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Fuzzy Linear FractionalProgramming Problem Using Trapezoidal Fuzzy Numbers

D. Archana
PG and Research Department of Mathematics, Seethalakshmi Ramaswami College(Autonomous), Tiruchirappalli, (Affiliated to Bharathidasan University, Tiruchirappalli)

Abstract: In order to solve Fully Fuzzy Linear Fractional Programming Problems (FFLFPP) with trapezoidal fuzzy integers as the objective function and constraints, this study aims to develop a novel trapezoidal fuzzy number ranking function. The simplex method and crisp linear fractional programming serve as the foundation for the suggested approach. With the aid of a recently proposed ranking function, we first converted FFLFPP into a Crisp Linear Programming Problem. The resulting problem was then transformed into LPP. The suggested method is demonstrated using a numerical example.
Keywords: Fuzzy Fractional Linear Programming Problem (FFLPP), Trapezoidal Fuzzy Number, Simplex Method.

I. INTRODUCTION

B. Matrosin, a Hungarian Mathematician, developed this FLFPP (1960, 1964). There are numerous approaches to resolving the FFLPP issues. Tantawy (2007, 2008) recently proposed the idea of duality as a solution to a FLFPP. Mojtaba Borza and Azmin Sham Rambely [11] (2021) introduced a novel, efficient, and straightforward method for converting MOLFPP to LPP with reduced computing costs and sufficient accuracy. A method for resolving fully fuzzy linear fractional programming problems was introduced by T. Loganathan and K. Ganesan [6] in 2019. All of the parameters and variables in this instance are fuzzy triangular numbers. In operation research, one of the most crucial methods is the linear fractional programming (LFP) problem. It is possible to transform a variety of real-world problems into linear fractional programming problems. Uncertainty over the parameters may exist in real-world problems. In such a setting the parameters of linear programming problems may be expressed as fuzzy numbers. In Bellman and Zadeh [1] introduced fuzzy optimization. Using a ranking function, R. Maleki et al. [10] presented an effective technique for resolving linear programming problems involving fuzzy variables. A method for obtaining the estimated solution of fully fuzzy linear programming problems was created by Lotfi et al. [8]. Stanojevic etal. [13] suggested a method for solving the FFLP problems with equality constraints occurring in real-life scenarios, using the concept of crisp linearprogramming and ranking function. Graded Mean Integration was employed by Chen and Hsieh [5].Beaula Thangaraj, and V. Vijaya [2] (2012), "Critical Path in a project network using a new representation for Trapezoidal Fuzzy Numbers." International Journal of Mathematics Research 4.5:549-557. Beaula Thangaraj, and V. Vijaya [3] (2015), "A new method to find critical path from multiple paths in project networks." Int J Fuzzy Math Arch 9.2: 235-243. Beaula, T., and V. Vijaya [4] (2013), "A Study on Exponential Fuzzy Numbers Using α-Cuts." International Journal of Applied 3.2 : 1-13.1-13. Thangaraj Beaula and Vijaya[14] have adopted different fuzzy numbers and ranking methods to solve critical path problems. Vijaya et al [13,14] have used Pythagorean Fuzzy numbers and Neutrosophic Fuzzy numbers to find the solution of Decision making problem and critical path problems. Vijaya, V., and D. Rajalaxmi [15] (2022), "Decision making in fuzzy environment using Pythagorean fuzzy numbers." Mathematical Statistician and Engineering Applications 71.4: 846- 854. Vijaya, V., D. Rajalaxmi, and H. Manikandan [16] (2022), "Finding critical path in a fuzzy project network using neutrosophic fuzzy number." Advances and Applications in Mathematical Sciences 21.10: 5743-5753.Vijaya et al [17] (2025) discussed about Complex FermateanNeutrosophic Sets and their Applications in Decision Making.

A generalized fuzzy number with a representation. Charnes et al. [6], [7] proposed substituting at most two simple linear programming issues for any linear fractional programming. By transforming the Linear Fractional Programming (LFP) problem into a single Linear Programming (LP) problem, present a computer-oriented method for tackling the LFP problem. A fully fuzzy linear fractional programming problem using trapezoidal fuzzy numbers has been proposed in this study. wherein a new ranking function is added to help transform the objective function and constraints into a linear programming problem.The basic idea of fuzzy sets and trapezoidal fuzzy numbers with their mathematical features are covered in section 2, which is followed by the generic form of fractional linear programming and fully fuzzy fractional linear programming problems. This paper is broken into four sections. In section 4, we developed the technique to solve Fully Fuzzy Linear Fractional Programming Problems using the suggested new ranking function, which is provided in section 3.

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The present work proposed a new ranking function of trapezoidal fuzzy numbers for solving Problems involving Fully Fuzzy Linear Fractional Programming (FFLFPP) using a clear linear fractional programming problem and the simplex method.

II. PRELIMINARIES

Definition: 2.1

A fuzzy set Ã is defined by Ã = {(x, μ_A(x): x ϵ A, μ_A(x) ϵ [0,1]}. In the pair (x, μ_A(x)), the first element x belongs to the classical set A, the second element μ_A(x) belong to the interval [0,1] and is called **Membershipfunction.**

Definition: 2.2

A fuzzy set Ã on R must poses at least the following three properties to qualify as a fuzzy number.

- Ã must be a normal fuzzy set,
- $\alpha_{\tilde{A}}$ must be a closed interval for every α ∈ [0,1],
- The support of Ã,$\alpha_{\tilde{A}}$ must be bounded.

Definition: 2.3

The Fuzzy number Ã is a fuzzy set on the Real line R must satisfy the following conditions:

- $\mu_{\tilde{A}}\ (x_0)$ is piecewise continuous.
- There exists at least one x_0ϵR with $\mu_{\tilde{A}}$ (x_0) =1.
- (iii) Ã must be normal and convex.

Definition: 2.4

A trapezoidal membership function is specified by four parameters (a, b, c, d) as follows:

$$\mu_A(x) = \begin{cases} 0 & if\ x \leq a \\ x - a/b - a & if\ a \leq x \leq b \\ d - x/d - c & if c \leq x \leq d \\ 0 & if\ d \leq a \end{cases}$$

Definition: 2.5 (Arithmetic operation on Trapezoidal Fuzzy Numbers)

Trapezoidal fuzzy number $\bar{A}$ is a fuzzy set defined on R and is represented

By $\bar{A}$ = (b_1, b_2, b_3, b_4); $b_1 \leq b_2 \leq b_3 \leq b_4$

$$\mu_{\tilde{A}}(x) = \begin{cases} r(b_1 - x/b_1 - b_2) & , \quad b_1 \leq\ x \leq b_2 \\ r \quad , & b_2 \leq x \leq b_3 \\ r(b_4 - x/b_4 - b_3) & , \quad b_3 \leq x \leq b_4 \\ 0 & , \quad otherwise \end{cases}$$

Let $\tilde{E}$ = (e_1, e_2, e_3, e_4) and $\tilde{G}$= (g_1, g_2, g_3, g_4) be two trapezoidal fuzzy numbers, Where $e_1, e_2, e_3, e_4, g_1, g_2, g_3, g_4$∈ R. Then the arithmetic operations are defined by

[i] $\tilde{E}$ +$\tilde{G}$ =$(e_1 + g_1, e_2 + g_2, e_3 + g_3, e_4 + g_4)$

[ii] $\tilde{E} - \tilde{G}$ = $(e_1 - g_1, e_2 - g_2, e_3 - g_3, e_4 - g_4)$

[iii] $\tilde{E}.\tilde{G}$ = $(e_1.g_1, e_2.g_2, e_3.g_3, e_4.g_4)$

[iv] $\frac{\tilde{E}}{\tilde{G}} = \left(\frac{e_1}{g_1}, \frac{e_2}{g_2}, \frac{e_3}{g_3}, \frac{e_4}{g_4}\right)$

Definition: 2.6

The general form of Fractional Linear Programming Problems

$$\text{Max } f(x) = \frac{\sum_{j=1}^{l} R_j^T x_j + p}{\sum_{j=1}^{l} S_j^T x_j + q}$$

Subject to $\sum A_{ij} x_j \leq\ B_i$; i = 1………m; j = 1………l

$$x_j\ \geq 0,$$

where R, S, A and B are crisp numbers.

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Definition: 2.7

The general form of Fully Fuzzy Fractional Linear Programming Problems

$$\text{Max } f(x) = \frac{\sum_{j=1}^{l} \tilde{R}_j^T x_j + p}{\sum_{j=1}^{l} \tilde{S}_j^T x_j + q}$$

Subject to $\sum \tilde{A}_{ij} x_j \leq \tilde{B}_i$; i = 1……..m; j = 1……..l

where $\tilde{R} = (r_1, r_2, r_3, r_4)$ $\tilde{S} = (s_1, s_2, s_3, s_4)$

$\tilde{A} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ $\tilde{B} = (\beta_1, \beta_2, \beta_3, \beta_4)$

are trapezoidal fuzzy numbers.

Definition: 2.8

We proposed a signed distance ranking to rank fuzzy numbers.

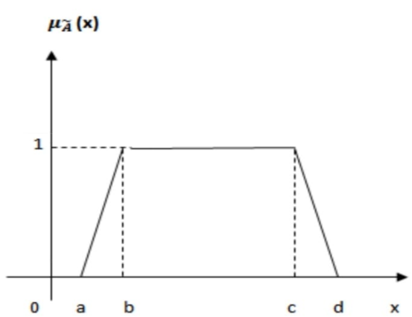
Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number is

$$R(\tilde{A}) = \frac{(a_1 + 4a_2 + 4a_3 + a_4)}{10}$$

Definition: 2.9

A fuzzy number $\tilde{A}$ = (a, b, c, d), $a \leq b \leq c \leq d$ (a, b, c, d> 0) is said to be a trapezoidal fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a, \\ \dfrac{x-a}{b-a}, & a < x \leq b, \\ 1, & b < x < c, \\ \dfrac{d-x}{d-c}, & c \leq x \leq d, \\ 0, & x > d, \end{cases}$$



Trapezoidal Fuzzy Number $\tilde{A} = (a, b, c, d)$

III. PROPOSED ALGORITHM TO SOLVE FFLPP PROBLEMS

Step 1:

Find an initial basic feasible solution of the given FLFP problem.

Step 2:

Compute the values of $\tilde{z}^1$, $\tilde{z}^2$ and $\frac{\tilde{z}^1}{\tilde{z}^2}$

Step 3:

Compute the values of $\tilde{z}_j^1 - \tilde{c}_j$ and $\tilde{z}_j^2 - \tilde{d}_j$ for all the non-basic vectors.

Step 4:

Check whether $\tilde{z}^1 = 0$ or $(\tilde{z}_j^2 - \tilde{d}_j) = 0$ for the non-vectors holds or not. If yes, go to Step 5; else, Step 6.

Step 5:

If either $\tilde{z}^1 = 0$ or $(\tilde{z}_j^2 - \tilde{d}_j) = 0$ for the non-vectors holds, then calculate $(\tilde{z}_j^1 - \tilde{c}_j)$ for all the non-basic vectors. If either of the above two holds, then calculate $(\tilde{z}_j^1 - \tilde{c}_j)$ for all the non-basic vectors.

If $(\tilde{z}_{j/}^{1} - \tilde{c}_j) < 0$, then the non-basic vector $\tilde{\alpha}_j \in \tilde{A}$ corresponding to $\text{Min}(\tilde{z}_j^1 - \tilde{c}_j)$ is selected as the incoming vector. Go to Step 7.

If$(\tilde{z}_j^1 - \tilde{c}_j) \geq 0$, for all non-basic vectors, then no further improvement is possible and the optimal solution is reached.

Step 6:

If neither $\tilde{z}^1 = 0$ nor $(\tilde{z}_j^2 - \tilde{d}_j) = 0$ for the non-basic vectors, then check whether $\tilde{z}_j^2 - \tilde{d}_j > 0$ or $\tilde{z}_j^2 - \tilde{d}_j < 0$.

Now, calculate $\tilde{R}_j = \frac{\tilde{z}_j^1 - \tilde{c}_j}{\tilde{z}_j^2 - \tilde{d}_j}$, for the non – basic vectors for which $(\tilde{z}_j^2 - \tilde{d}_j) \neq 0$.

Step 6a:

Check whether the conditions $(\tilde{z}_j^2 - \tilde{d}_j) < 0$ and $\tilde{R}_j > \frac{\tilde{z}^1}{\tilde{z}^2}$ for one or more non-basic vectors are satisfied or not. If yes, then the non-basic vector$\tilde{\alpha}_j \in \tilde{A}$ corresponding Min$\tilde{R}_j$ is selected as the incoming vector. Go to Step 7; else, Step 6(b).

Step 6b:

Check whether theconditions $(\tilde{z}_j^2 - \tilde{d}_j) > 0$ and $\tilde{R}_j < \frac{\tilde{z}^1}{\tilde{z}^2}$ for one or more non-basic vectors are satisfied or not. If yes, then the non-basic vector corresponding to Max $\tilde{R}_j$ is selected as the incoming vector. Go to Step 7; else, no further improvement is possible and the optimal solution is reached.

Step 7:

Using simplex method, the outgoing vector is selected and a new basic feasible solution is obtained. The process is continued till the criterion of optimality is satisfied.

IV. NUMBERICAL EXAMPLE

Consider the following FFLFP problem

$$\text{Max u} = \frac{(4,7,10,12)t_1 + (8,10,14,15)t_2 + (2.5,4,7.5,11.5)t_3 + (2,3,4,6)}{(10,14,20,22)t_1 + (20,23,27.5,29)t_2 + (18,20,25,28)t_3 + (5,10,18,20)}$$

Subject to

$[10,17,19,25]t_1 + [14,16,22,24]t_2 + [20,25,27,30]t_3 \leq [100,35,27,101.8]$

$[0.25,0.04,0.04,0.12]t_1 + [0.15,0.04,0.04,0.12]t_2 + [0.15,0.04,0.04,0.12]t_3 \leq [1.3,0.9,0,1.5]$

$[4,6,10,13]t_1 + [0,5,11,16]t_2 + [8,11,15,20]t_3 \leq [80,21.9,22,95]$

Non-negative condition

$t_1, t_2, t_3 \geq 0$

Solution:

Now using the proposed new Ranking Function

$$\text{R}(\bar{A}) = \frac{(b_1 + 4b_2 + 4b_3 + b_4)}{10}$$

and GMIR method. We have converted the above problem into a crisp fractional linear programming problem.

The given problem is now written as a crisp fractional linear programming problem.

$$\text{Max u} = \frac{8.4t_1 + 11.9t_2 + 6t_3 + 3.6}{16.8t_1 + 25.3t_2 + 22.6t_3 + 13.7}$$

Subject to

$17.9t_1 + 19t_2 + 25.8t_3 \leq 44.96$

$0.07t_1 + 0.06t_2 + 0.06t_3 \leq 0.64$

$8.1t_1 + 8t_2 + 13.2t_3 \leq 35$

The FFLPP problem in to standard form we get,

$$\text{Max u} = \frac{8.4t_1 + 11.9t_2 + 6t_3 + 0t_4 + 0t_5 + 0t_6 + 3.6}{16.8t_1 + 25.3t_2 + 22.6t_3 + 0t_4 + 0t_5 + 0t_6 + 13.7}$$

Subject to

$17.9□_1 + 19□_2 + 25.8□_3 + □_4 + 0□_5 + 0□_6 = 44.96$

$0.07t_1 + 0.06□_2 + 0.06□_3 + 0□_4 + □_5 + 0□_6 = 0.64$

$8.1□_1 + 8□_2 + 13.2□_3 + 0□_4 + 0□_5 + □_6 = 35$

Where $\square_4$, $\square_5$ and $\square_6$ are slack variables.

$$\square_1, \square_2, \square_3, \square_4, \square_5, \square_6 \geq 0.$$

Initial Table:

BV	$\tilde{\square}_{\square}$	$\tilde{\square}_{\square}$	$\tilde{\square}_{\square}$	$\square_{\square}$	$\square_{\square}$	$\square_{\square}$	$\square_{\square}$	$\square_{\square}$	$\square_{\square}$
$\square_{\square}$	0	0	44.96	17.9	19	25.8	1	0	0
$\square_{\square}$	0	0	0.64	0.07	0.06	0.06	0	1	0
$\square_{\square}$	0	0	35	8.1	8	13.2	0	0	1
			$(\tilde{\square}^{1}_{\square} - \tilde{\square}_{\square})$	-8.4	-11.9 ↑	-6	0	0	0
			$(\tilde{\square}^{2}_{\square} - \tilde{\square}_{\square})$	-16.8	-25.3	- 22.6	0	0	0

Iteration 1:

BV	$\tilde{C}_B$	$\tilde{D}_B$	$\tilde{X}_B$	t_1	t_2	t_3	t_4	t_5	t_6
$\square_{\square}$	11.9	25.3	2.4	0.9	1	1.4	0.1	0	0
$\square_{\square}$	0	0	0.22	0.77	0	0.02	0.01	-1	0
$\square_{\square}$	0	0	15.7	-0.9	0	-2	0.8	0	-1
			$(\tilde{\square}^{1}_{\square} - \tilde{\square}_{\square})$	2.31	0	10.66	1.19	0	0
			$(\tilde{\square}^{2}_{\square} - \tilde{\square}_{\square})$	5.97	0	12.82	2.53	0	0

The optimal solution is

$$t_1 = 0, t_2 = 11.9, t_3 = 0$$

$$\text{Max u} = \frac{8.4(0)+11.9(11.9)+6(0)+0+0+0+3.6}{16.8(0)+25.3(11.9)+0+0+0+0+13.7}$$

$$= \frac{141.61+3.6}{301.07+13.7}$$

$$= \frac{145.21}{314.77}$$

= 0.46

Therefore,

The optimal solution is

$$\square_{\square} = \square, \square_{\square} = \square\square.\square, \square_{\square} = \square$$

and Max u = 0.46

Using a new trapezoidal fuzzy number ranking function for addressing Fully Fuzzy Linear Fractional Programming Problems (FFLFPP) with trapezoidal fuzzy numbers as the objective function and constraints. The fuzzy optimal value of the given problem is **F(u) = 0.46.**

which is exactly as given in the optimal solution.

V. CONCLUSION

As stated by Moumita, D. and De, P. K. (2014), the proposed method has resolved the issue of maximizing Max u with three constrained variables. It was initially converted into a crisp fractional linear programming problem using the GMIR method and the proposed novel ranking function.

Then, using transformation techniques, it was changed into a linear programming problem. With the values of variables y1 = 0:0449, y2 = 0, and y3 = 0, the best solution to the issue is found to be F(u) = 0:43.

In this paper, we have suggested a novel trapezoidal fuzzy number ranking function and shown a productive method for resolving FFLFPP, whereby we first converted the original issue into a Crisp Linear Fractional Programming Issue using the suggested rating system.

Next, turn it into a linear programming problem instead of a crisp LFPP. then used the Simplex approach to achieve the best solution. The numerical example and its outcome Clearly demonstrate the value of the suggested approach.

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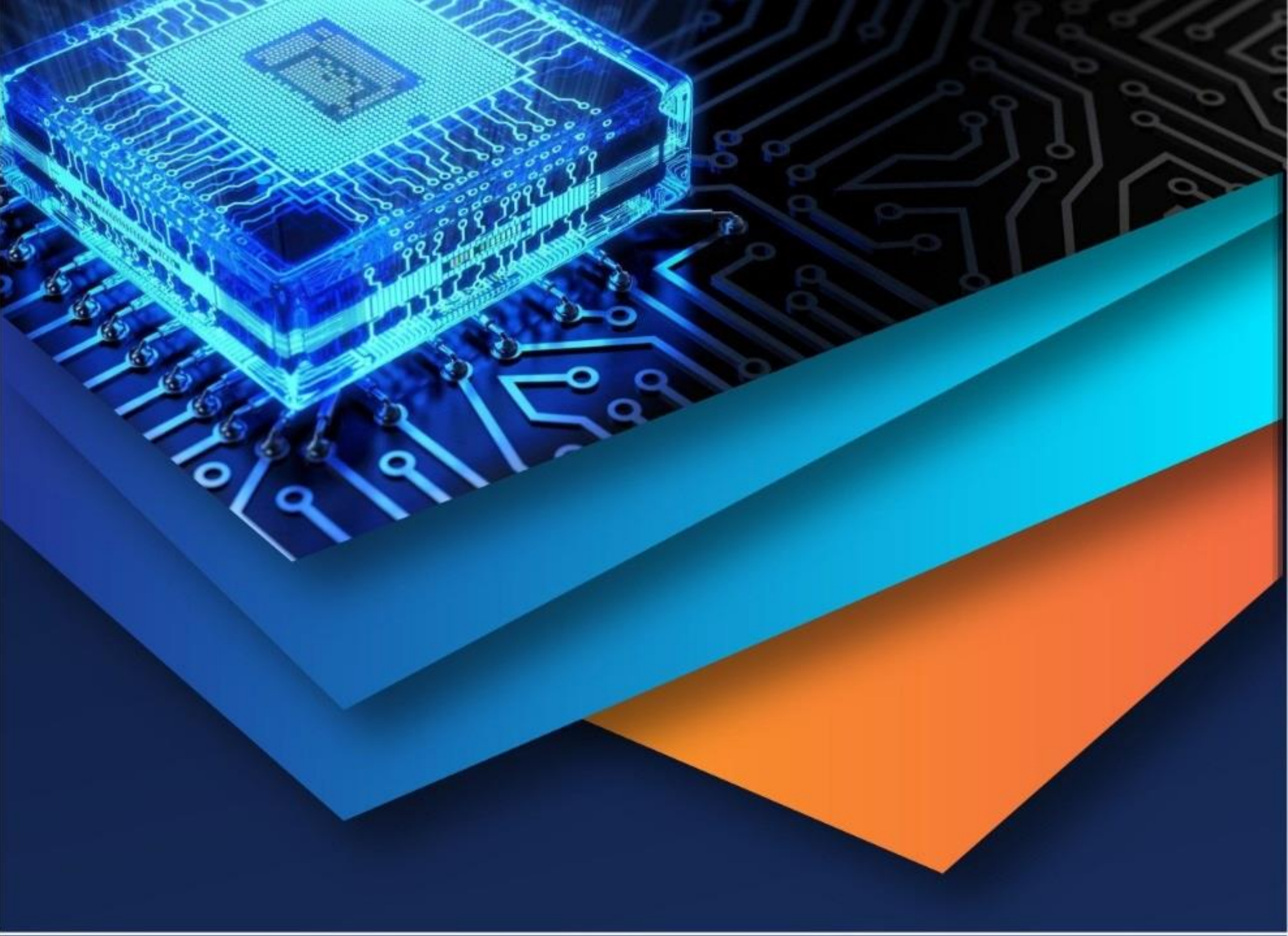

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