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Formation of Diophantine Triples Involving Heptagonal Pyramidal Numbers

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***Abstract:** In this paper, we seek for three specific polynomials with integer coefficients to such an extent that the product of any two numbers expanded by a non-zero number (or polynomials with integer coefficients) is a perfect square.*
***Keywords:** Diophantine triples, heptagonal pyramidal number, triples, perfect square, pyramidal number.*

Notation
$P_n^7 = \frac{n(n+1)}{6}(5n-2)$, heptagonal pyramidal number of rank n.

I. INTRODUCTION

A Diophantine equation is a polynomial equation in number theory where only the integer solutions are considered or searched for, typically with two or more unknowns [1-4]. The term "Diophantine" refers to the Greek mathematician Diophantus of Alexandria, who investigated similar situations and was one of the pioneers in introducing symbolism to variable-based mathematics in the third century.

The problem of the occurrence of Dio triples and quadruples with the property *D(n)* for any integer n as well as for any linear polynomial in *n* was studied by a number of mathematicians [5-8]. In this particular situation, one may turn to [9–16] for a thorough analysis of various difficulties on Diophantine triples. Half companion Diophantine triple sequences were studied in [17]. These results motivated us to examine for Diophantine triples with involving heptagonal pyramidal numbers. This paper aims at constructing Dio-Triples where the product of any two members of the triple with the addition of a non-zero integer or a polynomial with integer coefficients satisfies the required property. Also, we present three sections where in each of which we find the Diophantine triples from heptagonal pyramidal number of different ranks with their corresponding properties.

II. BASIC DEFINITION

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be Diophantine triple with property $D(n)$ if $a_i * a_j + n$ is a perfect square for all $1 \le i < j \le 3$, where n may be non zero or polynomial with integer coefficients.

III. METHOD OF ANALYSIS:

A. Section-A

Formation of Diophantine triples involving heptagonal pyramidal number of rank n and $n-1$

Let $a = 6p_n^7$ and $b = 6p_{n-1}^7$ be heptagonal pyramidal numbers of rank n and $n-1$ respectively. Then,

$$ab + (-5n^5 - 4n^4 + 5n^3 + 20n^2 - 8n + 1)$$
$$= 25n^6 - 50n^5 - 15n^4 + 50n^3 + 6n^2 - 8n + 1$$
$$= (5n^3 - 5n^2 - 4n + 1)^2$$

$$b + (-5n^5 - 4n^4 + 5n^3 + 20n^2 - 8n + 1) = (5n^3 - 5n^2 - 4n + 1)^2 = \alpha^2 \quad \text{(say)} \quad (1)$$

$$bc + (-5n^5 - 4n^4 + 5n^3 + 20n^2 - 8n + 1) = \beta^2 \quad (2)$$

$$ca + (-5n^5 - 4n^4 + 5n^3 + 20n^2 - 8n + 1) = \gamma^2 \quad (3)$$

Solving (2) & (3)

$$(b-a)(5n^5+4n^4-5n^3-20n^2+8n-1)=(a\beta^2-b\gamma^2) \quad (4)$$

Put, $\beta=x+by$ and $\gamma=x+ay$

Substituting β,γ in (4)

$$x^2(a-b)-(a-b)aby^2=-(5n^5+4n^4-5n^3-20n^2+8n-1)(a-b)$$

$$x^2=aby^2-5n^5-4n^4+5n^3+20n^2-8n+1$$

Put $y=1$,

$$x^2=25n^6-50n^5-15n^4+50n^3+6n^2-8n+1$$

$$x=5n^3-5n^2-4n+1$$

Now, $\beta=x+by$

$$\beta=10n^3-17n^2+3n+1$$

From equation (2)

bc $-5n^5-4n^4+5n^3+20n^2-8n+1=(10n^3-17n^2+3n+1)^2$

$\Rightarrow$ c $=20n^3-19n^2-3n+2$

$\Rightarrow$ c $=4(6p_n^7)-31n^2+5n+2$

Therefore, the triples

$\{a,b,c\}=\{6p_n^7,\ 6p_{n-1}^7, 4(6p_n^7)-31n^2+5n+2\}$ is a Diophantine triple with the property $D(-5n^5-4n^4+5n^3+20n^2-8n+1)$

Some numerical examples are given below in the following table.

TABLE 1

$S.No$	n	(a,b,c)	$property$
1	0	(0,0,2)	1
2	1	(6,0,0)	9
3	2	(48,6,80)	-119
4	3	(156,48,362)	-1247
5	4	(360,156,966)	-5535

B. Section -B

Formation of Diophantine triples involving heptagonal pyramidal number of rank n and $n-2$

Let $a=6p_n^7$ and $b=6p_{n-2}^7$ be heptagonal pyramidal numbers of rank n and $n-2$ respectively.

Now,

$a=6p_n^7$ and $b=6p_{n-2}^7$

$$ab+(-15n^5+6n^4+96n^2-60n+9)$$
$$=25n^6-120n^5+124n^4+78n^3-68n^2-12n+9$$
$$=(5n^3-12n^2-2n+3)^2$$

Equation (5) is a perfect square.

$$bc+(-15n^5+6n^4+96n^2-60n+9)=\beta^2 \quad (6)$$

$$ca+(-15n^5+6n^4+96n^2-60n+9)=\gamma^2 \quad (7)$$

Solving (6) & (7)

(b-c) $(15n^5-6n^4-96n^2+60n-9)=a\beta^2-b\gamma^2$ (8)

Put, $\beta = x + by$ and $\gamma = x + ay$,

Substituting β, γ in (8)

$x^2(a - b) - ab(a - b)y^2 = -(15n^5 - 6n^4 - 96n^2 + 60n - 9)(a - b)$

$x^2 = aby^2 - 15n^5 + 6n^4 + 96n^2 - 60n + 9$

Put, y=1,

$x^2 = 25n^6 - 120n^5 + 124n^4 + 78n^3 - 68n^2 - 12n + 9$

$x = 5n^3 - 12n^2 - 2n + 3$

Now , $\beta = x + by$

$\beta = 10n^3 - 39n^2 + 44n - 21$

From (6)

bc $-15n^5 + 6n^4 + 96n^2 - 60n + 9 = (10n^3 - 39n^2 + 44n - 21)^2$

$\Rightarrow$ c $= 20n^3 - 48n^2 + 40n - 18$

$\Rightarrow$ c $= 4(6p_n^7) - 60n^2 + 48n - 18$

Therefore, the triples

$\{a, b, c\} = \{6p_n^7 \ , \ 6p_{n-2}^7, 4(6p_n^7) - 60n^2 + 48n - 18\}$ is a Diophantine triples with the property $D(-15n^5 + 6n^4 + 96n^2 - 60n + 9)$.

Some numerical examples are given below in the following table.

TABLE 2

S.NO	n	(a, b, c)	property
1	0	(0, $-24, -18$)	9
2	1	(6,0, -6)	36
3	2	(48,0,30)	81
4	3	(156,6,210)	-360
5	4	(360,48,654)	-2151

C. Section -C

Formation of Diophantine triples involving heptagonal pyramidal number of rank n and $n - 3$

Let $a = 6p_n^7$ and $b = 6p_{n-3}^7$ be heptagonal pyramidal numbers of rank n and $n - 3$ respectively.

Now,

$a = 6p_n^7$ and $b = 6p_{n-3}^7$

$$ab + (-5n^5 + n^4 + n^3 + 232n^2 - 140n + 64)$$
$$= 25n^6 - 200n^5 + 440n^4 - 80n^3 - 304n^2 + 64n + 64$$
$$= (5n^3 - 20n^2 + 4n + 8)^2$$

$ab + (-5n^5 + n^4 + n^3 + 232n^2 - 140n + 64) = (5n^3 - 20n^2 + 4n + 8)^2 = \alpha^2$ (9)

Equation (9) is a perfect square.

$bc + (-5n^5 + n^4 + n^3 + 232n^2 - 140n + 64) = \beta^2$ (10)

$ca + (-5n^5 + n^4 + n^3 + 232n^2 - 140n + 64) = \gamma^2$ (11)

Solving (10) & (11)

$(b - a)\ (5n^5 - n^4 - n^3 - 232n^2 + 140n - 64) = a\beta^2 - b\gamma^2$ (12)

Put, $\beta = x + by$ and $\gamma = x + ay$, Substituting β & γ in (12)

$(a-b)\,x^2 - ab(a-b)y^2 = -(5n^5 - n^4 - n^3 - 232n^2 + 140n - 64)(a-b)$

Put, y=1,

$x^2 = 25n^6 - 200n^5 + 440n^4 - 80n^3 - 304n^2 + 64n + 64$

$x = 5n^3 - 20n^2 + 4n + 8$

Now , $\beta = x + by$

$\beta = 10n^3 - 62n^2 + 119n - 94$

From,(10)

bc $-5n^5 + n^4 + n^3 + 232n^2 - 140n + 64 = (10n^3 - 62n^2 + 119n - 94)^2$

$\Rightarrow$ c $= 20n^3 - 79n^2 + 121n - 86$

$\Rightarrow c = 4(6p_n^7) - 60n^2 + 48n - 18$

Therefore, the triples, $\{a,b,c\} = \{6p_n^7\, , 6p_{n-3,}^7 4(6p_n^7) - 60n^2 + 48n - 18\}$ is a Diophantine triples with the property $D(-5n^5 + n^4 + n^3 + 232n^2 - 140n + 64)$. Some numerical examples are given below in the following table.

TABLE 3

S.NO	n	(a,b,c)	$property$
1	0	$(0,-102,-86)$	64
2	1	$(6,-24,-24)$	153
3	2	(48,0,0)	576
4	3	(156,0,106)	625
5	4	(360,6,414)	-1584

IV. CONCLUSION

We have presented the Diophantine triples involving heptagonal pyramidal numbers. To conclude one may look for triples or quadruples for different numbers with their relating properties.

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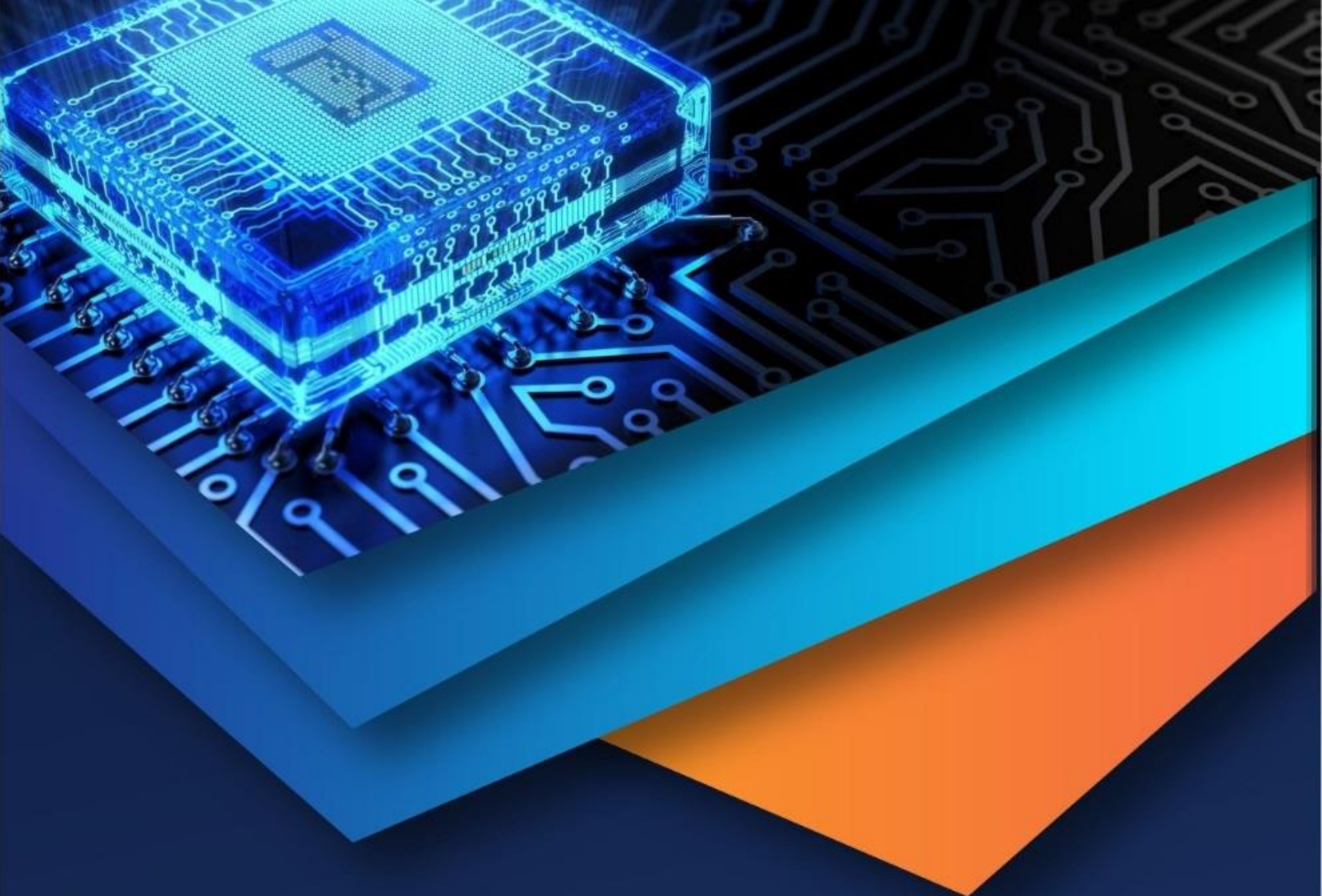

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