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Fuzzy Optimal Inventory and Shipment Policy on Non- Coordinated Supply Chain Under Quadratic Price Dependent Demand

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Abstract: Inventory management in the supply chain has been discovered to be the key to long-term success. Inventory is important in the supply chain because it balances supply characteristics and consumer demand. In this non-coordinated supply chain model, the merchant determines the appropriate order quantity based on the various expenses involved. Under quadratic price dependent demand, an extension of the lagrangian method is used to optimize fuzzy EOQ, total variable cost of the relevant retailer, and the supply chain. To support the model, a numerical case is solved.
Keywords: Globalization, supply chain, retailer.

I. INTRODUCTION

Globalization, or globalism, refers to the world's growing economy, culture, and population over the last few decades. Globalization has accelerated dramatically since the 18th century, thanks to advances in transportation and communication technology. As a result, many current globalization research projects focus on developing a model that aligns the role and notion of supply chain in such a way that it garners a lot of attention in both the academic and industry worlds. The current model focuses on the supply chain and its various characteristics. This study on supply chain is solved and optimized using an extension of the lagrangian approach, with cost parameters taken as pentagonal fuzzy numbers. To demonstrate the model, a numerical case is framed and solved.
Huang and Gangopadhyay (2004) created a simulation model to examine the impact of information sharing in a SC and discovered that information sharing contracts benefit both distributors and wholesalers. Furthermore, as evidenced by the literature, product demand is a critical factor in inventory decision-making. Retail pricing is a key aspect in maximising the SC's revenue/cost (He et al., 2009). Under trade credit phenomenon, Zhong and Zhou (2013) established a coordinated and non-coordinated two-level SC model for optimal inventory decisions and the length of the allowable wait time. They assumed that the retailer's storage space is limited, and that he is aware of inventory-dependent demand. Under revenue sharing contracts, Cao (2014) provided mathematical models for optimal price decisions and production methods. Kumar et al. (2016a, 2016b), Nagaraju et al. (2016a, 2016b), Kuntian et al. (2017), Lu and Zhou (2016) and Nagaraju et al. (2017) have all recently reported inventory and shipping decisions under-price dependent demand changes (2018). Unlike the other articles, this one proposes an inventory model for determining the optimal total relevant cost of non-coordinated Supply chain with single retailer. Product is a key component of the suggested paradigm. The retailer's unit selling price is used to calculate demand, which is written as a quadratic function. Model costs include ordering/setup costs, carrying costs, and shipping costs are considered for model development.

II. DEFINITIONS

A. *Fuzzy Set*

A fuzzy set $\tilde{B}$ in a universe of discourse X is defined as the following set of pairs $\tilde{B} = \{(x, \mu_{\tilde{B}}(x) : x \epsilon X)\}$.Here $\mu_{\tilde{B}} : X \rightarrow [0,1]$is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{B}$.

B. *Graded Mean Integration Representation Method*

If $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$is a pentagonal fuzzy number then the graded mean representation (GMIR) method of $\tilde{B}$ is defined as

$$\mathcal{P}(\tilde{B}) = \frac{1}{12}(b_1 + 3b_2 + 4b_3 + 3b_4 + b_5)$$

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C. *Pentagonal Fuzzy Number*

A fuzzy number $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ where $b_1 < b_2 < b_3 < b_4 < b_5$ are defined on R is called pentagonal fuzzy number if its membership function is

$$\mu_{\tilde{B}}(x) = \begin{cases} L_1(x) = \dfrac{x-a}{b-a}, a \le x \le b \\ L_2(x) = \dfrac{x-a}{b-a}, b \le x \le c \\ 1 \quad x = c \\ L_2(x) = \dfrac{d-x}{d-c}, c \le x \le d \\ L_2(x) = \dfrac{e-x}{e-d}, d \le x \le e \\ 0 \quad otherwise \end{cases}$$

D. *Arithmetic Operations under Function Principle*

The arithmetic operations between pentagonal fuzzy numbers proposed are given below.
Let us consider $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ be two pentagonal fuzzy numbers.

- $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$
- $\tilde{A} \ominus \tilde{B} = (a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1)$
- $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4, a_5 b_5)$
- $\tilde{A} \oslash \tilde{B} \left(\frac{a_1}{b_5}, \frac{a_2}{b_4}, \frac{a_3}{b_3}, \frac{a_4}{b_2}, \frac{a_5}{b_1}\right)$

E. *Extension of the Lagrangean Method*

Taha discussed how to solve the optimum solution of nonlinear programming problem with equality constraints by using Lagrangean Method, and showed how the Lagrangean method may be extended to solve inequality constraints. The general idea of extending the Lagrangean procedure is that if the unconstrained optimum the problem does not satisfy all constraints, the constrained optimum must occur at a boundary point of the solution space. Suppose that the problem is given by Minimize y = f(x) Sub to gi(x) ≥0, i = 1,2, …. m. The non-negativity constraints x ≥0 if any are included in them constraints. Then the procedure of the Extension of the Lagrangean method involves the following steps.

1) *Step 1:* Solve the unconstrained problem Min y = f(x) If the resulting optimum satisfies all the constraints, stop because all constraints are redundant. Otherwise, set k = 1 and go to step 2.
2) *Step 2:* Activate any k constraints (i. e., convert them into equality) and optimize f(x) subject to the k active constraints by the Lagrangean method. If the resulting solution is feasible with respect to the remaining constraints and repeat the step. If all sets of active constraints taken k at a time are considered without encountering a feasible solution, go to step 3.
3) *Step 3:* If K = m, stop; no feasible solution exists. Otherwise, set k = k + 1 and go to step 2.

F. *Assumptions*

- There will be no shortages.
- Demand is a quadratic function of the unit selling price at the store.

G. *Notations*

In this research paper, the notations considered as the mathematical model for two – echelon inventory system model,

$\mathcal{T}$ – The retailer annual demand rate (unit/year) = $v - \alpha m_j - \beta m_j^2$

where $v \gg 0, \alpha > 0, \beta > 0$

m_j - Unit selling price at the retailer (INR)

ℓ_i – Retailer cycle time, expressed in terms of a year.

s_j – ordering cost of the retailer per cycle time (INR/ order)

y_j – Unit cost of the retailer (INR/ unit)

k_j – Retailer fixed transportation cost to receive a consignment from the manufacturer

(INR/ shipment)

$\mathcal{R}_j$ – Shipment quality in each shipment to replenish the inventory at the retailer from the manufacturer for the cycle time ℓ_i

d – Carrying charge (in /INR / INR / year)

$TVC_j(\mathcal{R}_j)$ – The retailer's total variable cost expressed in terms of $\mathcal{R}_j$

H. *Formulation of the Crisp Model*

The retailer's annual ordering cost = $\frac{(v-\alpha m_j-\beta m_j{}^2)}{\mathcal{R}_j}\ s_j$

The retailer's annual transportation cost = $\frac{(v-\alpha m_j-\beta m_j{}^2)}{\mathcal{R}_j}\ k_j$

The retailer's annual carrying cost = $\frac{\mathcal{R}_j}{2}\ y_j\, d$

The retailer's annual TVC_j is the sum of the annual ordering cost, transportation cost and carrying cost is expressed as

$$TVC_j(\mathcal{R}_j) = \frac{(v-\alpha m_j-\beta m_j{}^2)}{\mathcal{R}_j}\ (s_j + k_j) + \frac{\mathcal{R}_j}{2}\ y_j\, d \quad \text{-------------------- (1)}$$

Differentiating (1) with respect to $\mathcal{R}_j$

$$\frac{\partial TVC_j}{\partial \mathcal{R}_j} = 0$$

$$\mathcal{R}_j{}^2 = \frac{2\,(v-\alpha m_j-\beta m_j{}^2)}{y_j\, d}\ (s_j + k_j)$$

$$\mathcal{R}_j = \sqrt{\frac{2\,(v-\alpha m_j-\beta m_j{}^2)}{y_j\, d}\ (s_j + k_j)} \quad \text{-------------------- (2)}$$

Equation (2) gives the required optimal order quantity of this model.

III. FORMULATION OF THE EQUATION IN FUZZY MODEL

Now fuzzify the total cost as

$$\widetilde{TVC_j(\mathcal{R}_j)} = \frac{(v-\alpha\widetilde{m}_j-\beta\widetilde{m}_j{}^2)}{\widetilde{\mathcal{R}}_j}\ (\widetilde{s}_j + \widetilde{k}_j) + \frac{\widetilde{\mathcal{R}}_j}{2}\ \widetilde{y}_j\, d \quad \text{-------------------- (3)}$$

Now we get,

$$\widetilde{m}_j = \left(m_{j_1}, m_{j_2}, m_{j_3}, m_{j_4}, m_{j_5}\right)$$

$$\widetilde{s}_j = \left(s_{j_1}, s_{j_2}, s_{j_3}, s_{j_4}, s_{j_5}\right)$$

$$\widetilde{k}_j = \left(k_{j_1}, k_{j_2}, k_{j_3}, k_{j_4}, k_{j_5}\right)$$

$$\widetilde{y}_j = \left(y_{j_1}, y_{j_2}, y_{j_3}, y_{j_4}, y_{j_5}\right)$$

$$\widetilde{\mathcal{R}}_j = \left(\mathcal{R}_{j_1}, \mathcal{R}_{j_2}, \mathcal{R}_{j_3}, \mathcal{R}_{j_4}, \mathcal{R}_{j_5}\right)$$

Are non- negative pentagonal fuzzy numbers

By graded mean integration formula

$$\mathcal{P}(\widetilde{B}) = \frac{1}{12}(b_1 + 3b_2 + 4b_3 + 3b_4 + b_5)$$

we write the fuzzified total cost as,

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$$\widetilde{TVC_j}(\mathcal{R}_j) = \left\{\begin{array}{l} \left(\frac{(v-\alpha m_{j_1}-\beta m_{j_1}^2)}{\mathcal{R}_{j_5}} \quad (s_{j_1}+k_{j_1}) + \frac{\mathcal{R}_{j_1}}{2}\, y_{j_1} d\right), \\ \left(\frac{(v-\alpha m_{j_2}-\beta m_{j_2}^2)}{\mathcal{R}_{j_4}} \quad (s_{j_2}+k_{j_2}) + \frac{\mathcal{R}_{j_2}}{2}\, y_{j_2} d\right), \\ \left(\frac{(v-\alpha m_{j_3}-\beta m_{j_3}^2)}{\mathcal{R}_{j_3}} \quad (s_{j_3}+k_{j_3}) + \frac{\mathcal{R}_{j_3}}{2}\, y_{j_3} d\right), \\ \left(\frac{(v-\alpha m_{j_4}-\beta m_{j_4}^2)}{\mathcal{R}_{j_2}} \quad (s_{j_4}+k_{j_4}) + \frac{\mathcal{R}_{j_4}}{2}\, y_{j_4} d\right), \\ \left(\frac{(v-\alpha m_{j_5}-\beta m_{j_5}^2)}{\mathcal{R}_{j_1}} \quad (s_{j_5}+k_{j_5}) + \frac{\mathcal{R}_{j_5}}{2}\, y_{j_5} d\right) \end{array}\right\}$$

Now defuzzify the total cost, now $p\left[\widetilde{TVC_j}(\mathcal{R}_j)\right]$ is defined as

$$p\left[\widetilde{TVC_j}(\mathcal{R}_j)\right] = \frac{1}{12}\left\{\begin{array}{l} \left(\frac{(v-\alpha m_{j_1}-\beta m_{j_1}^2)}{\mathcal{R}_{j_5}} \quad (s_{j_1}+k_{j_1}) + \frac{\mathcal{R}_{j_1}}{2}\, y_{j_1} d\right) + \\ 3\left(\frac{(v-\alpha m_{j_2}-\beta m_{j_2}^2)}{\mathcal{R}_{j_4}} \quad (s_{j_2}+k_{j_2}) + \frac{\mathcal{R}_{j_2}}{2}\, y_{j_2} d\right) + \\ 4\left(\frac{(v-\alpha m_{j_3}-\beta m_{j_3}^2)}{\mathcal{R}_{j_3}} \quad (s_{j_3}+k_{j_3}) + \frac{\mathcal{R}_{j_3}}{2}\, y_{j_3} d\right) + \\ 3\left(\frac{(v-\alpha m_{j_4}-\beta m_{j_4}^2)}{\mathcal{R}_{j_2}} \quad (s_{j_4}+k_{j_4}) + \frac{\mathcal{R}_{j_4}}{2}\, y_{j_4} d\right) + \\ \left(\frac{(v-\alpha m_{j_5}-\beta m_{j_5}^2)}{\mathcal{R}_{j_1}} \quad (s_{j_5}+k_{j_5}) + \frac{\mathcal{R}_{j_5}}{2}\, y_{j_5} d\right) \end{array}\right\}$$

------------------------ (4)

IV. SOLUTION METHODOLOGY USING EXTENSION OF LAGRANGIAN METHOD

1) Step1

$$p\left[\widetilde{TVC_j}(\mathcal{R}_j)\right] = \frac{1}{12}\left\{\begin{array}{l} \left(\frac{(v-\alpha m_{j_1}-\beta m_{j_1}^2)}{\mathcal{R}_{j_5}} \quad (s_{j_1}+k_{j_1}) + \frac{\mathcal{R}_{j_1}}{2}\, y_{j_1} d\right) + \\ 3\left(\frac{(v-\alpha m_{j_2}-\beta m_{j_2}^2)}{\mathcal{R}_{j_4}} \quad (s_{j_2}+k_{j_2}) + \frac{\mathcal{R}_{j_2}}{2}\, y_{j_2} d\right) + \\ 4\left(\frac{(v-\alpha m_{j_3}-\beta m_{j_3}^2)}{\mathcal{R}_{j_3}} \quad (s_{j_3}+k_{j_3}) + \frac{\mathcal{R}_{j_3}}{2}\, y_{j_3} d\right) + \\ 3\left(\frac{(v-\alpha m_{j_4}-\beta m_{j_4}^2)}{\mathcal{R}_{j_2}} \quad (s_{j_4}+k_{j_4}) + \frac{\mathcal{R}_{j_4}}{2}\, y_{j_4} d\right) + \\ \left(\frac{(v-\alpha m_{j_5}-\beta m_{j_5}^2)}{\mathcal{R}_{j_1}} \quad (s_{j_5}+k_{j_5}) + \frac{\mathcal{R}_{j_5}}{2}\, y_{j_5} d\right) \end{array}\right\}$$

-------------------- (5)

With $0< \mathcal{R}_{j_1} \le \mathcal{R}_{j_2} \le \mathcal{R}_{j_3} \le \mathcal{R}_{j_4} \le \mathcal{R}_{j_5}$, $\mathcal{R}_{j_2} - \mathcal{R}_{j_1} \ge 0$, $\mathcal{R}_{j_3} - \mathcal{R}_{j_2} \ge 0$, $\mathcal{R}_{j_4} - \mathcal{R}_{j_3} \ge 0$, $\mathcal{R}_{j_5} - \mathcal{R}_{j_4} \ge 0$ and $\mathcal{R}_{j_1} > 0$

Now let all the above partial derivatives in the equation (5) equal to zero, and solve $\mathcal{R}_{j_1}, \mathcal{R}_{j_2}, \mathcal{R}_{j_3}, \mathcal{R}_{j_4}, \mathcal{R}_{j_5}$ then we get

$$\mathcal{R}_{j_1} = \sqrt{\frac{2\left(v-\alpha m_{j_5}-\beta m_{j_5}^2\right)(s_{j_5}+k_{j_5})}{y_{j_1} d}} \qquad \mathcal{R}_{j_2} = \sqrt{\frac{2\left[3\left(v-\alpha m_{j_4}-\beta m_{j_4}^2\right)(s_{j_4}+k_{j_4})\right]}{3 y_{j_2} d}}$$

$$\mathcal{R}_{j_3} = \sqrt{\frac{2\left[4\left(v-\alpha m_{j_3}-\beta m_{j_3}^2\right)(s_{j_3}+k_{j_3})\right]}{4 y_{j_3} d}} \qquad \mathcal{R}_{j_4} = \sqrt{\frac{2\left[3\left(v-\alpha m_{j_2}-\beta m_{j_2}^2\right)(s_{j_2}+k_{j_2})\right]}{3 y_{j_4} d}}$$

$$\mathcal{R}_{j_5} = \sqrt{\frac{2\left(v-\alpha m_{j_1}-\beta m_{j_1}^2\right)(s_{j_1}+k_{j_1})}{y_{j_5} d}}$$

Now, $\mathcal{R}_{j_1} > \mathcal{R}_{j_2} > \mathcal{R}_{j_3} > \mathcal{R}_{j_4} > \mathcal{R}_{j_5}$

It does not satisfy the local optimum 0< $\mathcal{R}_{j_1} \leq \mathcal{R}_{j_2} \leq \mathcal{R}_{j_3} \leq \mathcal{R}_{j_4} \leq \mathcal{R}_{j_5}$

2) Step 2

Now fix the constraints as $L\left(\mathcal{R}_{j_1},\mathcal{R}_{j_2},\mathcal{R}_{j_3},\mathcal{R}_{j_4},\mathcal{R}_{j_5},\lambda\right) = \wp\left[\widetilde{TVC_j}(\mathcal{R}_j)\right] - \lambda\left(\mathcal{R}_{j_2} - \mathcal{R}_{j_1}\right)$

$$\wp\left[\widetilde{TVC_j}(\mathcal{R}_j)\right] - \lambda\left(\mathcal{R}_{j_2} - \mathcal{R}_{j_1}\right) = \frac{1}{12}\left\{\begin{array}{l} \left(\frac{\left(v-\alpha m_{j_1}-\beta m_{j_1}{}^2\right)}{\mathcal{R}_{j_5}}\left(s_{j_1}+k_{j_1}\right)+\frac{\mathcal{R}_{j_1}}{2}y_{j_1}d\right)+ \\ 3\left(\frac{\left(v-\alpha m_{j_2}-\beta m_{j_2}{}^2\right)}{\mathcal{R}_{j_4}}\left(s_{j_2}+k_{j_2}\right)+\frac{\mathcal{R}_{j_2}}{2}y_{j_2}d\right)+ \\ 4\left(\frac{\left(v-\alpha m_{j_3}-\beta m_{j_3}{}^2\right)}{\mathcal{R}_{j_3}}\left(s_{j_3}+k_{j_3}\right)+\frac{\mathcal{R}_{j_3}}{2}y_{j_3}d\right)+ \\ 3\left(\frac{\left(v-\alpha m_{j_4}-\beta m_{j_4}{}^2\right)}{\mathcal{R}_{j_2}}\left(s_{j_4}+k_{j_4}\right)+\frac{\mathcal{R}_{j_4}}{2}y_{j_4}d\right)+ \\ \left(\frac{\left(v-\alpha m_{j_5}-\beta m_{j_5}{}^2\right)}{\mathcal{R}_{j_1}}\left(s_{j_5}+k_{j_5}\right)+\frac{\mathcal{R}_{j_5}}{2}y_{j_5}d\right) \end{array}\right\} - \lambda\left(\mathcal{R}_{j_2} - \mathcal{R}_{j_1}\right) \quad (6)$$

Differentiating (6) with respect to $\mathcal{R}_{j_1},\mathcal{R}_{j_2},\mathcal{R}_{j_3},\mathcal{R}_{j_4},\mathcal{R}_{j_5},\lambda$ and equate to zero, we get

$$\text{Now } \mathcal{R}_{j_1} = \mathcal{R}_{j_2} = \sqrt{\frac{2\left(\left(v-\alpha m_{j_5}-\beta m_{j_5}{}^2\right)\left(s_{j_5}+k_{j_5}\right)+3\left(v-\alpha m_{j_4}-\beta m_{j_4}{}^2\right)\left(s_{j_4}+k_{j_4}\right)\right)}{y_{j_1}d+3y_{j_2}d}}$$

$$\mathcal{R}_{j_3} = \sqrt{\frac{2\times 4\left(v-\alpha m_{j_3}-\beta m_{j_3}{}^2\right)\left(s_{j_3}+k_{j_3}\right)}{4y_{j_3}d}} \qquad \mathcal{R}_{j_4} = \sqrt{\frac{2\times 3\left(v-\alpha m_{j_2}-\beta m_{j_2}{}^2\right)\left(s_{j_2}+k_{j_2}\right)}{3y_{j_4}d}}$$

$$\mathcal{R}_{j_5} = \sqrt{\frac{2\left(v-\alpha m_{j_1}-\beta m_{j_1}{}^2\right)\left(s_{j_1}+k_{j_1}\right)}{y_{j_5}d}}$$

Here, $\mathcal{R}_{j_3} > \mathcal{R}_{j_4}$, $\mathcal{R}_{j_4} > \mathcal{R}_{j_5}$

It does not satisfy the local optimum 0< $\mathcal{R}_{j_1} \leq \mathcal{R}_{j_2} \leq \mathcal{R}_{j_3} \leq \mathcal{R}_{j_4} \leq \mathcal{R}_{j_5}$

3) Step 3

Now fix the constraints as $L\left(\mathcal{R}_{j_1},\mathcal{R}_{j_2},\mathcal{R}_{j_3},\mathcal{R}_{j_4},\mathcal{R}_{j_5},\lambda_1,\lambda_2\right) = \wp\left[\widetilde{TVC_j}(\mathcal{R}_j)\right] - \lambda_1\left(\mathcal{R}_{j_2} - \mathcal{R}_{j_1}\right) - \lambda_2\left(\mathcal{R}_{j_3} - \mathcal{R}_{j_2}\right)$

$$\wp\left[\widetilde{TVC_j}(\mathcal{R}_j)\right] - \lambda_1\left(\mathcal{R}_{j_2} - \mathcal{R}_{j_1}\right) - \lambda_2\left(\mathcal{R}_{j_3} - \mathcal{R}_{j_2}\right) = \frac{1}{12}\left\{\begin{array}{l} \left(\frac{\left(v-\alpha m_{j_1}-\beta m_{j_1}{}^2\right)}{\mathcal{R}_{j_5}}\left(s_{j_1}+k_{j_1}\right)+\frac{\mathcal{R}_{j_1}}{2}y_{j_1}d\right)+ \\ 3\left(\frac{\left(v-\alpha m_{j_2}-\beta m_{j_2}{}^2\right)}{\mathcal{R}_{j_4}}\left(s_{j_2}+k_{j_2}\right)+\frac{\mathcal{R}_{j_2}}{2}y_{j_2}d\right)+ \\ 4\left(\frac{\left(v-\alpha m_{j_3}-\beta m_{j_3}{}^2\right)}{\mathcal{R}_{j_3}}\left(s_{j_3}+k_{j_3}\right)+\frac{\mathcal{R}_{j_3}}{2}y_{j_3}d\right)+ \\ 3\left(\frac{\left(v-\alpha m_{j_4}-\beta m_{j_4}{}^2\right)}{\mathcal{R}_{j_2}}\left(s_{j_4}+k_{j_4}\right)+\frac{\mathcal{R}_{j_4}}{2}y_{j_4}d\right)+ \\ \left(\frac{\left(v-\alpha m_{j_5}-\beta m_{j_5}{}^2\right)}{\mathcal{R}_{j_1}}\left(s_{j_5}+k_{j_5}\right)+\frac{\mathcal{R}_{j_5}}{2}y_{j_5}d\right) \end{array}\right\}$$

$$-\lambda_1\left(\mathcal{R}_{j_2} - \mathcal{R}_{j_1}\right) - \lambda_2\left(\mathcal{R}_{j_3} - \mathcal{R}_{j_2}\right) \quad (7)$$

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Differentiating (7) with respect to $\mathcal{R}_{j_1},\mathcal{R}_{j_2},\mathcal{R}_{j_3},\mathcal{R}_{j_4},\mathcal{R}_{j_5},\lambda_1,\lambda_2$ and equate to zero, we get

Now $\mathcal{R}_{j_1} = \mathcal{R}_{j_2} = \mathcal{R}_{j_3} =$

$$\sqrt{\frac{2\left(\left(v-\alpha m_{j_5}-\beta m_{j_5}{}^2\right)\left(s_{j_5}+k_{j_5}\right)+3\left(v-\alpha m_{j_4}-\beta m_{j_4}{}^2\right)\left(s_{j_4}+k_{j_4}\right)+4\left(v-\alpha m_{j_3}-\beta m_{j_3}{}^2\right)\left(s_{j_3}+k_{j_3}\right)\right)}{y_{j_1}d+3y_{j_2}d+4y_{j_3}d}}$$

$$\mathcal{R}_{j_4} = \sqrt{\frac{2\times 3\left(v-\alpha m_{j_2}-\beta m_{j_2}{}^2\right)\left(s_{j_2}+k_{j_2}\right)}{3y_{j_4}d}} \qquad \mathcal{R}_{j_5} = \sqrt{\frac{2\left(v-\alpha m_{j_1}-\beta m_{j_1}{}^2\right)\left(s_{j_1}+k_{j_1}\right)}{y_{j_5}d}}$$

Here, $\mathcal{R}_{j_4} > \mathcal{R}_{j_5}$

It does not satisfy the local optimum $0< \mathcal{R}_{j_1} \le \mathcal{R}_{j_2} \le \mathcal{R}_{j_3} \le \mathcal{R}_{j_4} \le \mathcal{R}_{j_5}$

4) Step 4

Now fix the constraints as $L\left(\mathcal{R}_{j_1},\mathcal{R}_{j_2},\mathcal{R}_{j_3},\mathcal{R}_{j_4},\mathcal{R}_{j_5},\lambda_1,\lambda_2,\lambda_3\right) = p\left[\widetilde{TVC_j(\mathcal{R}_j)}\right]-\lambda_1\left(\mathcal{R}_{j_2}-\mathcal{R}_{j_1}\right)-\lambda_2\left(\mathcal{R}_{j_3}-\mathcal{R}_{j_2}\right)-\lambda_3\left(\mathcal{R}_{j_4}-\mathcal{R}_{j_3}\right)$

$$p\left[\widetilde{TVC_j(\mathcal{R}_j)}\right]-\lambda_1\left(\mathcal{R}_{j_2}-\mathcal{R}_{j_1}\right)-\lambda_2\left(\mathcal{R}_{j_3}-\mathcal{R}_{j_2}\right)-\lambda_3\left(\mathcal{R}_{j_4}-\mathcal{R}_{j_3}\right)=$$

$$\frac{1}{12}\left\{\begin{array}{l}\left(\frac{\left(v-\alpha m_{j_1}-\beta m_{j_1}{}^2\right)}{\mathcal{R}_{j_5}}\left(s_{j_1}+k_{j_1}\right)+\frac{\mathcal{R}_{j_1}}{2}y_{j_1}d\right)+\\ 3\left(\frac{\left(v-\alpha m_{j_2}-\beta m_{j_2}{}^2\right)}{\mathcal{R}_{j_4}}\left(s_{j_2}+k_{j_2}\right)+\frac{\mathcal{R}_{j_2}}{2}y_{j_2}d\right)+\\ 4\left(\frac{\left(v-\alpha m_{j_3}-\beta m_{j_3}{}^2\right)}{\mathcal{R}_{j_3}}\left(s_{j_3}+k_{j_3}\right)+\frac{\mathcal{R}_{j_3}}{2}y_{j_3}d\right)+\\ 3\left(\frac{\left(v-\alpha m_{j_4}-\beta m_{j_4}{}^2\right)}{\mathcal{R}_{j_2}}\left(s_{j_4}+k_{j_4}\right)+\frac{\mathcal{R}_{j_4}}{2}y_{j_4}d\right)+\\ \left(\frac{\left(v-\alpha m_{j_5}-\beta m_{j_5}{}^2\right)}{\mathcal{R}_{j_1}}\left(s_{j_5}+k_{j_5}\right)+\frac{\mathcal{R}_{j_5}}{2}y_{j_5}d\right)\end{array}\right\}$$

$$-\lambda_1\left(\mathcal{R}_{j_2}-\mathcal{R}_{j_1}\right)-\lambda_2\left(\mathcal{R}_{j_3}-\mathcal{R}_{j_2}\right)-\lambda_3\left(\mathcal{R}_{j_4}-\mathcal{R}_{j_3}\right) \text{ ------------------------- } (8)$$

Differentiating (8) with respect to $\mathcal{R}_{j_1},\mathcal{R}_{j_2},\mathcal{R}_{j_3},\mathcal{R}_{j_4},\mathcal{R}_{j_5},\lambda_1,\lambda_2,\lambda_3$ and equate to zero,

Now, $\mathcal{R}_{j_1} = \mathcal{R}_{j_2} = \mathcal{R}_{j_3} = \mathcal{R}_{j_4} =$

$$\sqrt{\frac{2\left(\begin{array}{c}\left(v-\alpha m_{j_5}-\beta m_{j_5}{}^2\right)\left(s_{j_5}+k_{j_5}\right)+3\left(v-\alpha m_{j_4}-\beta m_{j_4}{}^2\right)\left(s_{j_4}+k_{j_4}\right)+4\left(v-\alpha m_{j_3}-\beta m_{j_3}{}^2\right)\left(s_{j_3}+k_{j_3}\right)+\\ 3\left(v-\alpha m_{j_2}-\beta m_{j_2}{}^2\right)\left(s_{j_2}+k_{j_2}\right)\end{array}\right)}{y_{j_1}d+3y_{j_2}d+4y_{j_3}d+3y_{j_4}d}}$$

$$\mathcal{R}_{j_5} = \sqrt{\frac{2\left(v-\alpha m_{j_1}-\beta m_{j_1}{}^2\right)\left(s_{j_1}+k_{j_1}\right)}{y_{j_5}d}}$$

Here, $\mathcal{R}_{j_1} > \mathcal{R}_{j_5}$

It does not satisfy the local optimum $0< \mathcal{R}_{j_1} \le \mathcal{R}_{j_2} \le \mathcal{R}_{j_3} \le \mathcal{R}_{j_4} \le \mathcal{R}_{j_5}$

5) Step 5

Now fix the constraints as $L\left(\mathcal{R}_{j_1},\mathcal{R}_{j_2},\mathcal{R}_{j_3},\mathcal{R}_{j_4},\mathcal{R}_{j_5},\lambda_1,\lambda_2,\lambda_3,\lambda_4\right) =$

$$p\left[\widetilde{TVC_j(\mathcal{R}_j)}\right]-\lambda_1\left(\mathcal{R}_{j_2}-\mathcal{R}_{j_1}\right)-\lambda_2\left(\mathcal{R}_{j_3}-\mathcal{R}_{j_2}\right)-\lambda_3\left(\mathcal{R}_{j_4}-\mathcal{R}_{j_3}\right)-\lambda_4\left(\mathcal{R}_{j_5}-\mathcal{R}_{j_4}\right)$$

Now,

$$p\left[\widetilde{TVC_j(\mathcal{R}_j)}\right]-\lambda_1\left(\mathcal{R}_{j_2}-\mathcal{R}_{j_1}\right)-\lambda_2\left(\mathcal{R}_{j_3}-\mathcal{R}_{j_2}\right)-\lambda_3\left(\mathcal{R}_{j_4}-\mathcal{R}_{j_3}\right)$$
$$-\lambda_4\left(\mathcal{R}_{j_5}-\mathcal{R}_{j_4}\right)=$$

$$\frac{1}{12}\left\{\begin{array}{l}\left(\frac{\left(v-\alpha m_{j_1}-\beta m_{j_1}^2\right)}{\mathcal{R}_{j_5}}\left(s_{j_1}+k_{j_1}\right)+\frac{\mathcal{R}_{j_1}}{2}y_{j_1}d\right)+\\ 3\left(\frac{\left(v-\alpha m_{j_2}-\beta m_{j_2}^2\right)}{\mathcal{R}_{j_4}}\left(s_{j_2}+k_{j_2}\right)+\frac{\mathcal{R}_{j_2}}{2}y_{j_2}d\right)+\\ 4\left(\frac{\left(v-\alpha m_{j_3}-\beta m_{j_3}^2\right)}{\mathcal{R}_{j_3}}\left(s_{j_3}+k_{j_3}\right)+\frac{\mathcal{R}_{j_3}}{2}y_{j_3}d\right)+\\ 3\left(\frac{\left(v-\alpha m_{j_4}-\beta m_{j_4}^2\right)}{\mathcal{R}_{j_2}}\left(s_{j_4}+k_{j_4}\right)+\frac{\mathcal{R}_{j_4}}{2}y_{j_4}d\right)+\\ \left(\frac{\left(v-\alpha m_{j_5}-\beta m_{j_5}^2\right)}{\mathcal{R}_{j_1}}\left(s_{j_5}+k_{j_5}\right)+\frac{\mathcal{R}_{j_5}}{2}y_{j_5}d\right)\end{array}\right\}$$
$$-\lambda_1\left(\mathcal{R}_{j_2}-\mathcal{R}_{j_1}\right)-\lambda_2\left(\mathcal{R}_{j_3}-\mathcal{R}_{j_2}\right)-\lambda_3\left(\mathcal{R}_{j_4}-\mathcal{R}_{j_3}\right)-\lambda_4\left(\mathcal{R}_{j_5}-\mathcal{R}_{j_4}\right)$$

------------------------ (9)

Differentiating (9) with respect to $\mathcal{R}_{j_1},\mathcal{R}_{j_2},\mathcal{R}_{j_3},\mathcal{R}_{j_4},\mathcal{R}_{j_5},\lambda_1,\lambda_2,\lambda_3,\lambda_4$ and equate to zero,

$$\widetilde{\mathcal{R}_j}^* = \sqrt{\frac{2\begin{pmatrix}\left(v-\alpha m_{j_5}-\beta m_{j_5}^2\right)\left(s_{j_5}+k_{j_5}\right)\\ +3\left(v-\alpha m_{j_4}-\beta m_{j_4}^2\right)\left(s_{j_4}+k_{j_4}\right)\\ +4\left(v-\alpha m_{j_3}-\beta m_{j_3}^2\right)\left(s_{j_3}+k_{j_3}\right)\\ +3\left(v-\alpha m_{j_2}-\beta m_{j_2}^2\right)\left(s_{j_2}+k_{j_2}\right)\\ +\left(v-\alpha m_{j_1}-\beta m_{j_1}^2\right)\left(s_{j_1}+k_{j_1}\right)\end{pmatrix}}{y_{j_1}d+3y_{j_2}d+4y_{j_3}d+3y_{j_4}d+y_{j_5}d}}$$

-------------------- (10)

Hence the equation (10) is the required equation of fuzzy optimal order quantity and equation (4) is the required fuzzy total cost.

V. NUMERICAL EXAMPLE

Consider the values in the below data to represent the proposed fuzzy inventory model

v=25,000	m_j=315
α=5	s_j=200
β=0.1	k_j=100
d=0.18	y_j=300

VI. SOLUTION IN CRISP MODEL

By using the given values in the data, we obtain the optimal order quantity in the crisp sense as $\mathcal{R}_j = 387.3$

By using the equation (1) in this model, we obtain the Total cost in crisp sense as

$TVC_j(\mathcal{R}_j) = 20{,}916.5$

VII. SOLUTION IN FUZZY MODEL

By using the given values in the data, we obtain the optimal order quantity in the fuzzy sense as $\widetilde{\mathcal{R}_j}^* = 387.25$

By using the equation (4) in this model, we obtain the Total cost in crisp sense as

$\widetilde{TVC_j(\mathcal{R}_j)}^* = 20{,}911.5$

VIII. CONCLUSION

The mathematical model proposed in this research is extremely valuable for industries that produce fast-moving consumer items while making inventory decisions. With the existing approach, management may make replenishment and shipment decisions to keep optimal stock levels at the SC's various entities. Especially when the store is dealing with quadratic price dependent demand. A mathematical model for non-coordinated Supply chain is constructed. Numerical example is performed to show that inventory decisions are optimal.

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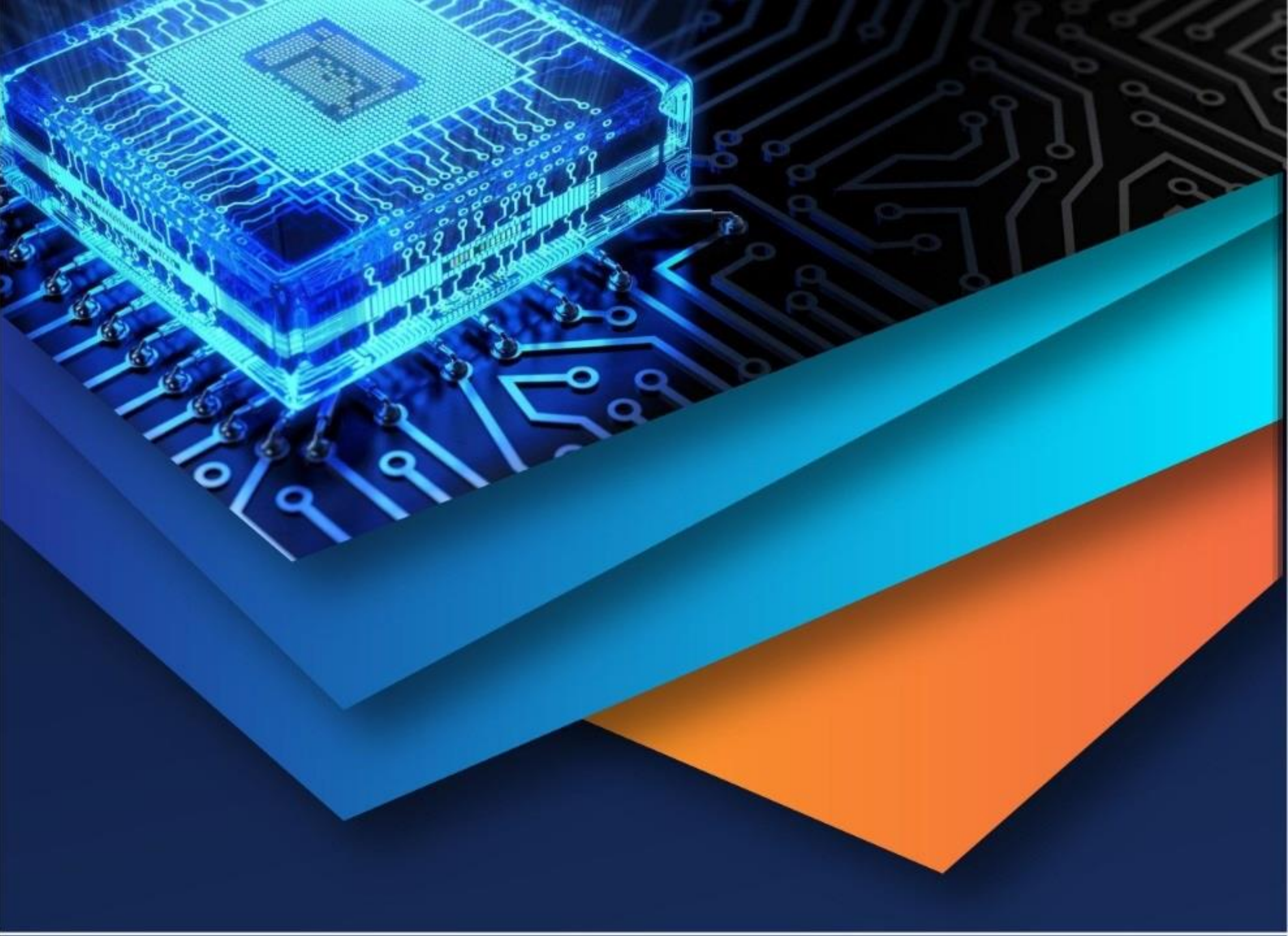

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