$$
$$\leq T\{K_1{}^n(K+l)/K \in f^{-1}(t) \text{ and } l \in f^{-1}(p)\}$$
$$\leq T\{S\{K_1{}^n(K), K_1{}^n(l)\}/K \in f^{-1}(t) \text{ and } l \in f^{-1}(p)\}$$
$$= S\{T\{K_1{}^n(K)/K \in f^{-1}(t)\}, T\{K_1{}^n(l)/l \in f^{-1}(p)\}\}$$
$$= S\{f(K_1{}^n)(t), f(K_1{}^n)(p)\}.$$

since D_1 is a fermat'z fuzzy ideal of A. Also

$$f(K_1{}^n)(t,p) = T\{K_1{}^n(\wp_2)/\wp_2 \in f^{-1}(t,p)\}$$
$$\leq T\{K_1{}^n(K,l)/K \in f^{-1}(t) \text{ and } l \in f^{-1}(p)\}$$
$$\leq T\{T\{K_1{}^n(K), K_1{}^n(l)\}/K \in f^{-1}(t) \text{ and } l \in f^{-1}(p)\}$$
$$= T\{(TK_1{}^n)(K)/K \in f^{-1}(t), (TK_1{}^n)(l)/l \in f^{-1}(p)\}.$$

since D_1 is a fermat'z fuzzy ideal of A.
Hence $f(D_1)$ is fermat'z fuzzy ideal of A'.

Theorem-4.3: Let $f: A \rightarrow A'$ be a lattice homomorphism and D_2 is a fermat'z fuzzy ideal of A', then $f^{-1}(D_2)$ is a fermat'z fuzzy ideal of A.

Theorem-4.4: Let $f: A \rightarrow A'$ be an onto mapping and D_1 and D_2 are fermat'z fuzzy subsets of the lattices A and A' respectively. Then,

(i) $f\left(f^{-1}(D_2)\right) = D_2$

(ii) $D_1 \subseteq f^{-1}\left(f(D_1)\right)$.

Proof: (i) We have

$$f\left(f^{-1}(J_2{}^n)(t)\right) = S\{f^{-1}(J_2{}^n)(\wp_2)/\wp_2 \in f^{-1}(t)\}$$
$$= S\{J_2{}^n\left(f(\wp_2)\right)/\wp_2 \in A, f(\wp_2) = t\}$$
$$= (J_2{}^n)(t)$$

Similarly, $f\left(f^{-1}(K_2{}^n)(t)\right) = (K_2{}^n)(t)$.
Therefore, $f\left(f^{-1}(D_2)\right) = D_2$.
(ii) Also we have,

$$f^{-1}\left(f(J_1{}^n)\right)(\wp_2) = f(J_1{}^n)\left(f(\wp_2)\right)$$
$$= S\{J_1{}^n(\wp_2)\,/\wp_2 \in f^{-1}\left(f(\wp_2)\right)\}$$
$$\geq J_1{}^n(\wp_2), \text{ and}$$
$$f^{-1}\left(f(K_1{}^n)\right)(\wp_2) = f(K_1{}^n)\left(f(\wp_2)\right)$$
$$= T\{K_1{}^n(\wp_2)\,/\wp_2 \in f^{-1}\left(f(\wp_2)\right)\}$$
$$\leq K_1{}^n(\wp_2)$$

Hence $D_1 \subseteq f^{-1}\left(f(D_1)\right)$.

Definition-4.5: Let $f: A \rightarrow A'$ be a function from a lattice A to another lattice A' and
$D_1 = \{(\wp_2, J(\wp_2))/\wp_2 \in A\}$ be a fermat'z fuzzy subset of A then D_1 is said to be f-invariant if
$f(\wp_2) = f(t) \Rightarrow J^n(\wp_2) = J^n(t)$ for all $\wp_2, t \in A$.

Proposition-4.6: If a fermat'z fuzzy set D_1 is f-invariant, then $f^{-1}\left(f(D_1)\right) .= D_1$.

Theorem-4.7: Let $f: A \to A'$ be a function from a lattice A to another lattice A'. Also, D_1, D_2 are two fermat'z fuzzy subsets of A and D'_1, D'_2 are two fermat'z fuzzy subsets of A'. Then

(i) $D_1 \subseteq D_2 \Rightarrow f(D_1) \subseteq f(D_2)$

(ii) $D'_1 \subseteq D'_2 \Rightarrow f^{-1}(D'_1) \subseteq f^{-1}(D'_2)$

Proof: Let $D_1 = \{(\wp_2, J_1(\wp_2), K_1(\wp_2) \,/\, \wp_2 \in A)\}$ and $D_2 = \{(\wp_2, J_2(\wp_2), K_2(\wp_2) \,/\, \wp_2 \in A)\}$ be two fermat'z fuzzy subsets of A. Then $D_1 \subseteq D_2 \Rightarrow {J_1}^n(\wp_2) \leq {J_2}^n(\wp_2)$ and ${K_1}^n(\wp_2) \geq {K_2}^n(\wp_2)$.

So, $f(D_1) = \{(m, f({J_1}^n)(x), f({K_1}^n)(x)) / m \in A'\}$ and

$f(D_2) = \{(m, f({J_2}^n)(x), f({K_2}^n)(x)) / m \in A'\}$.

Now, $f({J_1}^n)(t) = S\{{J_1}^n(\wp_2) \,/\, \wp_2 \in f^{-1}(t)\}$

$\leq S\{{J_2}^n(\wp_2) \,/\, \wp_2 \in f^{-1}(t)\}$

$= f({J_2}^n)(t)$ as ${J_1}^n(\wp_2) \leq {J_2}^n(\wp_2)$.

Also, $f({K_1}^n)(t) = T\{{K_1}^n(\wp_2) \,/\, \wp_2 \in f^{-1}(t)\}$

$\geq T\{{K_2}^n(\wp_2) \,/\, \wp_2 \in f^{-1}(t)\}$

$= f({K_2}^n)(t)$ as ${K_1}^n(\wp_2) \geq {K_2}^n(\wp_2)$.

Hence, $f(D_1) \subseteq f(D_2)$.

Likewise, we can demonstrate other condition (ii).

V. CONCLUSION

To summarize up, our study investigate the complex characteristics of fermat'z fuzzy ideals and sub lattices. We defined operations for these fuzzy ideals and proved their preservation in distributive lattices. We also performed a thorough examination of fermat'z fuzzy ideals homomorphic images and pre-images, which resulted in the creation of invariant Fermat's fuzzy sets. This work culminated in establishing a correspondence theorem that connects the f-invariant fermat'z fuzzy ideals of a lattice to its homomorphic image.

VI. FUTURE WORK

Our efforts aim to build on this basis work by extending various uncertainty sets to further develop Lattice theory, building on the foundations laid this study.

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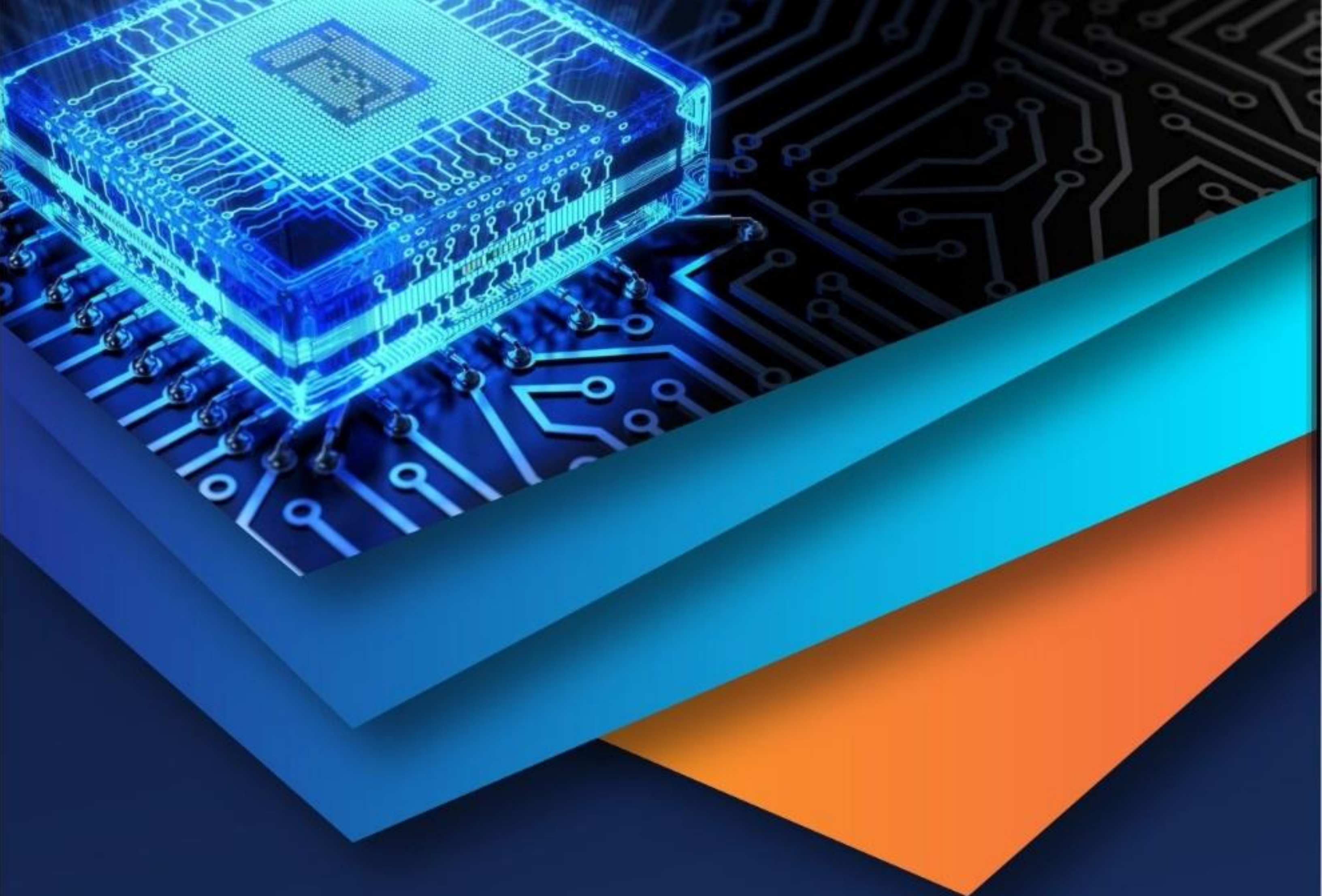

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