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Gompertz Log-Logistic Distribution: A Case Study of CASP-CUSUM Schemes

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Abstract: The acceptance sampling technique is one of the oldest methods of quality control and relates to inspection and decision-making regarding lots of goods. In this way, many optimal techniques were developed to amplify and manage the quality of the products. Consistent with the supposition that the quality characteristic variable is scattered according to certain probability laws. Based on this assumption, we optimized CASP-CUSUM schemes for continuous variables which used a Truncated Gompertz Log-Logistic distribution used in Statistical Quality Control and Reliability analysis. In particular, the distribution is intended to estimate the optimal truncated point and the probability of acceptance of lots. The operating characteristics and average run length values are shown. The results are illustrated with figures.
Keywords: CASP-CUSUM Schemes, optimal truncated point, Truncated Gompertz Log-Logistic Distribution.

I. INTRODUCTION

Quality has become one of the most important factors for consumers to choose competitive goods and services. Consumers can be individuals, industrial organizations, retail stores, banks, financial institutions or military defense projects, this phenomenon is everywhere. Therefore, implementing and improving features are key drivers of business performance.

Acceptance sampling procedures are used for statistical quality control. It is part of the continuity of operations management and quality of service. A quality management system is fundamental for industrial and commercial purposes. They are very vigilant about the quality of their products, so when consumers come to buy them, they can accept them without any problem.

Acceptance sampling is more likely to be used in situations where the test is destructive, 100% inspection would be extremely expensive, or 100% inspection is not technically feasible or would take so long that the production planning would be grossly artificial.

CUSUM charts are more effective than Shewhart charts for process monitoring because they can more quickly detect small disturbances in the middle, so CUSUM charts are increasingly popular with researchers.

Later on, Vance[12] provides a computer program for evaluating Average Run Length, Hawkins[9] gives consistency simple at the very accurate estimated equation for the evaluation of Average Run Length. Numerous Markov chain approaches have been used for the computation of A.R.L by Ewan and Kemp[7]. The integral equation is encountered in a variety of applications from many fields including continuum mechanics, mathematical economics, queuing theory, potential theory, geophysics, electricity and magnetism, organization, optimal control systems, communication theory, population genetics, medicine and so on.

The integral equation was provided by Page[14] and was used to approximate the ARLs of control chart by assuming a small shift in the mean. A computer program based on the integral equation procedure was given by Vance[12]. Goel and Wu[8] provided a nomogram for the fortitude of chart parameters of a CUSUM control chart. Lashkari and Rahim[11] and Chung[5] reported the economical design of CUSUM control charts.

Sarma and Akhtar[2] studied Continuous acceptance sampling plans based on the truncated negative exponential distribution for Optimizing CASP-CUSUM schemes by solving the integral equation using Gauss-Chebyshev integration method with facilitate of computer program. Lastly, the obtained results were compared at different values of the parameters.

Sainath et.al[15] Considered Continuous acceptance sampling plan Cumulative sum to determine ARL values through truncated two parameters Burr distribution. To determine Optimum CASP-CUSUM values, a computer program is generated to solve the integral equations. By executing the computer program is generated to solve the integral equations. By executing the computer program, thus obtained ARL values for CASP-CUSUM schemes.

Type-C OC curve values and ARL values are compared at different values of the parameters of the underlying probability distribution. They also determined an optimal CASP-CUSUM scheme at which the probability of acceptance is utmost.
Venkatesulu.G and Mohammed Akhtar.P determined Truncated Lomax Distribution[17] and Truncated Gompertz Distribution[16] and its Optimization of CASP-CUSUM Schemes by altering the values of the parameters and in conclusion critical comparisons have based on the obtained numerical fallout.
In the present paper, it is unwavering CASP-CUSUM Chart when the variable under study follows Truncated Gompertz Log-Logistic Distribution. Thus it is more worthwhile to study some fascinating characteristics of this distribution.

A. *Gompertz Log-Logistic Distribution*

Gompertz[13] introduced the Gompertz distribution to describe human transience and establish actuarial tables. It is well-known human lifetime model and has many applications for instance; biology, gerontology, and marketing science. The Log-Logistic distribution (branded as the Fisk distribution in economics) possesses a rather flexible functional form.
The log-logistic distribution is amongst the class of survival time parametric models where the hazard rate initially increases and then decreases and at times can be hump-shaped. The log-logistic distribution can be used as an appropriate substitute for Weibull distribution. It is, actually a mixture of Gompertz distribution with the value of the mean and the variance coincide equal to one. The log-logistic distribution as life testing model has its own standing; it is increasing failure rate (IFR) model and also is viewed as a weighted exponential distribution.
Definition: The non-negative random variable X is said to have Gompertz log-logistic distribution if its P.D.F is given by

$$\mathbf{f(x;\alpha,\beta,\gamma,\theta)} = \frac{\theta\beta\alpha^{\beta}x^{\beta-1}}{(x^{\beta}+\alpha^{\beta})^{2}}\left[1-\frac{1}{1+\left(\frac{x}{\alpha}\right)^{-\beta}}\right]^{-\gamma-1} e^{\frac{\theta}{\gamma}\left\{1-\left[1-\frac{1}{1+\left(\frac{x}{\alpha}\right)^{-\beta}}\right]^{-\gamma}\right\}} \quad \text{Where } \alpha,\beta,\gamma,\theta, x>0 \qquad \ldots\ldots (1.1)$$

B. *Truncated Gompertz Log-Logistic Distribution*

It is the ratio of probability density function of the Gompertz log-logistic distribution to their corresponding cumulative distribution function at the point B.

The random variable X is said to follow Truncated Gompertz log-logistic Distribution as

$$f_B(x) = \frac{\frac{\theta\beta\alpha^{\beta}x^{\beta-1}}{(x^{\beta}+\alpha^{\beta})^{2}}\left[1-\frac{1}{1+\left(\frac{x}{\alpha}\right)^{-\beta}}\right]^{-\gamma-1} e^{\frac{\theta}{\gamma}\left\{1-\left[1-\frac{1}{1+\left(\frac{x}{\alpha}\right)^{-\beta}}\right]^{-\gamma}\right\}}}{\left(1-\left[\frac{1}{1+\left(\frac{B}{\alpha}\right)^{-\beta}}\right]^{-\gamma}\right)} \quad \alpha>0,\ \beta,\ \gamma \text{ and } \theta>0 \qquad \ldots\ldots (1.2)$$

Where '**B**' is the upper truncated point of the Truncated Gompertz log-logistic Distribution.

C. *Applications Of Gompertz Log-Logistic Distribution*

1) The basic log-logistic distribution has either decreasing failure rate, or mixed decreasing-increasing failure rate, depending on the shape parameter.
2) The log-logistic distribution arises as a special case when the GOLL density function for selected parameter values the the corresponding Hazard Rate Functions for some particular values of the parameters.
3) The Gompertz distribution, named for Benjamin Gompertz, is a continuous probability distribution on $[0,\infty)[0,\infty)$ that has exponentially increasing failure rate. Unfortunately, the death rate of adult humans increases exponentially, so the Gompertz distribution is widely used in actuarial science.
4) We will start by giving the reliability function, since most applications of the Gompertz distribution deal with mortality. The basic Gompertz distribution with shape parameter a $\epsilon(0,\infty)$ is a continuous distribution on $[0,\infty)$ with reliability function .The special case a=1 gives the standard Gompertz distribution.

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II. DESCRIPTION OF CASP-CUSUM SCHEMES

Beattie[4] has suggested the method for constructing the continuous acceptance sampling plans. The scheme, suggested by him consists of chosen decision interval namely, "Return interval" with the length h', above the decision line is taken. We plot on the chart the sum $S_m = \sum (X_i - k\ ) X_i ' s (i = 1,2,3 ....)$ is distributed independently and k is the reference value. If the sum lies in the area of the normal chart, the product is accepted and if it lie on the return chart, then the product is rejected, subject to the following assumptions.

1) When the recently plotted a point on the chart touches the decision line, then the subsequent point to be plotted at the highest, i.e., h+h'
2) When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

   When the CUSUM falls in the return chart, a network or a change of specification may be engaged rather than outright rejection.

The procedure, in brief, is given underneath.

a) Start plotting the CUSUM at 0.
b) The product is accepted $S_m = \sum (X_i - k) < h;$ when $S_m < 0$, return cumulative to 0.
c) When h < S_m < h+h' the product is rejected: when S_m crossed h, i.e., when S_m > h+h' and continue rejecting product until S_m > h+h' return cumulative to h+h'

The type-C, OC function, which is defined as the probability of acceptance of an item as a function of incoming quality, when the sampling rate is same in acceptance and rejection regions. Then the probability of acceptance P (A) is given by

$$P(A) = \frac{L(0)}{L(0) + L'(0)} \qquad \text{...... (2.1)}$$

Where L (0) = Average Run Length in acceptance zone and
L' (0) = Average Run Length in rejection zone.
Page E.S.[11] has introduced the formulae for L (0) and L' (0) as

$$L(0) = \frac{N(0)}{1 - P(0)} \qquad \text{...... (2.2)}$$

$$L'(0) = \frac{N'(0)}{1 - P'(0)} \qquad \text{...... (2.3)}$$

Where P (0) =Probability for the test starting from zero on the normal chart,
N (0) = ASN for the test starting from zero on the normal chart,
P' (0) = Probability for the test on the return chart and
N' (0) = ASN for the test on the return chart
He further obtained integral equations for the quantities
P (0), N (0), and P' (0), N' (0) as follows:

$$P(z) = F(k - z) + \int_0^h P(y) f(y + k - z) dy, \qquad \text{...... (2.4)}$$

$$N(z) = 1 + \int_0^h N(y) f(y + k\ - z) dy, \qquad \text{...... (2.5)}$$

$$P'(z) = \int_{k_1 + z}^{B} f(y) dy + \int_0^h P'(y) f(-y + k\ + z) dy \qquad \text{....... (2.6)}$$

$$N'(z)=1+\int_{0}^{h}N'(y)f(-y+k\ \ +z)dy, \qquad \ldots\ldots (2.7)$$

$$F(x)=1+\int_{A}^{h}f(x)dx:$$

$$F(k\ \ -z)=1+\int_{A}^{k_1-z}f(y)dy$$

and z is the distance of the preliminary of the test in the normal chart from zero.

III. COMPUTATION OF ARL's AND P (A)

The equations (2.4), (2.5), (2.6) and (2.7) has to be solved to evaluate L(0) & L'(0), for this we have to integrate each and every equation by using Iterative procedure for the evaluations of P(z), N(z), P'(z) and N'(z). For this integration we use Gauss-Hermite Integration Method(GHIM) to evaluate the above equations and then we estimated the value of Probability of Acceptance and Average Run Length(ARL) of the distribution. We developed computer programs to solve the above equations and we obtain the following results given in the Tables (3.1) to (3.40).

TABLE-3.1

Values of ARL's AND TYPE-C OC CURVES when

α =0.2, β=2, γ= 0.2, θ=1, k=1, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
1.4	52.3709	1.0092535	0.9810931087
1.3	62.7697	1.0092826	0.9841753244
1.2	83.7223	1.0093192	0.9880880713
1.1	146.6136	1.0093660	0.9931625128
1.0	11972.3262	1.0094279	0.9999157190

TABLE-3.2

Values of ARL's AND TYPE-C OC CURVES when

α=0.2, β=2, γ=0.2, θ=1, k=1, h=0.50, h'=0.50

B	L(0)	L'(0)	P(A)
1.4	51.2432	1.0183308	0.9805147052
1.3	61.2590	1.0183872	0.9836475849
1.2	81.2846	1.0184582	0.9876255393
1.1	140.1762	1.0185496	0.9927862287
1.0	2968.0652	1.0186695	0.9996569157

TABLE-3.3

Values of ARL's AND TYPE-C OC CURVES when

α =0 .2, β=2, γ= 0.2, θ=1, k=1, h=0.75, h'=0.75

B	L(0)	L'(0)	P(A)
1.4	49.7407	1.0272194	0.9797663689
1.3	59.2160	1.0273013	0.9829474688
1.2	77.9283	1.0274045	0.9869875908
1.1	131.2258	1.0275370	0.9922305346
1.0	1305.7245	1.0277110	0.9992135167

TABLE-3.4

Values of ARL's AND TYPE-C OC CURVES when

α=0.2, β=2, γ=0.2, θ=1, k=1, h=1.00, h'=1.00

B	L(0)	L'(0)	P(A)
1.4	47.9179	1.0359074	0.9788390994
1.3	56.7316	1.0360128	0.9820658565
1.2	73.8560	1.0361457	0.9861648083
1.1	120.6787	1.0363163	0.9914857149
1.0	726.4708	1.0365402	0.9985752106

TABLE-3.5
Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.2, θ=1.2, k=1, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
1.4	52.7634	1.0048456	0.9813115597
1.3	63.2805	1.0048609	0.9843687415
1.2	84.5117	1.0048802	0.9882493019
1.1	148.5609	1.0049051	0.9932811856
1.0	42478.3086	1.0049376	0.9999763370

TABLE-3.6
Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.2, θ=1.2, k=1, h=0.50, h'=0.50

B	L(0)	L'(0)	P(A)
1.4	52.3227	1.0096443	0.9810687900
1.3	62.7051	1.0096745	0.9841532111
1.2	83.6178	1.0097126	0.9880687594
1.1	146.3341	1.0097616	0.9931468964
1.0	10641.0830	1.0098258	0.9999051094

TABLE-3.7
Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.2, θ=1.2, k=1, h=0.75, h'=0.75

B	L(0)	L'(0)	P(A)
1.4	51.7652	1.0143942	0.9807806015
1.3	61.9612	1.0144387	0.9838916063
1.2	82.4237	1.0144950	0.9878413677
1.1	143.1961	1.0145673	0.9929646850
1.0	4708.6772	1.0146623	0.9997845292

TABLE-3.8
Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.2, θ=1.2, k=1, h=1.00, h'=1.00

B	L(0)	L'(0)	P(A)
1.4	51.0970	1.0190934	0.9804456830
1.3	61.0581	1.0191520	0.9835825562
1.2	80.9495	1.0192257	0.9875656962
1.1	139.2520	1.0193205	0.9927332401
1.0	2635.7000	1.0194451	0.9996133447

TABLE-3.9
Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.4, θ=1.6, k=1, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
1.4	53.0747	1.0001564	0.9815042615
1.3	63.6706	1.0001569	0.9845346212
1.2	85.0763	1.0001575	0.9883806109
1.1	149.7822	1.0001583	0.9933668971
1.0	4193635.5000	1.0001594	0.9999997616

TABLE-3.10
Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.4, θ=1.6, k=1, h=0.50, h'=0.50

B	L(0)	L'(0)	P(A)
1.4	53.0663	1.0003127	0.9814984798
1.3	63.6603	1.0003136	0.9845297933
1.2	85.0624	1.0003150	0.9883769155
1.1	149.7558	1.0003166	0.9933646917
1.0	3354373.5000	1.0003186	0.9999997020

TABLE-3.11

Values of ARL's AND TYPE-C OC CURVES when $\alpha=0.2$, $\beta=2$, $\gamma=0.4$, $\theta=1.6$, k=1, h=0.75, h'=0.75

B	L(0)	L'(0)	P(A)
1.4	53.0576	1.0004690	0.9814926982
1.3	63.6499	1.0004705	0.9845249057
1.2	85.0482	1.0004723	0.9883731604
1.1	149.7294	1.0004748	0.9933624864
1.0	2395599.0000	1.0004780	0.9999995828

TABLE-3.12

Values of ARL's AND TYPE-C OC CURVES when $\alpha=0.2$, $\beta=2$, $\gamma=0.4$, $\theta=1.6$, k=1, h=1.00, h'=1.00

B	L(0)	L'(0)	P(A)
1.4	53.0488	1.0006253	0.9814868569
1.3	63.6391	1.0006273	0.9845199585
1.2	85.0335	1.0006298	0.9883694053
1.1	149.7030	1.0006330	0.9933602810
1.0	1676651.7500	1.0006373	0.9999994040

TABLE-3.13

Values of ARL's AND TYPE-C OC CURVES when $\alpha=0.2$, $\beta=2$, $\gamma=0.2$, $\theta=1$, k=2, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
2.4	319.5867	1.0091335	0.9968523383
2.3	397.9833	1.0091391	0.9974707961
2.2	552.4064	1.0091453	0.9981765151
2.1	994.8134	1.0091524	0.9989866018
2.0	13098.6553	1.0091608	0.9999229908

TABLE-3.14

Values of ARL's AND TYPE-C OC CURVES when $\alpha=0.2$, $\beta=2$, $\gamma=0.2$, $\theta=1$, k=2, h=0.50, h'=0.50

B	L(0)	L'(0)	P(A)
2.4	295.2009	1.0180978	0.9965630174
2.3	361.5660	1.0181085	0.9971920848
2.2	486.0731	1.0181205	0.9979097843
2.1	803.0931	1.0181346	0.9987338185
2.0	3244.1169	1.0181507	0.9996862411

TABLE-3.15

Values of ARL's AND TYPE-C OC CURVES when $\alpha=0.2$, $\beta=2$, $\gamma=0.2$, $\theta=1$, k=2, h=0.75, h'=0.75

B	L(0)	L'(0)	P(A)
2.4	262.7704	1.0268806	0.9961073399
2.3	314.5953	1.0268961	0.9967464209
2.2	405.7886	1.0269139	0.9974757433
2.1	607.7393	1.0269341	0.9983130693
2.0	1427.2377	1.0269575	0.9992809892

TABLE-3.16

Values of ARL's AND TYPE-C OC CURVES when $\alpha=0.2$, $\beta=2$, $\gamma=0.2$, $\theta=1$, k=2, h=1.00, h'=1.00

B	L(0)	L'(0)	P(A)
2.3	266.1873	1.0354906	0.9961249828
2.2	329.3061	1.0355134	0.9968653321
2.1	452.2198	1.0355396	0.9977153540
2.0	794.2744	1.0355698	0.9986978769
1.9	6564.6763	1.0356048	0.9998422861

TABLE-3.17

Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.2, θ=1.2, k=2, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
2.4	326.6801	1.0047822	0.9969336987
2.3	408.6088	1.0047852	0.9975469708
2.2	572.1119	1.0047885	0.9982467890
2.1	1056.7374	1.0047922	0.9990500808
2.0	46768.5586	1.0047966	0.9999785423

TABLE-3.18

Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.2, θ=1.2, k=2, h=0.50, h'=0.50

B	L(0)	L'(0)	P(A)
2.4	318.4915	1.0095193	0.9968402982
2.3	396.3344	1.0095252	0.9974592924
2.2	549.3011	1.0095316	0.9981655478
2.1	985.1000	1.0095391	0.9989762306
2.0	11635.5273	1.0095478	0.9999132156

TABLE-3.19

Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.2, θ=1.2, k=2, h=0.75, h'=0.75

B	L(0)	L'(0)	P(A)
2.4	306.5574	1.0142097	0.9967025518
2.3	378.4157	1.0142182	0.9973269701
2.2	516.3179	1.0142277	0.9980394840
2.1	886.7318	1.0142387	0.9988574982
2.0	5147.7788	1.0142516	0.9998030066

TABLE-3.20

Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.2, θ=1.2, k=2, h=1.00, h'=1.00

B	L(0)	L'(0)	P(A)
2.4	291.6678	1.0188512	0.9965189695
2.3	356.3284	1.0188624	0.9971488118
2.2	476.7844	1.0188750	0.9978675842
2.1	778.3740	1.0188895	0.9986926913
2.0	2880.5583	1.0189062	0.9996464252

TABLE-3.21

Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.4, θ=1.6, k=2, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
2.4	330.4890	1.0001544	0.9969828725
2.3	414.1065	1.0001544	0.9975906014
2.2	581.8461	1.0001545	0.9982840419
2.1	1086.1578	1.0001546	0.9990800023
2.0	***16774619.0000***	***1.0001549***	***0.9999999404***

TABLE-3.22

Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.4, θ=1.6, k=2, h=0.50, h'=0.50

B	L(0)	L'(0)	P(A)
2.4	330.4314	1.0003085	0.9969818592
2.3	414.0323	1.0003088	0.9975898266
2.2	581.7360	1.0003090	0.9982834458
2.1	1085.9194	1.0003092	0.9990797043
2.0	8386011.0000	1.0003095	0.9999998808

TABLE-3.23

Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.4, θ=1.6, k=2, h=0.75, h'=0.75

B	L(0)	L'(0)	P(A)
2.4	330.3674	1.0004628	0.9969807863
2.3	413.9479	1.0004630	0.9975889325
2.2	581.6057	1.0004634	0.9982827902
2.1	1085.6108	1.0004637	0.9990792871
2.0	4192356.0000	1.0004642	0.9999997616

TABLE-3.24

Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.4, θ=1.6, k=2, h=1.00, h'=1.00

B	L(0)	L'(0)	P(A)
2.4	330.2968	1.0006170	0.9969797134
2.3	413.8533	1.0006174	0.9975880384
2.2	581.4553	1.0006179	0.9982820749
2.1	1085.2322	1.0006183	0.9990788102
2.0	2395261.0000	1.0006188	0.9999995828

TABLE-3.25

Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.2, θ=1, k=3, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
3.4	940.0134	1.0091025	0.9989276528
3.3	1176.6113	1.0091044	0.9991431236
3.2	1624.1455	1.0091065	0.9993790984
3.1	2789.5771	1.0091087	0.9996383786
3.0	13319.7480	1.0091113	0.9999242425

TABLE-3.26

Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.2, θ=1, k=3, h=0.50, h'=0.50

B	L(0)	L'(0)	P(A)
3.4	768.9754	1.0180373	0.9986778498
3.3	921.8629	1.0180411	0.9988968968
3.2	1178.5465	1.0180451	0.9991369247
3.1	1697.8279	1.0180496	0.9994007349
3.0	3299.0474	1.0180545	0.9996914864

TABLE-3.27

Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.2, θ=1, k=3, h=0.75, h'=0.75

B	L(0)	L'(0)	P(A)
3.2	807.1396	1.0268040	0.9987294674
3.1	1023.2435	1.0268104	0.9989975095
3.0	1451.3357	1.0268176	0.9992930293
2.9	2699.2588	1.0268254	0.9996197224
2.8	58264.8555	1.0268341	0.9999823570

TABLE-3.28

Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.2, θ=1, k=3, h=1.00, h'=1.00

B	L(0)	L'(0)	P(A)
3.1	654.1954	1.0353801	0.9984198213
3.0	807.7458	1.0353892	0.9987198114
2.9	1090.6100	1.0353993	0.9990515113
2.8	1783.1997	1.0354106	0.9994196892
2.7	6084.0054	1.0354232	0.9998298287

TABLE-3.29

Values of ARL's AND TYPE-C OC CURVES when α =0.2, β=2, γ= 0.4, θ=1.4, k=3, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
3.4	1019.8920	1.0004249	0.9990200400
3.3	1301.3179	1.0004250	0.9992318153
3.2	1865.0002	1.0004251	0.9994638562
3.1	3556.7507	1.0004252	0.9997187853
3.0	5590026.0000	1.0004253	0.9999998212

TABLE-3.30

Values of ARL's AND TYPE-C OC CURVES when α=0.2, β=2, γ=0.4, θ=1.4, k=3, h=0.50, h'=0.50

B	L(0)	L'(0)	P(A)
3.4	1018.9627	1.0008494	0.9990187287
3.3	1299.9575	1.0008495	0.9992306828
3.2	1862.3429	1.0008497	0.9994629025
3.1	3548.4641	1.0008500	0.9997180104
3.0	1396911.7500	1.0008502	0.9999992847

TABLE-3.31

Values of ARL's AND TYPE-C OC CURVES when α =0.2, β=2, γ= 0.4, θ=1.4, k=3, h=0.75, h'=0.75

B	L(0)	L'(0)	P(A)
3.4	1017.6014	1.0012735	0.9990170002
3.3	1297.8945	1.0012738	0.9992291331
3.2	1858.4532	1.0012741	0.9994615316
3.1	3536.4729	1.0012745	0.9997169375
3.0	644453.9375	1.0012748	0.9999984503

TABLE-3.32

Values of ARL's AND TYPE-C OC CURVES when α=0.2, β=2, γ=0.4, θ=1.4, k=3, h=1.00, h'=1.00

B	L(0)	L'(0)	P(A)
4.4	2292.1489	1.0016950	0.9995631576
4.3	2948.7200	1.0016952	0.9996604323
4.2	4255.2661	1.0016953	0.9997646809
4.1	8122.5625	1.0016955	0.9998766780
4.0	364102.5938	1.0016958	0.9999972582

TABLE-3.33

Values of ARL's AND TYPE-C OC CURVES when α =0.2, β=2, γ= 0.2, θ=1, k=4, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
4.4	1955.6954	1.0090899	0.9994843006
4.3	2418.6521	1.0090909	0.9995829463
4.2	3241.0566	1.0090919	0.9996887445
4.1	5105.4556	1.0090928	0.9998024106
4.0	13405.9238	1.0090939	0.9999247193

TABLE-3.34

Values of ARL's AND TYPE-C OC CURVES when α=0.2, β=2, γ=0.2, θ=1, k=4, h=0.50, h'=0.50

B	L(0)	L'(0)	P(A)
4.2	1860.9618	1.0180167	0.9994532466
4.1	2360.5386	1.0180186	0.9995689392
4.0	3318.4446	1.0180207	0.9996933341
3.9	5898.7827	1.0180230	0.9998274446
3.8	36844.2031	1.0180254	0.9999723434

TABLE-3.35

Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.2, θ=1, k=4, h=0.75, h'=0.75

B	L(0)	L'(0)	P(A)
4.0	1459.8569	1.0267686	0.9992971420
3.9	1811.4023	1.0267719	0.9994334579
3.8	2447.8975	1.0267754	0.9995807409
3.7	3949.4473	1.0267793	0.9997400641
3.6	11805.4199	1.0267833	0.9999130368

TABLE-3.36

Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.2, θ=1, k=4, h=1.00, h'=1.00

B	L(0)	L' (0)	P(A)
3.7	1257.0552	1.0353397	0.99917703 87
3.6	1598.3186	1.0353451	0.99935263 40
3.5	2267.1841	1.0353509	0.99954354 76
3.4	4167.1914	1.0353572	0.99975162 74
3.3	49269.0156	1.0353642	0.99997895 96

TABLE-3.37

Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.4, θ=1.4, k=4, h=0.25, h'=0.25

B	L(0)	L'(0)	P(A)
4.4	2308.6584	1.0004243	0.9995668530
4.3	2975.0034	1.0004244	0.9996638298
4.2	4307.7559	1.0004244	0.9997678399
4.1	8306.1387	1.0004245	0.9998795986
4.0	5590030.5000	1.0004245	0.9999998212

TABLE-3.38

Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.4, θ=1.4, k=4, h=0.50, h'=0.50

B	L(0)	L'(0)	P(A)
4.4	2304.8225	1.0008482	0.99956595 90
4.3	2968.9998	1.0008482	0.99966299 53
4.2	4297.0957	1.0008483	0.99976712 47
4.1	8265.7646	1.0008484	0.99987894 30
4.0	1396914.000 0	1.0008485	0.99999928 47

TABLE-3.39

Values of ARL's AND TYPE-C OC CURVES when
α =0.2, β=2, γ= 0.4, θ=1.4, k=4, h=0.75, h'=0.75

B	L(0)	L'(0)	P(A)
4.4	2299.4172	1.0012717	0.9995647669
4.3	2960.3977	1.0012718	0.9996618629
4.2	4278.8174	1.0012720	0.9997660518
4.1	8205.6055	1.0012722	0.9998779893
4.0	644455.5625	1.0012723	0.9999984503

TABLE-3.40

Values of ARL's AND TYPE-C OC CURVES when
α=0.2, β=2, γ=0.4, θ=1.4, k=4, h=1.00, h'=1.00

B	L(0)	L'(0)	P(A)
4.4	2292.1489	1.0016950	0.99956315 76
4.3	2948.7200	1.0016952	0.99966043 23
4.2	4255.2661	1.0016953	0.99976468 09
4.1	8122.5625	1.0016955	0.99987667 80
4.0	364102.5938	1.0016958	0.99999725 82

IV. NUMERICAL RESULTS AND CONCLUSIONS

At the hypothetical values of the parameters α, β, γ, θ, k, h and h' are given at the top of each table, we determine optimum truncated point B at which P (A) the probability of accepting an item is greatest and also obtained ARL's values which represent the acceptance zone L(0) and rejection zone L'(0) values. The values of truncated point B of random variable X, L(0), L'(0) and the values for Type-C Curve, i.e. P (A) are given in columns I, II, III, and IV respectively.

From the above tables 3.1 to 3.40 we made the subsequent conclusions

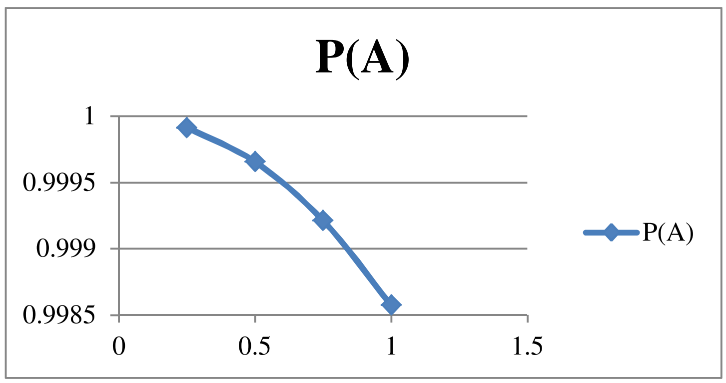
1) From the Table 3.1 to 3.40, it is observed that the values of P (A) are increased as the value of truncated point decreases thus the truncated point of the random variable and the various parameters for CASP-CUSUM are correlated.
2) From the Table 3.1 to 3.40, we observe that it can be maximized the truncated point B by increasing value of ***k***.
3) From Table 3.1 to 3.40, it is observed that at the maximum level of probability of acceptance P (A) the truncated point **'B'** from 5.0 to 1.0 as the value of ***h*** changes from 0.10 to 0.25.
4) From the Table 3.1 to 3.40, it was observed that the truncated point **'B'** changes from 4.0 to 1 .0 and P (A) are as $h \rightarrow 0.25$ maximum **i.e. 0.9999999404.** Thus truncated point B and k are inversely connected.
5) From Table 3.1 to 3.40 it is observed that the optimal truncated point changes from 2.4 to 2.0 as $h \rightarrow 1.00$.
6) From the Table 3.1 to 3.40, it is observed that the values of Maximum Probabilities increased as the increased values of **'θ'** at constant values of the constraint.
7) It is observed that the Table - 4.1 values of Maximum Probabilities increased as the increased values of **'k'** at constant values of the parameter as shown below the Figure - 4.1.

TABLE - 4.1

α=0.2, β=4, γ=0.2,θ=1,h=0.25,h'=0.25

k	P(A)
1	0.999915719
2	0.999922991
3	0.999924243
4	0.999924719

FIGURE - 4.1



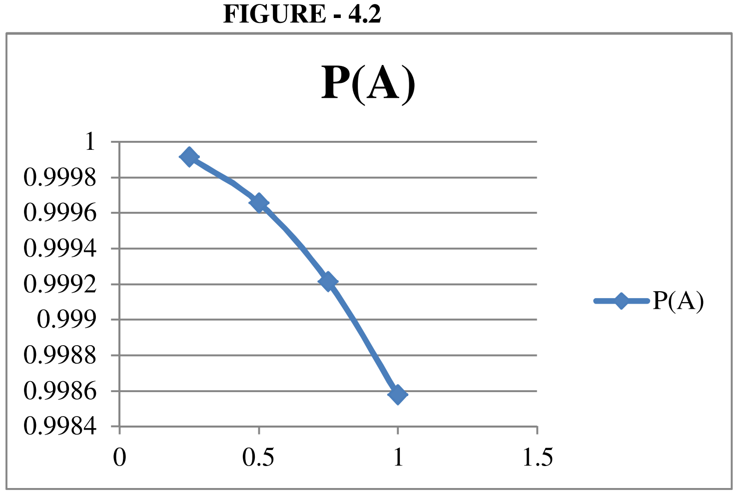
8) It is observed that the Table - 4.2 values of Maximum Probabilities increased as the decreased values of h and h' at constant values of the parameter as exposed below the Figure - 4.2.

TABLE - 4.2

α=0.2, β=4, γ=0.2,θ=1,h=0.25,h'=0.25 , B=1.0

h and h'	P(A)
0.25	0.999915719
0.5	0.999656916
0.75	0.999213517
1	0.998575211

FIGURE - 4.2



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9) The various relations exhibited among the ARL's and Type-C OC Curves with the parameters of the CASP-CUSUM based on the above table 3.1 to 3.40 are observed from the next Table.

TABLE - 4.3
Consolidated Table from the Tables (3.1) to (3.40)

B	α	β	γ	θ	k	h	h'	P(A)
1.0	0.2	2	0.2	1	1	0.25	0.25	0.9999157190
2.0	0.2	2	0.2	1	2	0.25	0.25	0.9999229908
3.0	0.2	2	0.2	1	3	0.25	0.25	0.9999242425
1.0	0.2	2	0.2	1	1	0.50	0.50	0.9996569157
1.0	0.2	2	0.2	1	1	0.75	0.75	0.9992135167
1.0	0.2	2	0.2	1	1	1.00	1.00	0.9985752106
2.0	0.2	2	0.2	1	2	0.50	0.50	0.9996862411
2.0	0.2	2	0.2	1	2	0.75	0.75	0.9992809892
2.0	0.2	2	0.2	1	2	1.00	1.00	0.9998422861
2.0	0.2	2	0.2	1.2	2	0.50	0.50	0.9999132156
3.0	0.2	2	0.2	1	3	0.50	0.5	0.9996914864
2.8	0.2	2	0.2	1	3	0.75	0.75	0.9999823570
2.7	0.2	2	0.2	1	3	1.00	1.00	0.9998298287
2.0	***0.2***	***2***	***0.4***	***1.6***	***2***	***0.25***	***0.25***	***0.9999999404***
3.6	0.2	2	0.2	1	4	0.75	0.75	0.9999130368
4.0	0.2	2	0.4	1.4	4	0.75	0.75	0.9999984503
3.3	0.2	2	0.2	1	4	1.00	1.00	0.9999789596
2.8	0.2	2	0.2	1	3	0.75	0.75	0.9999823570
3.8	0.2	2	0.2	1	4	0.50	0.50	0.9999723434

By observing the Table - 4.3, we can bring to a close that the optimum CASP-CUSUM Schemes evaluated with Truncated Gompertz Log-Logistic distribution, which have the values of ARL and P(A) reach their maximum i.e., *16774619, 0.9999999404* correspondingly, is

$$\begin{bmatrix} B = 2.0 \\ \alpha = 0.2 \\ \beta = 2.0 \\ \gamma = 0.4 \\ \theta = 1.6 \\ k = 2.0 \\ h = 0.25 \\ h' = 0.25 \end{bmatrix}$$

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