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Generalized Investigated in Topological Space

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***Abstract:** In this paper, some characterizations and proportion of notion a investigated. Throughout this paper (X, τ) and (Y, σ) (simply, X and Y) represent topological spaces on which separation axioms are assumed unless otherwise mentioned. We introduce a new class of sets called regular generalized open sets which is properly placed in between the class of open sets and the class of - open sets. Throughout this paper (X, τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl (A) and int (A) denote the closure of A and the interior of A respectively. X/A or Ac denotes the complement of A in X. introduced and investigated semi open sets, generalized closed sets, regular semi open sets, weakly closed sets, semi generalized closed sets , weakly generalized closed sets, strongly generalized closed sets, generalized pre - regular closed sets, regular generalized closed sets, and generalized α-generalized closed sets respectively.*

***Keywords:** Topological space, Cluster Point, Open and Closed set, β** - **Continuous, Subset, .Regular open closed set, Separation axioms**

I. INTRODUCTION

A function $f : (X, \tau) \to (Y, \sigma)$ is called super-continuous (resp. a-continuous α-continuous pre-continuous δ - smi - continuous Z - continuous γ - continuous continuous Z^*- continuous , β - continuous e^*- continuous) if $f^{-1}(V)$ is δ - open (resp. a-open, α-open, per open, δ-semiopen, Z-open, γ-open, e- open, Z^* - open, β-open, e^*-open) in X, for each $V \in \sigma$. the notion of β-open sets and β-continuity in topological space. The concepts of Z^*- open set and Z *- continuity introduced by Mubarki. The purpose of this paper introduce and study the notions of β^*- open sets, β^* - continuous functions and (β^*- open sets. For a subset A of a (X, τ), $cl(A)$, $int(A)$ and $X \setminus A$ denote the closure of A, the interior of A and the complement of A, respectively. A subset A of a topological space (X, τ) is called regular open (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). A point x of X is called δ - cluster point of A if $int(cl(U)) \cap A = \phi$, for every open set U of X containing x. The set of all δ-cluster points of A is called δ-closure of A and is denoted $cl\ \delta(A)$. A set A is δ-closed if and if $A = cl\ \delta(A)$. The complement of a a-open (resp. α-open, δ-semiopen, δ-preopen, Z-open, γ-open, e-open, Z^*-open, β-open, e^*-open) sets is called a-close (resp. α-closed δ-semi-closed , δ-pre-closed Z-closed γ-closed , e - closed Z^*-closed β-closed , e^*- closed The intersection of all δ - preclosed (resp. β-closed) set containing A is called the δ - preclosure (resp. β-closure) of A and is denoted by δ - pcl (A) (resp. β-$cl(A)$). The union of all δ-preopen (resp. β-open) sets contained in A is called the δ- pre - interior (resp. β - interior) of A and is denoted by δ-$pint(A)$ (resp. β-$int(A)$). The family of all δ-open (resp. δ-semiopen, δ- preopen, Z^*- open, β-open, e^*- open) sets is denoted by $\delta O\ (X)$ (resp. $\delta SO\ (X)$, $\delta PO\ (X)$, $Z^*\ O(X)$, $\beta O\ (X)$, $e^*O(X)$).

Lemma 1.1

Let A be a subset of a space (X, τ). Then:

(1) δ -$pint(A) = A \cap int(cl\delta(A))$ and δ-$pcl(A) = A \cup cl\ (int\ \delta(A))$,

(2) β – $Int(A) = A \cap cl\ (int(cl(A)))$ and $\beta – cl\ (A) = A \cup int\ (cl(int\ (A)))$.

β* - ***Open sets***

Definition 2.1

A subset A of a topological space (X, τ) is said to be:

(1) a β * - open set if $A \subseteq cl(int(cl(A))) \cup int(cl\delta(A))$,

(2) a β * - closed set if $in\ t\ (cl(int(A))) \cap cl\ (int\ \delta\ c\ (A)) \subseteq A$.

(3) The family of all β * - open (resp. β^*- closed) subsets of a space (X, τ) will be as always denoted by $\beta^*\ O(X)$ (resp. $\beta^*\ C(X)$).

Remark 3.1
The following diagram holds for each a subset A of X.
None of these implications are reversible as shown in the following examples

Example 4.1
Let $X = \{a, b, c, d\}$, with topology $\tau = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X\}$. Then:

(1) A subset $\{b, c\}$ of X is β^* - open but it is not β-open,
(2) A subset $\{b, d\}$ of X is e^* - open but it is not β^* - open,

Example 4.2
Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Then $\{a, e\}$ is β^*- open but it is not Z^* - open.

Remark 3.3
By the following example we show that the intersection of any two β^*-open sets is not β^* - open.

Example 4.4
Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then $A = \{a, c\}$ and $B = \{b, c\}$ are β^*-open sets. But, $A \cap B = \{c\}$ is not β^*- open
.

Definition 2.2
Let (X, τ) be a topological space. Then:

(1) The union of all β^* - open sets of contained in A is called the β^*-
(2) interior of A and is denoted by β^*-int (A),
(3) The intersection of all β^* - closed sets of X containing A is called
(4) The β^* - closure of A and is denoted by β^* - cl (A).

Theorem 5.1
Let A, B be two subsets of a topological space (X, τ). Then the following are hold:

(1) $\beta^*\text{- } int(X) = X$ and $\beta^*\text{- } int(\phi) = \phi$,
(2) $B^*\text{- } int(A) \subseteq A$,
(3) If $A \subseteq B$, then $\beta^*\text{- } int(A) \subseteq \beta^*\text{- } int(B)$,
(4) $x \in \beta^*\text{- } int(A)$ if and only if there exist β^* - open W such that $x \in W \subseteq A$,
(5) A is β^* - open set if and only if $A = \beta^*\text{- } int(A)$,
(6) $B^*\text{- } int(\beta^*\text{- } int(A)) = \beta^*\text{- } int(A)$,
(7) $B^*\text{- } int(A \cap B) \subseteq \beta^*\text{- } int(A) \cap \beta^*\text{- } int(B)$,
(8) $\beta^*\text{- } int(A) \cup \beta^*\text{-}int(B) \subseteq \beta^*\text{-}int(A \cup B)$.

Example 4.5
Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$.

(1) If $A = \{a, c\}$, $B = \{b, c\}$, then $\beta^*\text{- } cl(A) = A$, $\beta^*\text{- } cl(B) = B$ and β^*-)
(2) $lc\ (A \cup B) = X$. Thus $\beta^*\text{- } cl\ (A \cup B) \not\subset \beta^*\text{- } cl(A) \cup \beta^*\text{- } cl(B)$,
(3) If $A = \{a, c\}$, $C = \{a, b\}$, then $\beta^*\text{- } cl(C) = X$, $\beta^*\text{- } cl\ (A) = A$ and β^*-
$(A \cap C) = \{a\}$. Thus $\beta^*\text{- } cl(A) \cap \beta^*\text{- } cl(C) \not\subset \beta^*\text{- } cl(A \cap C)$,
(4) If $E = \{c, d\}$, $F = \{b, d\}$, then $E \cup F = \{b, c, d\}$ and hence β^*-
$Int\ (E) = \phi$, $\beta^*\text{- } int\ (F) = F$ and $\beta^*\text{- } int\ (E \cup F) = \{b, c, d\}$. Thus β^* -
$Int\ (E \cup F) \not\subset \beta^*\text{- } int(E) \cup \beta^*\text{- } int(F)$.

Theorem 5.2

For a subset A in a topological space (X, τ), the following statements are true:

(1) $B*\text{-}cl(X/A) = X \setminus \beta*\text{-}int(A)$,

(2) $\beta*\text{-}int(X\setminus A) = X/\beta*\text{-}cl(A)$.

Proof.

It follows from the fact the complement of β *- open set is a β * - closed

And $\cap_i (X/A_i) = X/\cup_i A_i$.

Theorem 5.3

Let A be a subset of a topological space (X, τ). Then the following are Equivalent to :

(1) *A is a β * - open set,*

(2) $A = \beta\text{-}int(A) \cup pint\delta(A)$

.Proof

(1) ⇒ (2). Let A be a β*- open set. Then $A \subseteq cl(int(cl(A))) \cup int(cl\delta(A))$ and hence by (Lemma 1.1) $A \subseteq (A \cap cl(int(cl(A)))) \cup (A \cap int(cl\delta(A))) = \beta\text{-}int(A) \cup pint\delta(A) \subseteq A$, (2) ⇒ (1).

Theorem 2.4

For a subset A of space (X, τ).Then the following are equivalent:

(1) *A is a β * - closed set,*

(2) $A = \beta\text{-}cl(A) \cap pcl\ \delta(A)$,

Proof

Theorem 5.5

β* * - Continuous function

Example 4.6

Let X $\{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X\}$. Then, the function $f: (X, \tau) \to (X, \tau)$ defined by $f(a) = a$, $f(b) = f(c) = c$ and $f(d) = d$ is β * - continuous but it is not β * - continuous. The function $f: (X, \tau) \to (X, \tau)$ defined by $f(a) = d$, $f(b) = a$, $f(c) = c$ and $f(d) = b$ is e*- continuous but it is not β*- continuous .

Example 4.7

Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Then function $f: (X, \tau) \to (X, \tau)$ which defined by $f(a) = a$, $f(b) = e$, $f(c) = c$, $f(d) = d$ and $f(e) = b$ is β*- continuous but it is not Z*- continuous.

Theorem 5.5

Let $f: (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are Equivalent :

(1) *f is β * - continuous,*

(2) *For each $x \in X$ and $V \in \sigma$ containing f(x), there exists $U \in \beta * O(X)$*

(3) *containing x such that $f(U) \subseteq V$,*

*The inverse image of each closed set in Y is β * - closed in X,*

(4) $int(cl(int(f-1(B)))) \cap cl(int\,\delta(f-1(B))) \subseteq f-1(cl(B))$, *for each* $B \subseteq Y$,

(5) $f-1(int(B)) \subseteq cl(int(cl(f-1(B)))) \cup int(cl\delta(f-1(B)))$, *for each* $B \subseteq Y$,

(6) β*- *cl (f −1(B))* ⊆ *f − 1(cl(B)), for each B* ⊆ *Y,*

(7) *f (β * - cl (A))* ⊆ *cl(f(A)), for each A* ⊆ *X,*

(8) *f − 1(int(B))* ⊆ *β * - int(f −1(B)), for each B* ⊆ *Y.*

Proof

(1) ⇔ (2) and (1) ⇔ (3) are obvious,

(3) ⇒ (4). Let $B \subseteq Y$. Then by (3) $f{-1}(cl(B))$ is β*-closed. This means $f{-1}(cl(B)) \supseteq int(cl(int(f{-1}(cl(B))))) \cap cl\,(int\,\delta(f{-1}(cl(B)))) \supseteq int(cl(int(f{-1}(B)))) \cap cl\,(int\,\delta(f-1(B)))$,

(4) ⇒ (5). By replacing Y/B instead of B in (4), we have

$Int\,(cl\,(int(f{-1}\,(Y/B)))) \cap cl\,(int\,\delta(f{-1}(Y\backslash B))) \subseteq f{-1}(cl\,(Y/B))$, and therefore $f{-1}(int(B)) \subseteq cl(int(cl(f{-1}(B)))) \cup int\,(cl\delta(f{-1}(B)))$, for each $B \subseteq Y$,

(5) ⇒ (1). Obvious,

(3) ⇒ (6). Let $B \subseteq Y$ and $f{-1}(cl(B))$ be β * - closed in X. Then

B*- $cl\,(f{-1}(B)) \subseteq \beta$*- $cl\,(f{-1}(cl\,(B))) = f{-1}(cl\,(B))$,

(7). Let $A \subseteq X$. Then $f(A) \subseteq Y$. By (6), we have $f - 1\,(cl(f(A))) \supseteq \beta$*- $Cl\,(f{-1}\,(f(A))) \supseteq \beta$* - $cl\,(A)$. Therefore, $cl\,(f(A)) \supseteq f f{-1}\,(cl\,(f(A))) \supseteq f\,(\beta$* - $cl(A))$,

(7) ⇒ (3). Let $F \subseteq Y$ be a closed set. Then, $f{-1}(F) = f{-1}(cl(F))$. Hence by (7), $f\,(\beta$*- $cl\,(f{-1}\,(F))) \subseteq cl\,(f\,(f{-1}(F))) \subseteq (F) = F$, thus, β*- $cl(f{-1}(F)) \subseteq f{-1}(F)$, so, $f{-1}(F) = \beta$*-$cl(f{-1}(F))$. Therefore, $f{-1}(F) \in \beta$*$C(X)$,

$Int\,(f - 1(int\,(B))) \subseteq \beta$*- $int\,(f{-1}(B))$. Therefore, $f{-1}\,(int\,(B)) \subseteq \beta$*- $int\,(f{-1}\,(B))$,

(8) ⇒ (1). Let $U \subseteq Y$ be an open set. Then $f - 1(U) = f - 1(int(U)) \subseteq \beta$*-$int\,(f{-1}(U))$.

Hence, $f - 1(U)$ is β* - open in X. Therefore, f is β* - continuous.

Remarks 3.3

The composition of two β * - continuous functions need not be β* - continuous as show by the following example.

Example 4.7

Let $X = Y = Z = \{a, b, c, d\}$ with topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$

. Let f: $X \to Y$ and g:$Y \to Z$ be functions defined by $f(a) = b$, $f(b) = b$, $f(c) = c$, $f(d) = d$ and $g(a) = a$, $g(b) = c$,$g(c) = a$, $g(d) = d$, respectively. Then f and g are β *- continuous but $g \circ f$ is not β*- continuous.

Theorem 5.6

*If f: (X, τ) → (Y, σ) is a β * - continuous function and A is δ - open in X, then the restriction f \A: (A, τ) → (Y, σ) is β*- continuous.*

Proof.

Let V be an open set of Y. Then by hypothesis $f^{-1}(V)$ is β* - open in X. we have $(f\backslash A)^{-1}(V) = f^{-1}(V) \cap A$

β* ∈ $O\,(A)$. Thus, it follows that f/A is β*- continuous.

Lemma 1.2

Let A and B be two subsets of a space (X, τ). If A ∈ δO(X) and B ∈β O(A), the A ∩ B ∈β*O(X).*

Theorem 5.7

Let (X, τ) → (Y, σ) be a function and {Gi: i ∈I s} be a cover of X by δ-open sets of (X, τ). If F /Gi : (Gi,τ Gi) → (Y, σ) is β- continuous for each i ∈ I, then f is β*- continuous.*

Proof.

Let V be an open set of (Y, σ). Then by $(V) = X \cap (V) = \cup \{Gi \cap f^{-1}(V){:}i \in I\} = \cup \{(f\backslash Gi)^{-1}(V){:}\ i \in I\}$. Since $f \backslash Gi$ is β*- Continuous for each $i \in I$, then $(f\ /\ Gi)^{-1}(V) \in YO\,(Gi)$ for each $i \in I$. we s $(f\backslash Gi)^{-1}(V)$ is β* - continuous in X. Therefore, f is β* - continuous in (X, τ).

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Definition 2.3 The β*- s frontier of a subset *A* of *X*, denoted by β*- *Fr* (*A*), is defined by β*- *Fr* (*A*) = β*- *cl* A) ∩ β*- *Cl* (*X* /*A*) equivalently β* - *Fr* (*A*) = β*- *cl* (*A*) / β*- *int* (*A*)
.

Theorem 5.8

The set of all points x of X at which a function f: (X, τ) → (Y, σ) is not β* - continuous is identical with the union of the β* - frontiers of the inverse images of open sets containing f (x).

Proof. Necessity. Let x be a point of X at which f is not β*- continuous. Then, there is an open set V of Y containing $f(x)$ such that $U \cap (X \backslash f^{-1}(V)) \neq \phi$, for every $U \in \beta^*O(X)$ containing x. Thus, we have $x \in \beta^*\text{-}\ cl\ (X/f^{-1}(V)) = X\ /\ \beta^*\text{-}\ int\ (f^{-1}(V))$ and $x \in f^{-1}(V)$. Therefore, we have $x \in \beta^*\text{-}\ Fr\ (f^{-1}(V))$.Sufficiency. Suppose that $x \in \beta^*\text{-}\ Fr\ (f^{-1}(V))$, for some V is open set containing $f(x)$. Now, we assume that f is β*- continuous at $x \in X$. Then there exists $U \in \beta^*\ O(X)$ containing x such that $f(U) \subseteq V$. Therefore, we have $x \in U \subseteq f^{-1}(V)$ and hence $x \in \beta^*\text{-}\ int\ (f^{-1}(V)) \subseteq X/\ \beta^*\ \text{-}\ Fr(f^{-1}(V))$. This is a contradiction. This means that f is not β* - continuous at $x\ \varepsilon\ V$.

Theorem 5.9

If f: (X, τ) s→ (Y, σ) is a β- continuous injection and (Y, σ) is Ti, then (X, τ) is β*- Ti, where i = 0, 1, 2.*

Proof.

We prove that the theorem for $i = 1$. Let Y be T_1 and x, y be distinct points in X. There exist open subsets U, V in Y such that $f(x) \in U$, $f(y) \notin U$, $f(x) \notin V$ and $f(y) \in V$. Since f is β * - continuous, f^{-1} (U) and f^{-1} (V) are β* - open subsets of X such that $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$, $x \notin f^{-1}(V)$ and $y \in f^{-1}(V)$. Hence, X is β * - T_1.

Theorem 5.10

If f: (X, τ) → (Y, σ) is β - continuous, g:(X, τ) → (Y, σ) is super-continuous and Y is Hausdorff, then the set {x ∈X: f(x) = G (x)} is β * - closed in X. f is β * - continuous in (X, τ).*

Proof :

Let $A = \{x \in X:\ f(x) = g(x)\}$ and $x \notin A$. Then $f(x) \neq g(x)$. Since Y is Hausdorff, there exist open sets U and V of Y such
That $f(x) \in U$, $g(x) \in V$ and $U \cap V = \phi$. Since f is β* - continuous, there exists a β * - open set G containing x such
that $f(G) \subseteq U$. Since g is super - continuous, there exist an δ-open set H of X containing x such that $g(H) \subseteq V$. Now,
put $W = G \cap H$, we have W is a β * - open set containing x and $f(W) \cap g(W) \subseteq U \cap V = \phi$. Therefore, we obtain $W \cap A = \phi$ and hence x ε s β * - *cl* (*A*). This shows that *A* is β * - closed in *X*. Let $f : X \rightarrow Y$ be a function. The subse $\{(x, f(x)) : x \in X\}$ of the product space $X \times Y$ is called the graph of f and is denoted by $G(f)$.

Explain : Definition 2.4 A function $f : X \rightarrow Y$ has a (β^*, τ) - graph if for each $(x, y) \in (X \times Y) / G(f)$, there exist a β *- open *U* of *X* containing *x* and an open set *V* of *Y* containing *y* such that $(U \times V) \cap G(f) = \phi$.
Proof: It follows readily from the above definition.

Theorem 5.11

*If f : X → Y is a β * - continuous function and Y is Hausdorff, then f has a (β*, τ) s - graph.*

Proof.

Let $(x, y) \in X \times Y$ such that $y \neq f(x)$. Then there exist open see that $y \in U$, $f(x) \in V$ and $V \cap U = \phi$. Since f is β * - continuous, there exists β * - open W containing x such that $f(W) \subseteq V$. This I $(W) \cap U \subseteq V \cap U = \phi$.

Definition 2.5

A space *X* is said to be β*- compact if every β * - open cover of *X* has a finite subcover.

Theorem 5.12

If $f: (X, \tau) \to (Y, \sigma)$ has a (β^*, τ) - graph, then $f(K)$ is closed in (Y, σ) for each subset K which is β^* - compact relative to (X, τ).

Proof.

Suppose that $y \varepsilon f(K)$.Then $(x, y) \varepsilon G(f)$ for each $x \in K$. Since $G(f)$ is (β^*, τ) - graph, there exist a β^*- open set U x containing x and an open set V x of Y containing y such that $f(U x) \cap V x = \phi$. The family $\{U s x: x \in K\}$ is a cover of K by β^*- open sets. Since K is β^*-compact relative to (X, τ), there exists a finite subset that $K \subseteq \cup \{U x: x \in K_0\}$. Let $V = \cap \{V x : x \in K_0\}$. have $f(K) \cap V \subseteq (\cup_{x \in K0} f(U_x)) \cap V \subseteq \cup_{x \in K0} (f(u_x)) \cap V = \phi$. It follows that, $y \varepsilon cl(f(K))$. Therefore, $f(K)$ is closed in (Y, σ).

Corollary 6.1

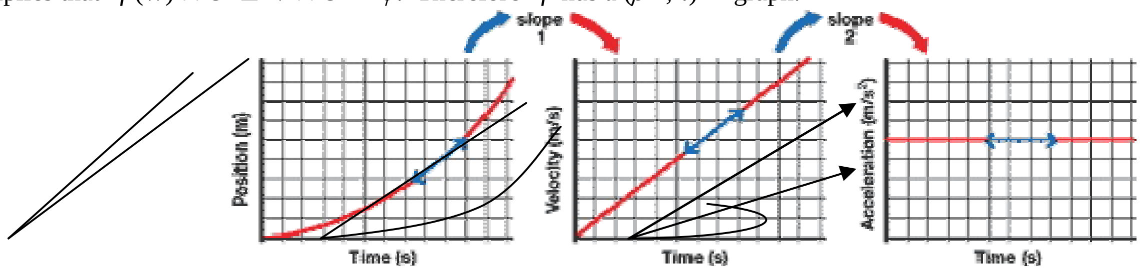
If $f: (X, \tau) \to (Y, \sigma)$ is β^*-continuous function and Y is Hausdorff, then $f(K)$ is closed in (Y, σ) for each subset K which is β^*-compact relative to (X, τ).

Theorem 5.13

If $f: X \to Y$ is a β^* - continuous function and Y is a Hausdorff space, then f has a (β^*, τ) - graph.

Proof

Let $(x, y) \in X \times Y$ such that $y \neq f(x)$ and Y be a Hausdorff space. Then there sexist two open sets U and V such that $y \in U$, $f(x) \in V$ and $V \cap U = \phi$. Since f is β^* - continuous, there exists a β^*-open set W containing x such that $f(W) \subseteq V$. This implies that $f(W) \cap U \subseteq V \cap U = \phi$. Therefore f has a (β^*, τ) - graph.



Corollary 6.2

If $f: X \to Y$ is β^* - continuous and Y is Hausdorff, then $G(f)$ is β^*- closed in $X \times Y$.

Theorem 5.14

If $f: X \to Y$ has a (β^*, τ)-graph and $g: Y \to Z$ is a β^*- continuous function, then the set $\{(x, y): f(x) = g(y)\}$ is β^*- closed in $X \times Y$.

Proof:

Let $A = \{(x, y): f(x) = g(y)\}$ and (x, y). We have $f(x) \neq g(y)$ and then $(x, g(y)) \in (X \times Z) \setminus G(f)$. Since f has a (β^*, τ) - graph, then there exist a β^* - open set U and an set V containing x and $g(y)$, respectively such that $f(U) \cap V = \phi$. Since g is a β^*- continuous function, then there exist an β^*- open set G containing y such that $g(G) \subseteq V$. We have $f(U) \cap g(G) = \phi$. This implies that $(U \times G) \cap A = \phi$. Since $U \times G$ is β^*- open, then (x, y) β^*- $cl(A)$. Therefore, A is β^*- closed in $X \times Y$.

Corollary 6.3

If $f: X \to Y$, $g: Y \to Z$ are β^*-continuous functions and Z is Hausdorff, then the Set $\{(x, y): f(x) = g(y)\}$ is β^* - closed in $X \times Y$.

Theorem 5.15

If $f: X \to Y$ is a β^*- continuous function and Y is Hausdorff, then the set $\{(x, y) \in X \times X: f(x) = f(y)\}$ is β^*- closed in $X \times X$.

Proof:

Let $A = \{(x, y): f(x) = f(y)\}$ and let $(x, y) \in (X \times X)\backslash A$. Then $f(x) \neq f(y)$. Since Y is Hausdorff, then there exist open sets U and V containing $f(x)$ and $f(y)$, respectively, such that $U \cap V = \phi$. But, f is β^*-continuous, then there exist β^* - open sets H and G in X containing x and y, respectively, such that $f(H) \subseteq U$ and $f(G) \subseteq V$. This implies $(H \times G) \cap A = \phi$ we have $H \times G$ is a β^* - open set in $X \times X$ containing (x, y). Hence, A is β^* - closed in $X \times X$.

Theorem 5.16

If f: $(X, \tau) \to (Y, \sigma)$ is β^*- continuous and S is closed in $X \times Y$, then υx $(S \cap G(f))$ is β^*- closed in X, where υxrepresents the projection of $X \times Y$ onto X.

Proof.

Let S be a closed subset of $X \times Y$ and $x \in \beta^*$-cl Let $U \in \tau$ containing x and $V \in \sigma$ containing $f(x)$. Since f is β^*-continuous, $x \in f^{-1}(V)$ $\subseteq$ β^*- $int(f^{-1}(V))$. Then $U \cap \beta^*$- $\mathrm{int}(f^{-1}(V)) \cap \upsilon\, x\ (S \cap G(f))$ contains some point z of X. This implies that $(z, f(z))$ S and $f(z) \in V$. Thus we have $(U \times V) \cap S \neq \phi$ and hence $(x, f(x))$ $cl(S)$. Since A is closed, then $(x, f(x))$ $S \cap G(f)$ and $x \in \upsilon\, x\ (S \cap G(f))$. Therefore $\upsilon\, x\ (S \cap G(f))$ is β^*-closed in (X, τ).

II. CONCLUSION

A topological space (X, τ) is said to be β^*-connected if it is not the union of two nonempty disjoint β^*- open sets. *If (X, τ) is a β^* - connected space and f:$(X, \tau) \to (Y, \sigma)$ has a (β^*, τ)-graph and β^*- continuous function, the constant.* Suppose that f is not constant. There exist disjoint points $x, y \in X$ such that $f(x) = f(y)$. Since $(x, f(x))$ $G(f)$, then $y \neq f(x)$, hence by, there exist open sets U and V containing x and $f(x)$ respectively such that $f(U) \cap V = \phi$. Since f is β * - continuous, there exist a β * - open sets G containing y such that $f(G) \subseteq V$. Since U and V are disjoint β^*- open sets of (X, τ), it follows that (X, τ) is not β^*- connected Therefore, f is constant. *Let $(X1, \tau1)$, $(X2, \tau2)$ and (X, τ) be topological spaces. Define a function f: $(X, \tau) \to (X1 \times X2, \tau1 \times \tau2)$ by $f(x) = (f(x1), f(x2))$. Then fi: $X \to (Xi, \tau i)$, where $(i = 1, 2)$ is β^*-continuous if f is β^*- continuous.*

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