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Generalized Investigated in Topological Space

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Abstract: In this paper, some characterizations and proportion of notion a investigated. Throughout this paper (X, τ) *and* (Y, σ) *(simply,* X *and* Y) *represent topological spaces on which separation axioms are assumed unless otherwise mentioned. We introduce a new class of sets called regular generalized open sets which is properly placed in between the class of open sets and the class of - open sets. Throughout this paper* (X, τ) *represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl (A) and int (A) denote the closure of A and the interior of A respectively. X/A or Ac denotes the complement of A in X. introduced and investigated semi open sets, generalized closed sets, regular semi open sets, weakly closed sets, semi generalized closed sets , weakly generalized closed sets, strongly generalized closed sets, generalized pre - regular closed sets, regular generalized closed sets, and generalized α-generalized closed sets respectively.*

Keywords: Topological space, Cluster Point, Open and Closed set, β* - Continuous, Subset, .Regular open closed set, Separation axioms

I.   INTRODUCTION

A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called super-continuous (resp. a-continuous α-continuous pre-continuous δ - smi - continuous Z - continuous γ - continuous continuous Z^*- continuous , β - continuous e^*- continuous) if

$f^{-1} (V)$ is δ - open (resp. a-open, α-open, per open, δ-semiopen, Z-open, γ-open, e- open, Z^* - open, β-open, e^*-open) in X, for each $V \in \sigma$. the notion of β-open sets and β-continuity in topological space. The concepts of Z^*- open set and Z^*- continuity introduced by   Mubarki. The purpose of this paper introduce and study the notions of β^*- open sets, β^* - continuous functions and (β^*- open sets. For a subset A of a (X, τ), $cl(A)$, $int(A)$ and $X \setminus A$ denote the closure of A, the interior of A and the complement of A, respectively. A subset A of a topological space (X, τ) is called regular open (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). A point x of X is called δ - cluster point of A if $int(cl(U)) \cap A = \phi$, for every open set U of X containing x.

The set of all δ-cluster points of A is called δ-closure of A and is denoted $cl\, \delta(A)$. A set A is δ-closed if and if $A = cl\, \delta(A)$. The complement of a a-open (resp. α-open, δ-semiopen, δ-preopen, Z-open, γ-open, e-open, Z^*-open, β-open, e^*-open) sets is called a-close (resp. α-closed δ-semi-closed , δ-pre-closed Z-closed γ-closed , e - closed Z^*-closed β-closed , e^*- closed The intersection of all δ - preclosed (resp. β-closed) set containing A is called the δ - preclosure (resp. β-closure) of A and is denoted by δ - pcl (A) (resp. β-$cl(A)$). The union of all δ-preopen (resp. β-open) sets contained in A is called the δ- pre - interior (resp. β - interior) of A and is denoted by δ-$pint(A)$ (resp. β-$int(A)$). The family of all δ-open (resp. δ-semiopen, δ- preopen, Z^*- open, β-open, e^*- open) sets is denoted by δO (X) (resp. δSO (X), δPO (X), $Z^* O(X)$, βO (X), $e^* O(X)$).

Lemma 1.1

Let A be a subset of a space (X, τ). Then:

(1) δ -$pint(A) = A \cap int(cl\delta(A))$ and δ-$pcl(A) = A \cup cl$ (int $\delta(A)$),

(2) $\beta - Int(A) = A \cap cl$ (int(cl(A))) and $\beta - cl$ (A) = A $\cup$ int (cl(int (A))).

β* - *Open sets*

Definition 2.1

A subset A of a topological space (X, τ) is said to be:

(1)   a β^* - open set if $A \subseteq cl(int(cl(A))) \cup int(cl\delta(A))$,

(2)   a β^* - closed set if $int (cl(int(A))) \cap cl$ (int δc (A)) $\subseteq$ A.

(3)   The family of all β^* - open (resp. β^*- closed) subsets of a space (X, τ) will be as always denoted by $\beta^* O(X)$ (resp. $\beta^* C(X)$).

Remark 3.1

The following diagram holds for each a subset *A* of *X*.

None of these implications are reversible as shown in the following examples

Example 4.1

Let $X = \{a, b, c, d\}$, with topology $\tau = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\},$
$\{a, c, d\}, X\}$. Then:

    (1)   A subset $\{b, c\}$ of *X* is β^* - open but it is not β-open,

    (2)   A subset $\{b, d\}$ of *X* is e^* - open but it is not β^* - open,

Example 4.2

Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Then $\{a, e\}$ is β^*- open but it is not Z^* - open.

Remark 3.3

By the following example we show that the intersection of any two β^*-open sets is not β^* - open.

Example 4.4

Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then $A = \{a, c\}$ and $B = \{b, c\}$ are β^*-open sets. But, $A \cap B = \{c\}$ is not β^*- open

.

Definition 2.2

Let (X, τ) be a topological space. Then:

    (1)   The union of all β^* - open sets of contained in *A* is called the β^*-

    (2)   interior of A and is denoted by β^*-*int* (*A*),

    (3)   The intersection of all β^* - closed sets of *X* containing *A* is called

    (4)   The β^* - closure of *A* and is denoted by β^* - *cl* (*A*).

Theorem 5.1

Let A, B be two subsets of a topological space (X, τ). Then the following are hold:

    (1) β^*- *int(X) = X and* β^* - *int(φ) = φ,*

    (2) B^* - *int(A)* $\subseteq A$,

    (3)   *If* $A \subseteq B$*, then* β^*- *int(A)* $\subseteq \beta^*$ - *int (B),*

    (4) $x \in \beta^*$- *int(A) if and only if there exist* β^* - *open W such that* $x \in W \subseteq A$,

    (5)   *A is* β^* - *open set if and only if* $A = \beta^*$ - *int(A),*

    (6) B^* - *int(* β^* - *int(A)) =* β^* - *int(A),*

    (7) B^* - *int(A* $\cap$ *B)* $\subseteq \beta^*$ - *int(A)* $\cap \beta^*$ - *int(B),*

    (8) β^*- *int(A)* $\cup \beta^*$-*int(B)* $\subseteq \beta^*$-*int(A* $\cup$ *B).*

Example 4.5

Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$.

    (1)   If $A = \{a, c\}$, $B = \{b, c\}$, then β^*- *cl(A) = A*, β^*- *cl(B) = B* and β^*-)

    (2)   *lc* (*A* $\cup$ *B*) = *X*. Thus β^*- *cl* (*A* $\cup$ *B*) $\not\subseteq \beta^*$ - *cl(A)* $\cup \beta^*$ - *cl(B)*,

    (3)   If $A = \{a, c\}$, $C = \{a, b\}$, then β^* - *cl(C) = X*, β^* - *cl* (*A*) = *A* and β^*-

            (*A∩C*) = $\{a\}$. Thus β^* - *cl(A)* $\cap \beta^*$- *cl(C)* $\not\subseteq \beta^*$- *cl(A* $\cap$ *C),*

    (4)   If $E = \{c, d\}$, $F = \{b, d\}$, then $E \cup F = \{b, c, d\}$ and hence β^*-

            Int (*E*) = ϕ, β^* - *int* (*F*) = *F* and β^* - *int* (*E* $\cup$ *F*) = $\{b, c, d\}$. Thus β^* -

            Int (*E* $\cup$ *F*) $\not\subseteq \beta^*$ - *int(E)* $\cup \beta^*$ - *int(F).*

Theorem 5.2

For a subset A in a topological space (X, τ), the following statements are true:

   (1) $B * - cl(X / A) = X \setminus \beta * - int(A)$,

   (2) $\beta * - int(X \backslash A) = X / \beta * - cl(A)$.

Proof.

It follows from the fact the complement of β *- open set is a β * - closed

And $\cap_i (X / A_i) = X / \cup_i A_i$.

Theorem 5.3

Let A be a subset of a topological space (X, τ). Then the following are

 Equivalent to :

   (1) A *is a* β * - *open set*,

   (2) $A = \beta - int(A) \cup pint\delta (A)$

.Proof

(1) $\Rightarrow$ (2). Let A be a β*- open set. Then $A \subseteq cl \ (int(cl \ (A))) \cup int \ (cl\delta(A))$ and hence by (Lemma 1.1) $A \subseteq (A \cap cl \ (int(cl(A)))) \cup (A \cap int \ (cl\delta(A)))$ = $\beta - int \ (A) \cup pint\delta \ (A) \subseteq A$, (2) $\Rightarrow$ (1).

Theorem 2.4

For a subset A of space (X, τ).Then the following are equivalent:

   (1) A *is a* β * - *closed set*,

   (2) $A = \beta - cl(A) \cap pcl \ \delta(A)$,

Proof

Theorem 5.5

  $\beta*$ * - Continuous function

Example 4.6

Let $X \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X\}$. Then, the function $f: (X, \tau) \rightarrow (X, \tau)$ defined by $f(a) = a$, $f(b) = f(c) = c$ and $f(d) = d$ is $\beta *$ - continuous but it is not $\beta *$ - continuous. The function $f: (X, \tau) \rightarrow (X, \tau)$ defined by $f(a) = d, f(b) = a, f(c) = c$ and $f(d) = b$ is $e*$- continuous but it is not $\beta*$- continuous .

Example 4.7

Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Then function $f : (X, \tau) \rightarrow (X, \tau)$ which defined by $f(a) = a$, $f(b) = e$, $f(c) = c$, $f(d) = d$ and $f(e) = b$ is $\beta*$- continuous but it is not $Z*$- continuous.

Theorem 5.5

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then the following statements are

Equivalent :

   (1) f *is* β * - *continuous*,

   (2)   *For each $x \in X$ and $V \in \sigma$ containing f(x), there exists $U \in \beta * O(X)$*

   (3)   *containing x such that $f(U) \subseteq V$,*

            *The inverse image of each closed set in Y is β * - closed in X,*

   (4)   *$int \ (cl(\ int \ (f-1(B)))) \cap cl \ (int \ \delta \ (f-1(B))) \subseteq f-1(cl(B))$, for each $B \subseteq Y$,*

   (5)   *$f -1(int(B)) \subseteq cl(int(cl(f-1(B)))) \cup int(cl\delta \ (f-1(B)))$, for each $B \subseteq Y$,*

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(6) β*- $cl (f - 1(B)) \subseteq f - 1(cl(B))$, for each $B \subseteq Y$,

(7) $f (\beta * - cl (A)) \subseteq cl(f(A))$, for each $A \subseteq X$,

(8) $f - 1(int(B)) \subseteq \beta * - int(f - 1(B))$, for each $B \subseteq Y$.

Proof

$(1) \Leftrightarrow (2)$ and $(1) \Leftrightarrow (3)$ are obvious,

$(3) \Rightarrow (4)$. Let $B \subseteq Y$. Then by (3) $f - 1(cl(B))$ is β*-closed. This

means $f - 1(cl(B)) \supseteq int(cl(int(f - 1(cl(B))))) \cap cl (int \delta(f - 1(cl(B)))) \supseteq int(cl(int(f - 1(B)))) \cap cl (int \delta(f - 1(B)))$,

$(4) \Rightarrow (5)$. By replacing Y/B instead of B in (4), we have

$Int (cl (int(f - 1 (Y/B)))) \cap cl (int \delta(f - 1(Y\backslash B))) \subseteq f - 1(cl (Y/B))$, and

therefore $f - 1(int(B)) \subseteq cl(int(cl(f - 1(B)))) \cup int (cl\delta(f - 1(B)))$, for each $B \subseteq Y$,

$(5) \Rightarrow (1)$. Obvious,

$(3) \Rightarrow (6)$. Let $B \subseteq Y$ and $f - 1(c l (B))$ be $\beta * - $ closed in X. Then

B*- $c l (f - 1 (B)) \subseteq \beta$*- $cl (f - 1(cl (B))) = f - 1(cl (B))$,

(7). Let $A \subseteq X$. Then $f (A) \subseteq Y$. By (6), we have $f - 1 (cl(f(A))) \supseteq \beta$*-

$Cl (f - 1 (f(A))) \supseteq \beta * - cl (A)$. Therefore, $cl (f(A)) \supseteq f f - 1 (cl (f(A))) \supseteq f (\beta * - cl(A))$,

$(7) \Rightarrow (3)$. Let $F \subseteq Y$ be a closed set. Then, $f - 1(F) = f - 1(cl(F))$. Hence by

$(7), f (\beta$*- $cl (f - 1 (F))) \subseteq cl (f(f - 1(F))) \subseteq (F) = F$, thus, β*- $cl(f - 1(F)) \subseteq f - 1(F)$,

so, $f - 1(F) = \beta$*-$cl(f - 1(F))$. Therefore, $f - 1(F) \in \beta$*$C(X)$,

$Int (f - 1(int (B))) \subseteq \beta$*- $int (f - 1(B))$. Therefore, $f - 1 (int (B)) \subseteq \beta$*- $int (f - 1 (B))$,

$(8) \Rightarrow (1)$. Let $U \subseteq Y$ be an open set. Then $f - 1(U) = f - 1(int(U)) \subseteq \beta$*-$int (f - 1(U))$.

Hence, $f - 1(U)$ is β* - open in X. Therefore, f is β* - continuous.

Remarks 3.3

The composition of two $\beta * - $ continuous functions need not be β* - continuous as show by the following example.

Example 4.7

Let $X = Y = Z = \{a, b, c, d\}$ with topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$

. Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions defined

by $f(a) = b, f(b) = b, f(c) = c, f(d) = d$ and $g(a) = a, g(b) = c, g(c) = a, g(d) = d$,

respectively. Then f and g are β*- continuous but $g \circ f$ is not β*- continuous.

Theorem 5.6

If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a $\beta * - $ continuous function and A is δ - open in X, then the restriction $f \backslash A: (A, \tau) \rightarrow (Y, \sigma)$ is β*-continuous.

Proof.

Let V be an open set of Y. Then by hypothesis $f - 1 (V)$ is β* - open in X. we have $(f \backslash A) - 1 (V) = f - 1(V) \cap A$

β* $\in O (A)$. Thus, it follows that f / A is β*- continuous.

Lemma 1.2

Let A and B be two subsets of a space (X, τ). If $A \in \delta O(X)$ and $B \in \beta$* $O(A)$, the $A \cap B \in \beta$*$O(X)$.

Theorem 5.7

Let $(X, \tau) \rightarrow (Y, \sigma)$ be a function and $\{Gi: i \in I$ s$\}$ be a cover of X by δ-open sets of (X, τ). If $F / Gi : (Gi, \tau Gi) \rightarrow (Y, \sigma)$ is β*-continuous for each $i \in I$, then f is β*- continuous.

Proof.

Let V be an open set of (Y, σ). Then by $(V) = X \cap (V) = \cup \{Gi \cap f - 1(V): i \in I\} = \cup \{(f \backslash Gi) - 1(V): i \in I\}$. Since $f \backslash Gi$ is β*-Continuous for each $i \in I$, then $(f / Gi) - 1 (V) \in YO (Gi)$ for each $i \in I$. we s $(f \backslash Gi) - 1 (V)$ is β* - continuous in X. Therefore, f is β* - continuous in (X, τ).

Definition 2.3 The β^*- s frontier of a subset A of X, denoted by β^*- Fr (A), is defined by β^*- Fr $(A) = \beta^*$- cl $\qquad A) \cap \beta^*$- Cl (X/A) equivalently β^* - Fr $(A) = \beta^*$- cl (A) / β^*- int (A)

.

Theorem 5.8

The set of all points x of X at which a function f: $(X, \tau) \to (Y, \sigma)$ is not β^* - continuous is identical with the union of the β^* - frontiers of the inverse images of open sets containing f (x).

Proof. Necessity. Let x be a point of X at which f is not β^*- continuous. Then, there is an open set V of Y containing $f(x)$ such that $U \cap (X \backslash f^{-1}(V)) \neq \phi$, for every $U \in \beta^* O(X)$ containing x. Thus, we have $x \in \beta^*$- cl $(X/f^{-1}(V)) = X$ / β^*- int $(f^{-1} (V))$ and $x \in f^{-1}(V)$. Therefore, we have $x \in \beta^*$- Fr $(f^{-1} (V))$.Sufficiency. Suppose that $x \in \beta^*$- Fr $(f^{-1}(V))$, for some V is open set containing $f(x)$. Now, we assume that f is β^*- continuous at $x \in X$. Then there exists $U \in \beta^* O(X)$ containing x such that $f(U) \subseteq V$. Therefore, we have $x \in U \subseteq f^{-1}(V)$ and hence $x \in \beta^*$- int $(f^{-1} (V)) \subseteq X/ \beta^*$ - $Fr(f^{-1}(V))$. This is a contradiction. This means that f is not β^* - continuous at $x \varepsilon V$.

Theorem 5.9

If f: (X, τ) s$\to (Y, \sigma)$ is a β^*- continuous injection and (Y, σ) is Ti, then (X, τ) is β^*- Ti, where i = 0, 1, 2.

Proof.

We prove that the theorem for $i = 1$. Let Y be T_1 and x, y be distinct points in X. There exist open subsets U, V in Y such that $f(x) \in U$, $f(y) \notin U, f(x) \notin V$ and $f(y) \in V$. Since f is β^* - continuous, $f^{-1} (U)$ and $f^{-1} (V)$ are β^* - open subsets of X such that $x \in f^{-1}(U)$, $y \notin f^{-1}(U), x \notin f^{-1}(V)$ and $y \in f^{-1}(V)$. Hence, X is β^* - T_1.

Theorem 5.10

If f: $(X, \tau) \to (Y, \sigma)$ is β^* - continuous, g:$(X, \tau) \to (Y, \sigma)$ is super-continuous and Y is Hausdorff, then the set $\{x \in X: f(x) = G (x)\}$ is β^* - closed in X. f is β^* - continuous in (X, τ).

Proof :

Let $A = \{x \in X : f(x) = g(x)\}$ and $x \notin A$. Then $f(x) \neq g(x)$. Since Y is Hausdorff, there exist open sets U and V of Y such That $f(x) \in U$, $g(x) \in V$ and $U \cap V = \phi$. Since f is β^* - continuous, there exists a β^* - open set G containing x such that $f(G) \subseteq U$. Since g is super - continuous, there exist an δ-open set H of X containing x such that $g(H) \subseteq V$. Now, put $W = G \cap H$, we have W is a β^* - open set containing x and $f(W) \cap g(W) \subseteq U \cap V = \phi$. Therefore, we obtain $W \cap A = \phi$ and hence $x \varepsilon s \beta^*$ - cl (A). This shows that A is β^* - closed in X. Let $f : X \to Y$ be a function. The subse $\{(x, f(x)) : x \in X\}$ of the product space $X \times Y$ is called the graph of f and is denoted by $G (f)$.

Explain : Definition 2.4 A function $f : X \to Y$ has a (β^*, τ) - graph if for each $(x, y) \in (X \times Y) / G (f)$, there exist a β^*- open U of X containing x and an open set V of Y containing y such that $(U \times V) \cap G(f) = \phi$.
Proof: It follows readily from the above definition.

Theorem 5.11

If $f : X \to Y$ is a β^* - continuous function and Y is Hausdorff, then f has a (β^*, τ) s - graph.

Proof.

Let $(x, y) \in X \times Y$ such that $y \neq f(x)$. Then there exist open see that $y \in U$, $f(x) \in V$ and $V \cap U = \phi$. Since f is β^* - continuous, there exists β^* - open W containing x such that $f(W) \subseteq V$. This I $(W) \cap U \subseteq V \cap U = \phi$.

Definition 2.5

A space X is said to be β^*- compact if every β^* - open cover of X has a finite subcover.

Theorem 5.12

If f: (X, τ) → (Y, σ) has a (β*, τ) - graph, then f (K) is closed in (Y, σ) for each subset K which is β * - compact relative to   (X, τ).

Proof.

Suppose that $y \varepsilon f(K)$.Then $(x, y) \varepsilon G (f)$ for each $x \in K$. Since $G (f)$ is $(β*, τ)$ - graph, there exist a $β*$- open set U

x containing x and an open set V x of Y containing y such that $f(U\ x) \cap V\ x = \phi$. The family $\{U\ s\ x: x \in K\}$ is a cover of K by $β*$- open sets. Since K is $β*$-compact relative to $(X, τ)$, there exists a finite subset   that $K \subseteq \cup \{U\ x: x \in K_0\}$.   Let $V = \cap \{V\ x$: $x \in K_0\}$. have $f(K) \cap V \subseteq$ $(\cup_{x \in K_0} f(U_x)) \cap V \subseteq \cup_{x \in K_0}(f(u_x)) \cap V = \phi$. It follows that, $y \varepsilon cl(f(K))$. Therefore, $f (K)$ is closed in $(Y, σ)$.
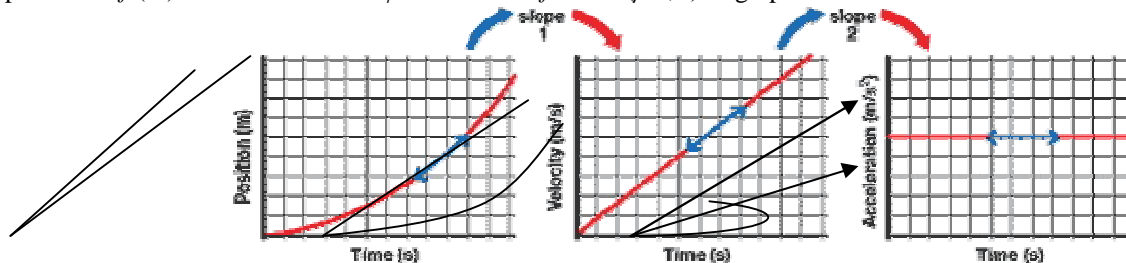
Corollary 6.1

If   f: (X, τ) → (Y, σ) is β*-continuous function and Y is Hausdorff, then f(K) is closed in (Y, σ) for each subset K which is β*- compact relative to (X, τ).

Theorem 5.13

If f: X → Y is a β * - continuous function and Y is a Hausdorff space, then f has a (β *, τ) - graph.

Proof

Let $(x, y) \in X \times Y$ such that $y \neq f(x)$ and Y be a Hausdorff space. Then there sexist two open sets U and V such

that $y \in U, f(x) \in V$ and $V \cap U = \phi$. Since f is $β *$ - continuous, there exists a $β*$-open set W containing x such that $f (W) \subseteq V$. This implies that $f (W) \cap U \subseteq V \cap U = \phi$. Therefore f has $a (β *, τ)$ - graph.

Corollary 6.2

If f: X → Y is β* - continuous and Y is Hausdorff, then G(f) is β*- closed in X × Y.

Theorem 5.14

If f: X → Y has a (β*, τ)-graph and g: Y → Z is a β*- continuous function, then the set {(x, y): f(x) = g(y)} is β*- closed in X × Y.

Proof:

Let $A = \{(x, y): f(x) = g(y)\}$ and (x, y) . We have $f(x) \neq g(y)$ and then $(x, g(y)) \in (X \times Z) \setminus G (f)$. Since f has a $(β*, τ)$ - graph, then there exist a $β*$ - open set U and an set V containing x and $g(y)$, respectively such that $f(U) \cap V = \phi$. Since g is a $β*$- continuous function, then there exist an $β*$- open set G containing y such that $g(G) \subseteq V$.   We have $f(U) \cap g(G) = \phi$. This   implies that $(U \times G) \cap A = \phi$. Since $U \times G$ is $β*$- open, then (x, y) $β*$- $cl (A)$. Therefore, A is $β*$- closed in $X \times Y$.

Corollary 6.3

If f: X → Y, g: Y → Z are β*-continuous functions and Z is Hausdorff, then the Set {(x, y): f(x) = g(y)} is β * - closed in X × Y.

Theorem 5.15

If f: X → Y is a β*- continuous function and Y is Hausdorff, then the set {(x, y) ∈ X × X: f(x) = f(y)} is β*- closed in X × X.

Proof:

Let $A = \{(x, y): f(x) = f(y)\}$ and let $(x, y) \in (X \times X)\backslash A$. Then $f(x) \neq f(y)$. Since Y is Hausdorff, then there exist open sets U and V containing $f(x)$ and $f(y)$, respectively, such that $U \cap V = \phi$. But, f is β^*-continuous, then there exist β^* - open sets H and G in X containing x and y, respectively, such that $f(H) \subseteq U$ and $f(G) \subseteq V$. This implies $(H \times G) \cap A = \phi$ we have $H \times G$ is a β^* - open set in $X \times X$ containing (x, y). Hence, A is β^* - closed in $X \times X$.

Theorem 5.16

If $f: (X, \tau) \to (Y, \sigma)$ is β^*- continuous and S is closed in $X \times Y$, then υx $(S \cap G(f))$ is β^*- closed in X, where υx represents the projection of $X \times Y$ onto X.

Proof.

Let S be a closed subset of $X \times Y$ and $x \in \beta^*$-cl Let $U \in \tau$ containing x and $V \in \sigma$ containing $f(x)$. Since f is β^*-continuous, $x \in f^{-1}(V) \subseteq \beta^*$- $int(f^{-1}(V))$. Then $U \cap \beta^*$- $int(f^{-1}(V)) \cap \upsilon x$ $(S \cap G(f))$ contains some point z of X. This implies that $(z, f(z))$ S and $f(z) \in V$. Thus we have $(U \times V) \cap S \neq \phi$ and hence $(x, f(x))$ $cl(S)$. Since A is closed, then $(x, f(x))$ $S \cap G(f)$ and $x \in \upsilon x$ $(S \cap G(f))$. Therefore υx $(S \cap G(f))$ is β^*-closed in (X, τ).

II. CONCLUSION

A topological space (X, τ) is said to be β^*-connected if it is not the union of two nonempty disjoint β^*- open sets. *If (X, τ) is a β^* - connected space and $f:(X, \tau) \to (Y, \sigma)$ has a (β^*, τ)-graph and β^*- continuous function, the constant.* Suppose that f is not constant. There exist disjoint points $x, y \in X$ such that $f(x) = f(y)$. Since $(x, f(x))$ $G(f)$, then $y \neq f(x)$, hence by, there exist open sets U and V containing x and $f(x)$ respectively such that $f(U) \cap V = \phi$. Since f is β^* - continuous, there exist a β^* - open sets G containing y such that $f(G) \subseteq V$. Since U and V are disjoint β^*- open sets of (X, τ), it follows that (X, τ) is not β^*- connected Therefore, f is constant. *Let $(X1, \tau1), (X2, \tau2)$ and (X, τ) be topological spaces. Define a function $f: (X, \tau) \to (X1 \times X2, \tau1 \times \tau2)$ by $f(x) = (f(x1), f(x2))$. Then $fi: X \to (Xi, \tau i)$, where $(i = 1, 2)$ is β^*-continuous if f is β^*- continuous.*

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