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International Journal For Research in  
Applied Science and Engineering Technology



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# **INTERNATIONAL JOURNAL FOR RESEARCH**

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

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**Volume: 10    Issue: II    Month of publication: February 2022**

**DOI: <https://doi.org/10.22214/ijraset.2022.40273>**

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# Characteristics of Fuzzy Petersen Graph and Platonic Graph with Fuzzy Rule

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**Abstract:** Graph theory is the concepts used to study and model various application in different areas. In this paper, we consider the The Petersen and also the Platonic graph using if-then-rules fuzzy numbers. The results are related to the find the degree of odd vertices and even vertices are same by applying if-then-rules through the paths described by fuzzy numbers.

**Keywords:** Degree of Vertex, Incident Graph, Peterson Graph, Platonic graph, Fuzzy IF-THEN rule.

## I. INTRODUCTION

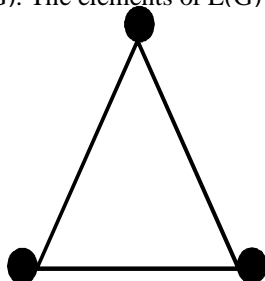
The first definition of fuzzy graph was introduced by Kaufmann (1973), based on Zadeh's fuzzy relations (1971). A more elaborate definition is due to Azriel Rosenfeld who considered fuzzy relation on fuzzy sets and developed the theory of fuzzy graph in 1975. During the same time Yeh and Bang have also introduced various connectedness concepts in fuzzy graph. Till now fuzzy graphs have been witnessing a tremendous growth and finds applications in many branches of engineering and technology.

Fuzzy systems based on fuzzy if-then rules have been successfully applied to various theorems in the field of fuzzy control. Fuzzy Rule based system has high comprehensibility because human users can easily understand the meaning of each fuzzy if-then rule through its linguistic interpretation. Graph theory has numerous applications to problems in system analysis, operation research, transportation and economics. In many cases, however some aspects of graph theoretic problem may be uncertain. For example, the vehicle travel time or vehicle capacity on a road network may not be known exactly. In such cases, it is natural to deal with the uncertainty using fuzzy set theory. The concepts of a fuzzy graph are a natural generalization of crisp graphs, given by Rosenfeld [Zadeh et al., 1975].

## II. BASIC CONCEPT

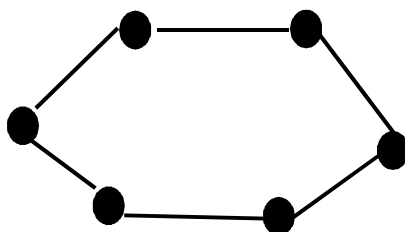
### A. Definition 2.1

A graph  $G$  consists of a pair  $G: (V, E)$  where  $V(G)$  is a non empty finite set whose elements are called points or vertices and  $E(G)$  is a set of unordered pairs of distinct elements of  $V(G)$ . The elements of  $E(G)$  are called lines or edges of the graph  $G$ .



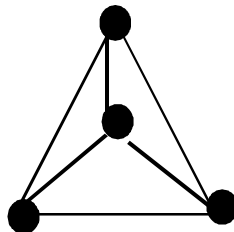
### B. Definition 2.2: Cycle Graph

In graph theory, a cycle graph or circular graph is a graph that consists of a single cycle, or in other words, some number of vertices (at least 3) connected in a closed chain. The cycle graph with  $n$  vertices is called  $C_n$ . The number of vertices in  $C_n$  equals the number of edges, and every vertex has degree 2; that is, every vertex has exactly two edges incident with it.



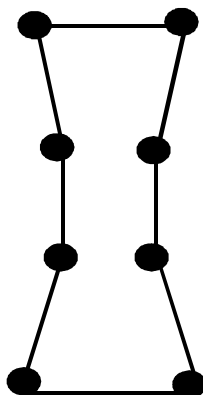
*C. Definition 2.3*

A wheel graph is obtained from a cycle graph  $C_{n-1}$  by adding a new vertex. That new vertex is called a Hub which is connected to all the vertices of  $C_{n-1}$ .



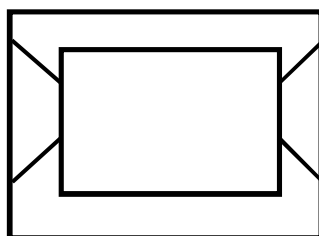
*D. Definition 2.4: Hamilton Cycle*

A path that contains every vertex of  $G$  is called a Hamilton path of  $G$ . similarly a Hamilton cycle of  $G$  is a cycle that contains every vertex of  $G$



*E. Definition 2.5: Hamilton graph*

A graph Hamiltonian if it contains a Hamilton cycle is called Hamilton graph.



*F. Definition 2.6*

Let  $V$  be a non- empty set. A fuzzy graph is a pair of function  $G : (\sigma, \mu)$ , where  $\sigma$  is a fuzzy subset of  $V$  and  $\mu$  is a symmetric fuzzy relation on  $\sigma$ . i.e.,  $\sigma : V \rightarrow [0,1]$  such that  $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$  for all  $u, v$  in  $V$ . where  $uv$  denotes the edge between  $u$  and  $v$  and  $\sigma(u) \wedge \sigma(v)$  denotes the minimum of  $\sigma(u)$  and  $\sigma(v)$ .

*G. Definition 2.7: Incident*

When a vertex  $\sigma(u_i)$  is an end vertex of some edges  $\mu(u_i, v_j)$  of any fuzzy graph  $G : (\sigma, \mu)$ . Then  $\sigma(u_i)$  and  $\mu(u_i, v_j)$  are said to be incident to each other.

#### H. Definition 2.8

The degree of any vertex  $\sigma(u_i)$  of a fuzzy graph is sum of degree of membership of all those edges which are incident on vertex  $\sigma(u_i)$  and is denoted by  $d[\sigma(u_i)]$ .

#### I. Definition 2.9: Fuzzy If-Then Rule

A fuzzy rule is defined as a conditional statement in the form:

IF  $x$  is  $A$ , THEN  $y$  is  $B$ ; where  $x$  and  $y$  are linguistic variable;  $A$  and  $B$  are linguistic values determined by fuzzy sets on the universe of discourse  $X$  and  $Y$ , respectively.

- 1) A rule is also called a fuzzy implication.
- 2) “ $x$  is  $A$ ” is called the antecedent or premise.
- 3) “ $y$  is  $B$ ” is called the consequent or conclusion.

Example:

- IF pressure is high, THEN volume is small.
- IF the speed is high, THEN apply the brake a little.

### III. FUZZY PETERSEN GRAPH WITH FUZZY RULE

#### 1) Theorem

In fuzzy Petersen graph  $G$ , the sum of degrees of vertices of even degree is equal to twice the degree of membership of all the edges and the difference of the sum of degrees of vertices of odd degree.

**Proof:** Let  $G: (\sigma, \mu)$  is fuzzy Petersen graph. Consider 10-vertices  $\{\sigma(v_1), \sigma(v_2), \sigma(v_3), \sigma(v_4), \dots, \sigma(v_{10})\}$  of fuzzy Petersen graph  $G: (\sigma, \mu)$

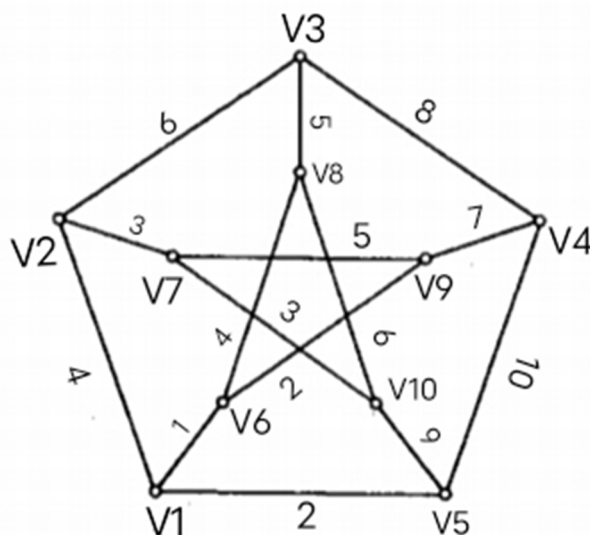


Figure: Petersen graph

IF the membership grades of edges which are incident on any degree of vertex  $\sigma(\ )$  are added, THEN the sum of corresponding membership value of vary.

$$d[\sigma(v_1)] = 2 + 4 + 1 = 7$$

$$d[\sigma(v_2)] = 4 + 6 + 3 = 13$$

$$d[\sigma(v_3)] = 6 + 8 + 5 = 19$$

$$d[\sigma(v_4)] = 8 + 10 + 7 = 25$$

$$d[\sigma(v_5)] = 10 + 2 + 9 = 21$$

$$d[\sigma(v_6)] = 1 + 2 + 4 = 7$$

$$d[\sigma(v_7)] = 3 + 3 + 5 = 11$$

$$d[\sigma(v_8)] = 5 + 6 + 4 = 15$$

$$d[\sigma(v_9)] = 7 + 5 + 2 = 14$$

$$d[\sigma(v_{10})] = 9 + 3 + 6 = 18$$

IF the membership grades of edges are added, THEN we find the degree of edges,

$$\sum_{i=1}^{11} \mu(u_i, v_{i+1}) = 2+4+6+8+10+1+3+5+7+9+2+4+5+3+6=75$$

$$\sum_{i=1}^{11} \mu(u_i, v_{i+1}) = 75$$

$$\sum_{i=1}^{11} d[\sigma(v_i)] = \text{Twice the sum of degree of membership of } (u_i, v_{i+1})$$

$$\text{Therefore, } \sum_{i=1}^n d(v_i) = 2 \sum_{i=1}^n \mu(u_i, v_{i+1})$$

But here,

The  $\deg[\sigma(v_i)]$  has been split into two parts.

$$\text{i.e., } \sum_{i=1}^k \deg(v_i) + \sum_{i=1}^n \deg(w_k) = 2 \sum_{i=1}^n \mu(u_i, v_{i+1})$$

$\sum_{i=1}^k d(v_i)$  denotes the sums over even degree vertices, i.e.,  $\sigma(v_2), \sigma(v_4), \sigma(v_6), \sigma(v_8), \sigma(v_{10})$ .

$$\sum_{i=1}^k d(v_i) = 13+25+7+15+18$$

$$\sum_{i=1}^k d(v_i) = 78$$

Now,  $\sum_{i=1}^n d(w_k)$  denotes the sum over odd degree vertices, i.e.,  $\sigma(v_1), \sigma(v_3), \sigma(v_5), \sigma(v_7), \sigma(v_9)$

$$\sum_{i=1}^n d(w_k) = 7+19+21+11+14$$

$$\sum_{i=1}^n d(w_k) = 72$$

Hence If the sum of degrees of vertices of even degree is 72, then it is equal to twice the degree of membership of all edges and difference of the sum of degrees of vertices of odd degree.

$$\sum_{i=1}^k d(v_i) = 2 \sum_{i=1}^n \mu(u_i, v_{i+1}) - \sum_{i=1}^n d(w_k)$$

$$72 = 2(75) - 78$$

$$= 150 - 78$$

$$72 = 72$$

Hence the theorem.

## 2) Theorem

In fuzzy Platonic graph G, the sum of degrees of vertices of even degree is equal to twice the degree of membership of all the edges and the difference of the sum of degrees of vertices of odd degree.

**Proof:** Let  $G: (\sigma, \mu)$  is fuzzy Platonic graph. Consider 4-vertices  $\{\sigma(v_1), \sigma(v_2), \sigma(v_3), \sigma(v_4)\}$  of fuzzy platonic graph  $G: (\sigma, \mu)$

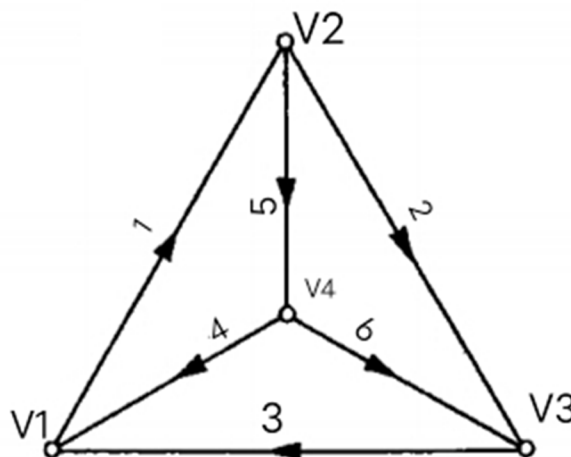


Figure: Fuzzy platonic graph

IF the membership grades of edges which are incident on any degree of vertex  $\sigma( )$  are added, THEN the sum of corresponding membership value of vary.

$$d[\sigma(v_1)] = 3+1+4=8$$

$$d[\sigma(v_2)] = 1+5+2=8$$



$$d[\sigma(v_3)] = 6+2+3=11$$

$$d[\sigma(v_4)] = 4+6+5=15$$

IF the membership grades of edges are added, THEN we find the degree of edges,

$$\sum_{i=1}^6 \mu(u_i, v_{i+1}) = 3+1+2+4+6+5=21$$

$$\sum_{i=1}^6 \mu(u_i, v_{i+1}) = 21$$

$$\sum_{i=1}^6 d[\sigma(v_i)] = \text{Twice the sum of degree of membership of } (u_i, v_{i+1})$$

$$\text{Therefore, } \sum_{i=1}^n d(v_i) = 2 \sum_{i=1}^n \mu(u_i, v_{i+1})$$

But here,

The  $\deg[\sigma(v_i)]$  has been split into two parts.

$$\text{i.e., } \sum_{i=1}^k \deg(v_i) + \sum_{i=1}^n \deg(w_k) = 2 \sum_{i=1}^n \mu(u_i, v_{i+1})$$

$$\sum_{i=1}^k d(v_i) \text{ denotes the sums over even degree vertices, i.e., } \sigma(v_2), \sigma(v_4).$$

$$\sum_{i=1}^k d(v_i) = 8+15=23$$

$$\sum_{i=1}^k d(v_i) = 23$$

$$\text{Now, } \sum_{i=1}^n d(w_k) \text{ denotes the sum over odd degree vertices, i.e., } \sigma(v_1), \sigma(v_3).$$

$$\sum_{i=1}^n d(w_k) = 8+11=19$$

$$\sum_{i=1}^n d(w_k) = 19$$

If the sum of degrees of vertices of even degree is 19, it is equal to twice the degree of membership of all edges and difference of the sum of degrees of vertices of odd degree.

$$\sum_{i=1}^k d(v_i) = 2 \sum_{i=1}^n \mu(u_i, v_{i+1}) - \sum_{i=1}^n d(w_k)$$

$$23 = 2(21) - 19$$

$$23 = 23$$

Hence the theorem.

#### IV. CONCLUSION

In this paper we obtain that Petersen graph and Platonic graph. We conclude that the odd degree of vertices is equal to the even degree of vertices, for both the above cases with the help of IF-THEN rules (i.e., L.H.S=R.H.S). Finally, IF-THEN rules applied through shortest paths are shown with different analysis.

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