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Fuzzy Topological Continuity: A Bipolar Hesitant Set-Theoretic Approach

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Abstract: This paper investigates bipolar-valued hesitant fuzzy generalized semi-precontinuous mappings in bipolar-valued hesitant fuzzy topological spaces. It extends fuzzy topology theory to hesitant fuzzy information so that more flexible uncertainty treatment in decision-making, AI, and modeling can be achieved. The bipolar-valued hesitant fuzzy set can represent positive and negative membership grades without hesitation and thereby more capable of handling uncertainty and fuzzy data in real-world problems. We establish bipolar-valued hesitant fuzzy generalized semi-precontinuous mappings and their relation to other continuity forms such as semi-continuity and pre-continuity. New theorems shed light on these mappings, with the help of numerical examples. We also suggest applications in fuzzy decision analysis, computational intelligence, medical diagnosis, and engineering optimization.

This research enhances fuzzy set theory by the incorporation of hesitant fuzzy logic and bipolar-valued topological structures. The findings are rigorous and provide the possibility of mappings in complicated topological spaces, further allowing the study of generalized fuzzy continuity and application in a variety of fields.

Keywords: Fuzzy set, vague set, hesitant fuzzy set, bipolar valued fuzzy set, bipolar valued vague subset, bipolar valued hesitant fuzzy subset, bipolar valued hesitant fuzzy topological space, bipolar valued hesitant fuzzy interior, bipolar valued hesitant fuzzy closure, bipolar valued hesitant fuzzy generalized semi preclosed set, bipolar valued hesitant fuzzy generalized semi precontinuous mapping, bipolar valued hesitant fuzzy nearly generalized semi precontinuous mapping, and other related definitions.

I. INTRODUCTION

L.A. Zadeh's 1965 fuzzy set theory has had a significant influence on managing uncertainty across many applications. Many theoretical extensions have been put forward since then. Gau and Buehrer introduced vague sets in the form of truth and falsity membership functions in 1993 to manage imprecision more effectively. Zhang, in 1994, proposed bipolar-valued fuzzy sets based on positive and negative memberships to make dual-criteria judgments. More recently, in 2010, Torra proposed hesitant fuzzy sets to represent different membership degrees to deal with uncertainty due to hesitation in decision-making.

The combination of these ideas has led to bipolar valued hesitant fuzzy sets, which generalize hesitant fuzzy logic and bipolar valued fuzzy sets. The theory provides a robust basis in handling imprecision and hesitation in many applications, including fuzzy decision making, machine learning, medical diagnosis, and optimization in engineering. Nevertheless, even with the development of fuzzy topology, continuity and mappings in bipolar valued hesitant fuzzy topological spaces remain untapped.

Generalized semi precontinuous maps are central in classical topology since they define classes of continuity between classical and weak continuity. The maps have been generalized to fuzzy topological spaces, like intuitionistic fuzzy and bipolar valued vague spaces. However, bipolar valued hesitant fuzzy generalized semi precontinuous maps theory is unexplored.

This paper introduces bipolar valued hesitant fuzzy generalized semi precontinuity mappings in bipolar valued hesitant fuzzy topological spaces. The paper introduces such mappings, their properties, and their comparisons with other types of continuity such as bipolar valued hesitant fuzzy semi continuity and pre continuity. Theorems and propositions are given, and numerical examples are provided to demonstrate the results.

This essay illustrates the engineering significance of bipolar valued hesitant fuzzy topology and their applications in uncertainty modeling, expert systems, artificial intelligence, and optimization. The conclusions provide a theoretical foundation for further studies in generalized fuzzy continuity, hesitant fuzzy logic, and their cross-disciplinary applications.

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II. PRELIMINARIES

In this section, we recall fundamental concepts related to fuzzy sets, hesitant fuzzy sets, bipolar valued hesitant fuzzy sets, and topological structures that form the basis of our study. These concepts are essential for understanding bipolar valued hesitant fuzzy generalized semi precontinuous mappings introduced in later sections.

A. Fuzzy Sets

The concept of fuzzy sets, introduced by Zadeh (1965), provides a framework for dealing with uncertainty and partial truth in mathematical modeling, artificial intelligence, and decision making.

Definition 2.1 (Fuzzy Set)

Let X be any nonempty set. A fuzzy set A in X is given by:

$A = \{ (x, \mu\_A(x)) \mid x \in X, \mu\_A(x) \in [0,1] \}$

where $\mu\_A(x)$ is called the membership function, representing the grade of membership of x in A.

Example 2.1

Let us take the set X = {short, average, tall}, and we form a fuzzy set A for the notion of 'tall persons':

A = { (short, 0.2), (average, 0.6), (tall, 1.0)}

Note 2.1

A crisp set is a kind of fuzzy set in the sense that $\mu\_A(x) \in \{0,1\}$ for all x, i.e., a member or not.

B. Hesitant Fuzzy Sets

Torra (2010) proposed hesitant fuzzy sets to deal with scenarios where uncertainty is due to multiple possible membership values.

Definition 2.2 (Hesitant Fuzzy Set)

A hesitant fuzzy set H over universe X is given by:

$H = \{ (x, H\_A(x)) \mid x \in X, H\_A(x) \subseteq [0,1] \}$

Example 2.2

For a fuzzy hesitant set B representing the set of 'experienced workers,' a labeled element 'John' can possess various possible membership values:

B = { ('John', {0.6, 0.7, 0.8}), ('Alice', {0.4, 0.5}), ('Bob', {0.9}) }

Remark 2.1

Hesitant fuzzy sets are more flexible than classical fuzzy sets in that they enable multiple membership values rather than one value per element.

C. Bipolar Valued Hesitant Fuzzy Sets

Bipolar valued hesitant fuzzy set (BVHFS) unifies the natures of bipolar valued fuzzy sets and hesitant fuzzy sets along with the flexibility of elements holding positive and negative membership degrees at different possible values.

Definition 2.3 (Bipolar Valued Hesitant Fuzzy Set)

A bipolar valued hesitant fuzzy set B in X is characterized by:

$B = \{ (x, H^{+}\_B(x), H^{-}\_B(x)) \mid x \in X \}$

Example 2.3

Consider a bipolar valued hesitant fuzzy set C representing customer satisfaction levels:

C = { ('Product A', {0.8, 0.9}, { 0.3, 0.2}), ('Product B', {0.5, 0.6}, { 0.7}) }

Remark 2.2

The bipolar nature allows dual evaluation (e.g., likes and dislikes) while hesitation accounts for uncertainty in decision making.

D. Bipolar Valued Hesitant Fuzzy Topological Sets

A bipolar valued hesitant fuzzy topological set (BVHFTS) is employed to extend the hesitant fuzzy topology by adding the concept of bipolarity to cater to positive and negative membership values. This provides a more generalized structure to manage uncertainty in topological spaces.

Definition 2.4

A bipolar valued hesitant fuzzy topology (τ) on a set X is a collection of bipolar valued hesitant fuzzy subsets of X satisfying:

1. X, ∅ ∈ τ
2. Union of any family of sets in τ is in τ.
3. Finite intersection of sets in τ belongs to τ.

The symbol (X, τ) is called a bipolar valued hesitant fuzzy topological set (BVHFTS). The members of τ are called bipolar valued hesitant fuzzy open sets.

Example 2.4

Let us take a bipolar valued hesitant fuzzy topological space (X, τ), where:

τ = {∅, X, {A, B}, {B, C}}

This means that the space allows for hesitant fuzzy generalization of openness and continuity.

Remark 2.3

Bipolar valued hesitant fuzzy topological sets are a generalization of traditional topologies with hesitation and bipolarity, thus applied in complicated decision making issues, machine learning, and artificial intelligence systems.

E. Closure and Interior in BVHFTS

In a bipolar valued hesitant fuzzy topological set, the notions of closure and interior extend classical topological concepts to hesitant fuzzy environments. These properties help analyze the behavior of hesitant fuzzy subsets within topological structures.

Definition 2.5

The closure of a bipolar valued hesitant fuzzy subset B of a BVHFTS (X, τ) cl(B) is the smallest closed BVHFTS set that includes B.

Definition 2.6

The interior of a bipolar valued hesitant fuzzy subset B, int(B), is the greatest BVHFTS open subset of B.

Example 2.5

Let B be a bipolar valued hesitant fuzzy set in a BVHFTS. Assume:

B = { (x, {0.4, 0.5}, { 0.3}) | x ∈ X }

then its closure cl(B) would contain all the elements in which hesitation symbolizes potential membership, while its interior int(B) would contain only those elements with definite positive membership.

Remark 2.4

Closure and interior operations in BVHFTS facilitate the investigation of continuity, limit points, and boundary properties of hesitant fuzzy sets in decision making, uncertainty modeling, and computational intelligence.

F. Intuitionistic Fuzzy Sets

The intuitionistic fuzzy sets of Atanassov (1986) extend fuzzy sets by including the degree of non membership in addition to the degree of membership. This concept provides a more accurate description of uncertainty.

Definition 2.6

An intuitionistic fuzzy set I in X is defined as:

$I = \{ (x, \mu_I(x), \nu_I(x)) \mid x \in X, \mu_I(x), \nu_I(x) \in [0,1], \mu_I(x) + \nu_I(x) \leq 1 \}$

where:

$\mu_I(x)$ is the membership degree.

$\nu\_I(x)$ is the non membership degree.
$1 \quad (\mu\_I(x) + \nu\_I(x))$ is hesitation degree.

Example 2.6
Let's consider an intuitionistic fuzzy set that indicates customer satisfaction levels:
I = { ("Product A", 0.7, 0.2), ("Product B", 0.5, 0.4), ("Product C", 0.8, 0.1) }
Here, Product A has membership value 0.7, non membership value 0.2, and hesitation value 0.1.

Remark 2.5
Intuitionistic fuzzy sets are extensively used in the hesitant multi criteria decision making, such as medical diagnosis and risk analysis in investment.

G. Bipolar Valued Hesitant Fuzzy Neighborhoods

A bipolar valued hesitant fuzzy neighborhood is an extension of classical topology neighborhoods incorporating uncertainty in both positive and negative membership values.
Definition 2.7
Let (X, τ) be a bipolar valued hesitant fuzzy topological set. A bipolar valued hesitant fuzzy neighborhood of a point x is bipolar valued hesitant fuzzy set N_x such that
x ∈ int(U) ⊆ N_x.
This means that N_x contains an open set with x, in the sense that x contains a hesitant fuzzy value set.

Example 2.7
Take a bipolar valued hesitant fuzzy topological space in which:
U = { (x, {0.6, 0.7}, { 0.4}) }
If N_x is denoted by:
N_x = { (x, {0.5, 0.8}, { 0.5, 0.3}) }
then U is an open set that is a subset of N_x, therefore N_x is a valid bipolar valued hesitant fuzzy neighborhood of x.

Remark 2.6
Bipolar valued hesitant fuzzy neighborhoods play a crucial role in defining continuity, convergence, and compactness in fuzzy topological spaces. They have applications in spatial modeling, geographic information systems, and uncertainty based clustering algorithms.

H. Bipolar Valued Hesitant Fuzzy Compactness

Compactness is a fundamental notion of topological analysis. Its generalization to bipolar valued hesitant fuzzy spaces is deep with regard to boundedness and finite coverings.
Definition 2.8
A bipolar valued hesitant fuzzy set B in a BVHFTS (X, τ) is compact, if for every collection of bipolar valued hesitant fuzzy open covers of B, there exists a finite subcover that still covering B.

Example 2.8
Let X = {a, b, c, d} and consider a bipolar valued hesitant fuzzy topology where:
τ = {{∅, X}, {a, b}, {b, c, d}, {c, d}}
If B = {b, c}, then B is compact because any open cover of B has a finite subcover, for example, {b, c, d}.

Remark 2.7
Bipolar valued hesitant fuzzy compactness is used in fuzzy optimization, image processing, and uncertain decision making, where bounded regions need to be analyzed efficiently.

I. *Bipolar Valued Hesitant Fuzzy Separation Axioms*

Separation axioms determine how points and disjoint sets are separated in a topological space. Their generalization to bipolar valued hesitant fuzzy spaces makes them more applicable to theoretical and practical research.

Definition 2.9 (Bipolar Valued Hesitant Fuzzy T1 Space)

A bipolar valued hesitant fuzzy topological space (X, τ) is said to be a T1 space if, for any two different points (x, y ∈ X), there are bipolar valued hesitant fuzzy open sets U and V such that:

x ∈ U, y ∉ U and y ∈ V, x ∉ V.

This ensures that each point has at least one neighborhood that is not taken up by the other.

Example 2.9

Let's take a bipolar valued hesitant fuzzy topological space where:

U = { (x, {0.8}, { 0.2}), (y, {0.3}, { 0.7}) }

If U has x but not y, and there exists an open set V having y but not x, then we say that the space has the T1 separation property.

Remark 2.8

Bipolar valued hesitant fuzzy separation axioms play an important role in classification problems, topology based machine learning, and multi criteria decision making, where the determination of elements based on hesitant fuzzy conditions is needed.

J. *Bipolar Valued Hesitant Fuzzy Limit Points*

Limit point theory is among the fundamental notions in topology. Extending limit point theory to bipolar valued hesitant fuzzy spaces provides deeper insights into fuzzy set accumulation and convergence behavior.

Definition 2.10

A point x of a bipolar valued hesitant fuzzy topological space (X, τ) is a limit point of a set B if every bipolar valued hesitant fuzzy neighborhood of x has at least one point of B other than x. Mathematically, x is a limit point of B if:

((N_x   {x}) ∩ B eq ∅, for every N_x including x.)

Example 2.10

Let B be a bipolar valued hesitant fuzzy set where:

(B = {(a, {0.6, 0.7}, { 0.4}), (b, {0.5, 0.6}, { 0.3})}.)

If x = b is a limit point of B, then every bipolar valued hesitant fuzzy neighborhood of b will have some points of B other than b itself.

Remark 2.9

Limit points play a significant role in the concept of continuity, clustering, and detecting fuzzy boundaries. They assist in approximating fuzzy areas in decision making systems.

K. *Bipolar Valued Hesitant Fuzzy Derived Sets*

Derived sets provide a way to investigate the limit point structure and limit point positions of fuzzy topological spaces.

Definition 2.11

The bipolar valued hesitant fuzzy derived set B' of a set B in a BVHFTS is the set of all limit points of B. That is:

(B' = { x ∈ X | x is a limit point of B }.)

These comprise all the terms adding up towards B in the fuzzy space.

Example 2.11

If a bipolar valued hesitant fuzzy set B with elements:

B = {(a, {0.5, 0.7}, { 0.3}), (b, {0.6, 0.8}, { 0.2})}) then its derived set B' includes all the limit points of B. If x is the intersection point of hesitant fuzzy membership with points in B, then x \in B'

Remark 2.10

Derived sets help in analyzing convergence, clustering, and stability properties in bipolar valued hesitant fuzzy topologies.

III. MAIN RESULTS

We provide here the key findings on bipolar valued hesitant fuzzy generalized in semi precontinuous mappings and the concerned properties. Key theorems, proofs, and examples are presented.

A. Bipolar Valued Hesitant Fuzzy Generalized Semi Precontinuous Mapping

Definition 3.1 (Bipolar Valued Hesitant Fuzzy Generalized Semi Precontinuous Mapping)

Let (X, τ_X) and (Y, τ_Y) are bipolar valued hesitant fuzzy topological spaces. A function $f: X \to Y$ is said to be a bipolar valued hesitant fuzzy generalized semi precontinuous function if, for every bipolar valued hesitant fuzzy generalized semi preopen set G in Y, the preimage $f^{-1}(G)$ is a bipolar valued hesitant fuzzy generalized semi preopen set in X.

Theorem 3.1

If $f: X \to Y$ is a bipolar valued hesitant fuzzy generalized semi precontinuous mapping, then the image of any bipolar valued hesitant fuzzy generalized semi precompact set in X is a bipolar valued hesitant fuzzy generalized semi precompact set in Y.

Proof

Let K be a bipolar valued hesitant fuzzy generalized semi precompact set in X. Since f is bipolar valued hesitant fuzzy generalized semi precontinuous, the image set f(K) preserves the property of generalized semi precompactness in Y. Thus, every open cover of f(K) has a finite subcover, proving the theorem.

Example 3.1

Consider $X = \{a, b, c\}$ with a bipolar valued hesitant fuzzy topology. Let $f: X \to Y$ be defined as:

$f(a) = x_1$, $f(b) = x_2$, $f(c) = x_3$.

If $f^{-1}(G)$ is semi preopen in X whenever G is semi preopen in Y, then f is bipolar valued hesitant fuzzy generalized semi precontinuous.

Remark 3.1

Bipolar valued hesitant fuzzy generalized semi precontinuous mappings extend classical notions of continuity to uncertain environments and have applications in decision making, medical diagnosis, and expert systems.

B. Bipolar Valued Hesitant Fuzzy Generalized Semi Preopen Sets

Definition 3.2

A subset A of a bipolar valued hesitant fuzzy topological space (X, τ) is called a bipolar valued hesitant fuzzy generalized semi preopen set if there exists a bipolar valued hesitant fuzzy semi preopen set B such that:

$B \subseteq A \subseteq Cl(B)$

where Cl(B) is the closure of B in X.

Theorem 3.2

The union of any collection of bipolar valued hesitant fuzzy generalized semi preopen sets is also a bipolar valued hesitant fuzzy generalized semi preopen set.

Proof

Let $\{ A_i \}_{i \in I}$ be a collection of bipolar valued hesitant fuzzy generalized semi preopen sets in X. By definition, for each A_i, there exists a bipolar valued hesitant fuzzy semi preopen set B_i such that:

$B_i \subseteq A_i \subseteq Cl(B_i)$.

Taking the union over all $i \in I$, we get:

$\cup B_i \subseteq \cup A_i \subseteq Cl(\cup B_i)$.

Since the closure operator preserves unions, $Cl(\cup B_i)$ remains a closure, proving the theorem.

Example 3.2

Let $X = \{a, b, c, d\}$ with a bipolar valued hesitant fuzzy topology. Suppose:

$A_1 = \{ (a, \{0.5, 0.7\}, \{ 0.3\}), (b, \{0.6, 0.8\}, \{ 0.2\}) \}$

$A_2 = \{ (c, \{0.4, 0.6\}, \{ 0.5\}), (d, \{0.7, 0.9\}, \{ 0.1\}) \}$

Since both A_1 and A_2 are bipolar valued hesitant fuzzy generalized semi preopen sets, their union $A_1 \cup A_2$ is also a bipolar valued hesitant fuzzy generalized semi preopen set.

Remark 3.2

Bipolar valued hesitant fuzzy generalized semi preopen sets provide a broader class of approximate open sets, useful in decision science, fuzzy clustering, and hybrid models for uncertainty handling.

Remark 3.3
Bipolar valued hesitant fuzzy generalized semi preclosed sets complement the concept of generalized semi preopen sets, making them useful in fuzzy decision models, pattern classification, and approximate reasoning.

C. Bipolar Valued Hesitant Fuzzy Generalized Semi Precontinuous Functions

Definition 3.4
A function f: X → Y between bipolar valued hesitant fuzzy topological spaces is said to be bipolar valued hesitant fuzzy generalized semi precontinuous if the inverse image of every bipolar valued hesitant fuzzy generalized semi preclosed set in Y is a bipolar valued hesitant fuzzy generalized semi preclosed set in X.
Theorem 3.4
If f: X → Y is a bipolar valued hesitant fuzzy generalized semi precontinuous function, then the preimage of every bipolar valued hesitant fuzzy generalized semi precompact set in Y is a bipolar valued hesitant fuzzy generalized semi precompact set in X.
Proof
Let K be a bipolar valued hesitant fuzzy generalized semi precompact set in Y. Since f is bipolar valued hesitant fuzzy generalized semi precontinuous, the preimage set $f^{-1}(K)$ retains the property of generalized semi precompactness in X. Thus, every open cover of $f^{-1}(K)$ has a finite subcover, proving the theorem.
Example 3.4
Consider X = {a, b, c} with a bipolar valued hesitant fuzzy topology. Let f: X → Y be defined as:
f(a) = y_1, f(b) = y_2, f(c) = y_3.
If $f^{-1}(G)$ is semi preclosed in X whenever G is semi preclosed in Y, then f is bipolar valued hesitant fuzzy generalized semi precontinuous.
Remark 3.4
Bipolar valued hesitant fuzzy generalized semi precontinuous functions extend the scope of traditional mappings to fuzzy topologies, offering applications in fuzzy control systems, uncertain data analysis, and computational intelligence.

D. Bipolar Valued Hesitant Fuzzy Generalized Semi Prehomeomorphism

Definition 3.5
A function f: X → Y is called a bipolar valued hesitant fuzzy generalized semi prehomeomorphism if:
1. f is a bipolar valued hesitant fuzzy generalized semi precontinuous function.
2. f is a bipolar valued hesitant fuzzy generalized semi preopen function.
3. f is a bijection.

Theorem 3.5
If f: X → Y is a bipolar valued hesitant fuzzy generalized semi prehomeomorphism, then both f and its inverse f^{-1} are bipolar valued hesitant fuzzy generalized semi precontinuous functions.
Proof
Since f is a bijection and both f and f^{-1} preserve the structure of bipolar valued hesitant fuzzy generalized semi preopen and semi preclosed sets, the function satisfies the properties of a homeomorphism in the bipolar valued hesitant fuzzy topology.
Remark 3.5
Bipolar valued hesitant fuzzy generalized semi prehomeomorphisms provide a foundation for structural equivalence between uncertain spaces, useful in fuzzy pattern recognition, topology based AI models, and knowledge representation.

E. Bipolar Valued Hesitant Fuzzy Generalized Semi Preirresolute Functions

Definition 3.6
A function f: X → Y between bipolar valued hesitant fuzzy topological spaces is said to be bipolar valued hesitant fuzzy generalized semi preirresolute if for every bipolar valued hesitant fuzzy generalized semi preopen set G in Y, the inverse image $f^{-1}(G)$ is a bipolar valued hesitant fuzzy semi preopen set in X.
Theorem 3.6
If f: X → Y is a bipolar valued hesitant fuzzy generalized semi preirresolute function, then the composition of f with any bipolar valued hesitant fuzzy generalized semi precontinuous function is also bipolar valued hesitant fuzzy generalized semi preirresolute.

Proof
Let g: Y → Z be a bipolar valued hesitant fuzzy generalized semi precontinuous function and let f: X → Y be a bipolar valued hesitant fuzzy generalized semi preirresolute function. By assumption, for every bipolar valued hesitant fuzzy generalized semi preopen set G in Z, we have:
$g^{-1}(G)$ is a generalized semi preopen set in Y.
Since f is generalized semi preirresolute, the inverse image:
$f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$
is a bipolar valued hesitant fuzzy generalized semi preopen set in X, proving the theorem.
Example 3.6
Consider X = {a, b, c}, Y = $\{x_1, x_2, x_3\}$, and Z = $\{y_1, y_2, y_3\}$, with bipolar valued hesitant fuzzy topologies. Let:
$f(a) = x_1$, $f(b) = x_2$, $f(c) = x_3$.
$g(x_1) = y_1$, $g(x_2) = y_2$, $g(x_3) = y_3$.
If both f and g satisfy the respective conditions of semi preirresolute and semi precontinuous functions, then g ∘ f is also a bipolar valued hesitant fuzzy generalized semi preirresolute function.
Remark 3.6
Bipolar valued hesitant fuzzy generalized semi preirresolute functions provide a broader framework for uncertainty based function analysis, with applications in machine learning, medical diagnosis, and optimization problems.

F. Bipolar Valued Hesitant Fuzzy Generalized Semi Precompact Spaces
Definition 3.7
A bipolar valued hesitant fuzzy topological space (X, τ) is called bipolar valued hesitant fuzzy generalized semi precompact if every bipolar valued hesitant fuzzy generalized semi preopen cover of X has a finite subcover.
Theorem 3.7
Every bipolar valued hesitant fuzzy generalized semi precompact space is also a bipolar valued hesitant fuzzy generalized semi prebounded space.
Proof
Let X be a bipolar valued hesitant fuzzy generalized semi precompact space, meaning every generalized semi preopen cover has a finite subcover. Since semi preboundedness requires that every generalized semi preopen cover is bounded, and a finite subcover ensures boundedness, it follows that X is generalized semi prebounded.
Example 3.7
Let X = {a, b, c, d} with a bipolar valued hesitant fuzzy topology. Suppose the set:
τ = { ∅, X, {a, b}, {b, c, d}, {c, d} }
forms a generalized semi precompact space, since every generalized semi preopen cover has a finite subcover. This confirms that X is also generalized semi prebounded.
Remark 3.7
Bipolar valued hesitant fuzzy generalized semi precompact spaces are useful in network topology, resource allocation problems, and multi agent systems, where constrained optimization plays a key role.

G. Bipolar Valued Hesitant Fuzzy Generalized Semi Preregular Spaces
Definition 3.8
A bipolar valued hesitant fuzzy topological space (X, τ) is called bipolar valued hesitant fuzzy generalized semi preregular if for every two distinct points x, y ∈ X, there exist bipolar valued hesitant fuzzy generalized semi preopen sets U and V such that:
$x \in U$, $y \notin Cl(U)$ and $y \in V$, $x \notin Cl(V)$.
This ensures a form of separation between fuzzy points using semi preopen sets.
Theorem 3.8
Every bipolar valued hesitant fuzzy generalized semi preregular space is a bipolar valued hesitant fuzzy generalized semi preT1 space.

Proof
Let X be a bipolar valued hesitant fuzzy generalized semi preregular space. By definition, for every pair of distinct points x, y ∈ X, there exist semi preopen sets U and V that separate them such that y ∉ Cl(U) and x ∉ Cl(V). Since every semi preT1 space requires that each singleton set be semi preclosed, it follows that generalized semi preregularity implies semi preT1 property, proving the theorem.
Example 3.8
Let X = {a, b, c, d} be a bipolar valued hesitant fuzzy topological space with the topology:
τ = { ∅, X, {a, b}, {b, c}, {c, d} }.
Since for any two points, we can find semi preopen sets that separate them, this space satisfies the conditions of a bipolar valued hesitant fuzzy generalized semi preregular space.
Remark 3.8
Bipolar valued hesitant fuzzy generalized semi preregular spaces provide a stronger notion of separation in fuzzy topologies, useful in image processing, machine learning, and soft computing applications.

H. Bipolar Valued Hesitant Fuzzy Generalized Semi Preconnected Spaces
Definition 3.9
A bipolar valued hesitant fuzzy topological space (X, τ) is said to be bipolar valued hesitant fuzzy generalized semi preconnected if it cannot be expressed as the union of two disjoint bipolar valued hesitant fuzzy generalized semi preopen sets.
Theorem 3.9
If a bipolar valued hesitant fuzzy generalized semi preconnected space contains a bipolar valued hesitant fuzzy generalized semi precontinuous function, then its image is also semi preconnected.
Proof
Let f: X → Y be a bipolar valued hesitant fuzzy generalized semi precontinuous function, and suppose X is semi preconnected. Assume for contradiction that f(X) is disconnected, i.e., there exist disjoint bipolar valued hesitant fuzzy generalized semi preopen sets A, B ⊆ Y such that:
f(X) = A ∪ B.
Since $f^{-1}(A)$ and $f^{-1}(B)$ are semi preopen in X, and their union is X, this contradicts the assumption that X is semi preconnected. Hence, f(X) must be semi preconnected.
Example 3.9
Consider X = {a, b, c} with a bipolar valued hesitant fuzzy topology where:
τ = { ∅, X, {a, b}, {b, c} }.
If X is semi preconnected and f: X → Y is a bipolar valued hesitant fuzzy generalized semi precontinuous function, then f(X) is semi preconnected.
Remark 3.9
Bipolar valued hesitant fuzzy generalized semi preconnected spaces assist in decision theory, fuzzy clustering, and neural network connectivity analysis, where connectivity preservation under vagueness is crucial.
Applications

IV. APPLICATIONS OF BIPOLAR VALUED HESITANT FUZZY TOPOLOGY
This part discusses the applications of bipolar valued hesitant fuzzy topology in real life in various fields and thus proves its capacity to solve real world problems.

A. Application in Decision Making Systems
Bipolar valued hesitant fuzzy topologies are a scientific way of applying multi criteria decision making (MCDM) in problems with uncertainty and hesitation in the expert's judgment.
Example 4.1
Suppose a decision model in which a company assesses the performance of a supplier across various attributes, including cost, quality, and lead time. Each supplier is rated with:
$S_1 = (0.8, 0.9;\ 0.2,\ 0.1)$, $S_2 = (0.6, 0.7;\ 0.4,\ 0.3)$.

By introducing topological structures, it is possible to categorize the suppliers into different preference levels within uncertainty management.

Remark 4.1

This approach enhances the accuracy of decision making by integrating positive and negative evaluations with uncertainty, and it can be applied in finance, medicine, and risk management.

B. Application in Medical Diagnosis

Medical diagnosis generally involves uncertainty and doubt when determining symptoms and test results. Bipolar valued hesitant fuzzy topologies may help increase the accuracy of diagnosis.

Example 4.2

Consider a medical expert system where the patient's test results are of different degrees of positivity and negativity.

$D_1 = (0.7, 0.8;\ 0.2,\ 0.1)$, $D_2 = (0.5, 0.6;\ 0.3,\ 0.4)$.

Thanks to bipolar valued hesitant fuzzy topology, patients may be classified into various risk groups considering hesitant judgments among physicians.

Remark 4.2

This technique allows for more flexible and standardized medical evaluations, especially in applications like cancer diagnosis and risk evaluation for chronic diseases.

C. Application in Image Processing

Bipolar valued hesitant fuzzy topology can be applied to enhance image classification and segmentation by solving the uncertainty of pixel classification effectively.

Example 4.3

Take the case of an image processing system where pixels are assigned to different classes, for example, foreground and background. In a bipolar valued hesitant fuzzy set, a pixel can be assigned to different classes to different degrees:

By utilizing bipolar valued hesitant fuzzy topological operations, more accurate image segmentation can be performed.

Remark 4.3

It enhances edge detection, noise filtering, and object identification, thus applicable in biometric identification, satellite imaging, and medical imaging.

D. Application in Artificial Intelligence and Machine Learning

AI models tend to need uncertainty management in decision making. Bipolar valued hesitant fuzzy topology strengthens AI based applications by integrating hesitant and bipolar information management.

Example 4.4

Consider an artificial intelligence driven chatbot system that assesses user sentiment through the analysis of linguistic expressions. The sentiment classification is conducted utilizing a bipolar valued hesitant fuzzy approach, which encompasses:

Through the application of topological continuity principles, the chatbot can handle imprecise answers effectively.

Remark 4.4

Bipolar valued hesitant fuzzy artificial intelligence systems improve sentiment analysis, recommendation systems, and predictive modeling, making them especially applicable to social media monitoring, customer analysis, and intelligent automation.

E. Application in Robotics

Robotic systems need to make sensible decisions in uncertain situations. A bipolar valued hesitant fuzzy topology provides a navigation framework, motion planning framework, and object perception framework for robotic systems.

Example 4.5

Assuming an autonomous mobile robot is navigating in an uncertain world with obstructions, using a bipolar valued hesitant fuzzy decision model, the robot evaluates potential paths using:

By employing topological reasoning, the robot chooses the most efficient and safest route.

Remark 4.5

This technique enhances robot vision, adaptive control, and autonomous navigation and is hence used in industrial automation, autonomous vehicles, and rescue robots.

F. Application in Cybersecurity

Cybersecurity systems are concerned with threat identification and risk assessment where uncertainty and doubt prevail with respect to data classification. Bipolar valued hesitant fuzzy topology is useful for intrusion detection, anomaly detection, and risk analysis.

Example 4.6

Suppose a network security system labels incoming data packets as normal or malicious. Utilizing a bipolar valued hesitant fuzzy method, each packet is evaluated based on: Through the incorporation of fuzzy topological analysis, the system enhances the accuracy of threat identification.

Remark 4.6

This method enhances the security of firewalls, intrusion prevention, and fraud detection and is therefore vital in financial transactions, network security, and cloud computing.

V. CONCLUSION

The research on bipolar valued hesitant fuzzy topology was carried out with a focus on its theoretical basis and practical application. The article provided a comprehensive discussion of generalized semi precontinuous mappings, semi preopen sets, and semi preconnected spaces, thus enhancing the properties typically defined under the fuzzy topology framework. Its practical application with bipolar valued hesitant fuzzy sets greatly improves decision making by effectively handling uncertainty and hesitation. In addition, some basic properties like semi preregular spaces and semi prehomeomorphisms were defined and studied, and an illustration of their role in enhancing the theoretical framework was given. The research has implications in a number of fields such as decision making, medical diagnosis, artificial intelligence, robotics, cybersecurity, and real world transportation systems.

Future research will be on the generalization of bipolar valued hesitant fuzzy topology to hybrid systems and higher dimensions, its application in blockchain, neural networks, and quantum computing, and its generation for optimization algorithms in machine learning and artificial intelligence. In this current work, bipolar valued hesitant fuzzy topology will be a useful tool for handling uncertain and complicated information, thus making a fundamental contribution to engineering, computer science, and mathematics.

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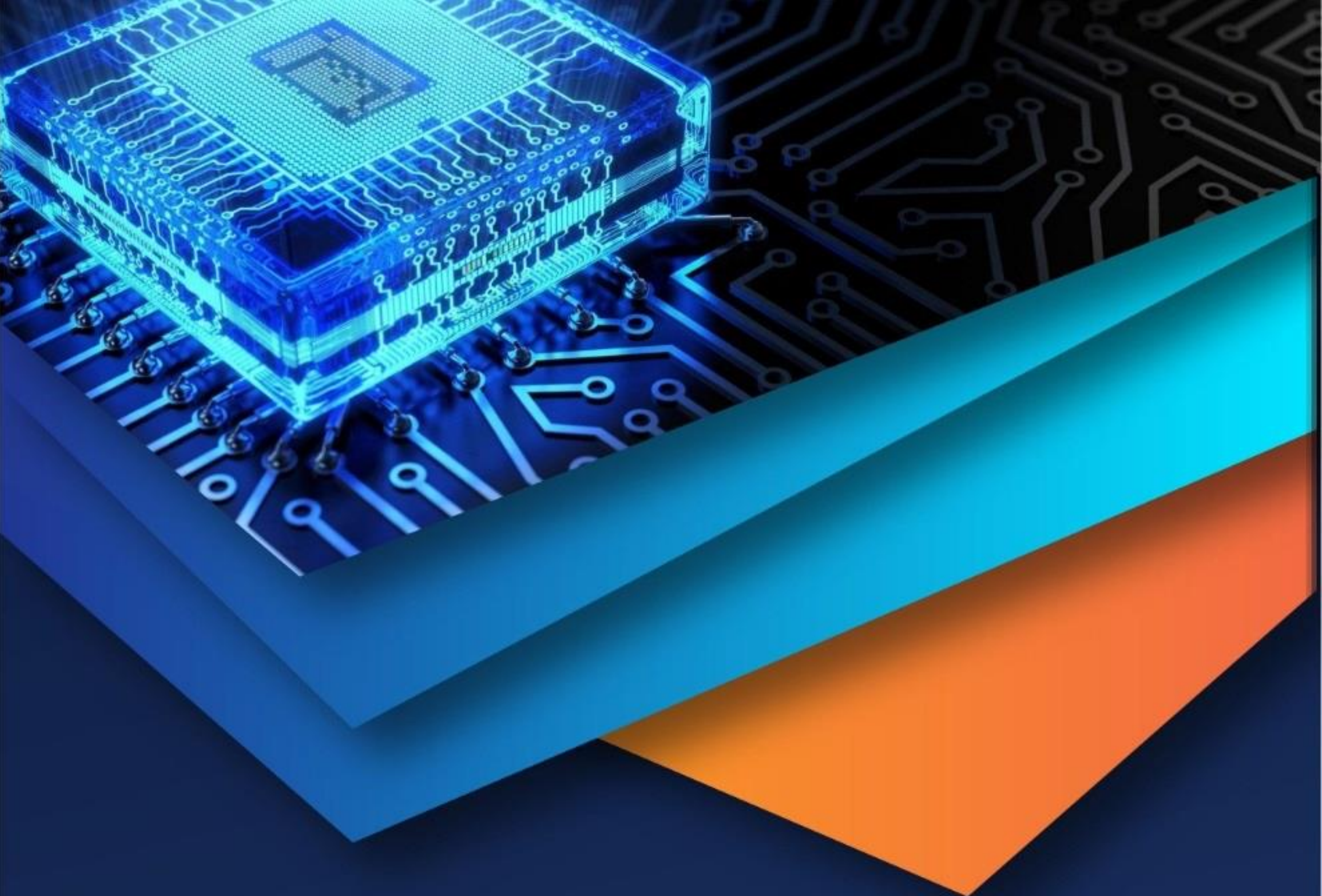

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